LOCAL CALCULATION OF HOFER'S METRIC AND APPLICATIONS TO BEAM PHYSICS

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Abstract. Hofer's metric is a very interesting way of measuring distances between compactly supported Hamiltonian symplectic maps. Unfortunately, it is not known yet how to compute it in general, for example for symplectic maps far away from each other. It is known that Hofer's metric is locally flat, and it can be computed by the so-called oscillation norm of the difference between the Poincare generating functions of symplectic maps close to identity. It is shown here that the same result holds for arbitrary extended generating function types and symplectic maps, as long as the respective generating functions are well defined for the given symplectic maps. This result plays a crucial role is formulating and solving the optimal symplectic approximation problem in Hamiltonian nonlinear dynamics. Applications to beam physics are oulined.

1. Introduction

Often, in Hamiltonian dynamics, it is necessary to study the quality of the approximation of the real dynamics by some approximate numerical or analytical method, since the equation of the motion cannot be solved analytically in closed form. Lately, the method of choice is the symplectic integration method, which is a category of the so-called geometric integrators [10]. For very complicated weakly nonlinear Hamiltonian systems, as for example large particle accelerators, a variant of the symplectic integration methods is utilized, called symplectification of one-turn maps. To this end, an approximate functional relationship is calculated for trajectories of particles one turn around the accelerator, and the long-term behavior of the beam is obtained by iterating this map. The one-turn map's approximation is not exactly symplectic, and the method that transforms this map into a symplectic map is called symplectification [19]. The study of the symplectification in an optimal way lead to the consideration of Hofer's metric [11] for the formulation of the optimal symplectification conditions [6]. However, the same theory may be applied to symplectic integrators in the traditional sense. The