BEREZIN–TOEPLITZ QUANTIZATION OF THE MODULI SPACE OF FLAT $\mathrm{SU}(N)$ CONNECTIONS

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Communicated by Theodore Voronov

Abstract. The moduli space of flat $\mathrm{SU}(n)$ connections on Riemann surfaces is of fundamental importance in TQFT. There is an associated representation of the mapping class group on the space of covariantly constant sections of the Verlinde bundle with respect to the AdPW-H connection. J. Andersen showed that this representation is asymptotically faithful. In his proof the Berezin-Toeplitz quantization of compact Kähler manifolds is used. In this contribution the background and some ideas of Andersen's proof is sketched.

1. Introduction

This is a write-up of a talk presented at the XXVth Białowieża Workshop on Geometric Methods in Physics.

First, we recall the basics of the Berezin-Toeplitz quantization (operator and formal deformation quantization). Then we discuss the moduli space of flat $\mathrm{SU}(n)$ connections on a fixed Riemann surface in its different guises. Finally, we present recent results obtained by Andersen [3] showing the asymptotic faithfulness of the representations of the mapping class group (MCG, Teichmüller group) on the covariantly constant sections of the projectivized Verlinde bundle. In his approach he uses the Toeplitz operators and results on their correct semiclassical behavior as they will be presented in the first part.

As far as the Berezin-Toeplitz quantization is concerned the results were obtained by the author partly in joint works with Meinrenken, and Bordemann respectively with Karabegov [4], [10], [11], [12], [9].

Quite a number of mathematician (and physicists) were involved in the study of the moduli space of connections and the mapping class group. Instead giving references here, let me refer to the recent overviews by Jeffrey [6] and Masbaum [7].

The beautiful results on the asymptotic faithfulness presented are entirely due to Andersen [1]. For similar results in the $U(1)$ case, obtained by him, see [2].