

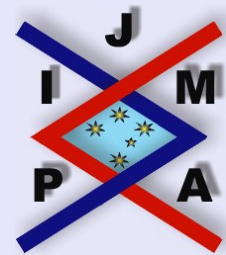
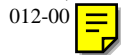
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ON A STRENGTHENED HARDY-HILBERT INEQUALITY

BICHENG YANG

Department of Mathematics
Guangdong Education College
Guangzhou
Guangdong 510303
PEOPLE'S REPUBLIC OF CHINA
EMail: bcyang@163.net

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Abstract

In this paper, a new inequality for the weight coefficient $W(n, r)$ of the form

$$\begin{aligned} W(n, r) &= \sum_{m=0}^{\infty} \frac{1}{m+n+1} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{r}} \\ &< \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{1}{13(n+1)(2n+1)^{1-\frac{1}{r}}} \quad (r > 1, n \in N_0 = N \cup \{0\}) \end{aligned}$$

is proved. This is followed by a strengthened version of the more accurate Hardy-Hilbert inequality.

2000 Mathematics Subject Classification: 26D15

Key words: Hardy-Hilbert inequality, Weight Coefficient, Hölder's inequality.

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1. Introduction

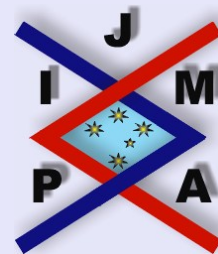
If $p > 1, \frac{1}{p} + \frac{1}{q} = 1, a_n, b_n \geq 0$, and $0 < \sum_{n=1}^{\infty} a_n^p < \infty, 0 < \sum_{n=1}^{\infty} b_n^q < \infty$ ($\lambda = 0, 1$), then the Hardy-Hilbert inequality is

$$(1.1) \quad \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_m b_n}{m+n+\lambda} < \frac{\pi}{\sin\left(\frac{\pi}{p}\right)} \left(\sum_{n=1}^{\infty} a_n^p\right)^{\frac{1}{p}} \left(\sum_{n=1}^{\infty} b_n^q\right)^{\frac{1}{q}},$$

where the constant $\pi/\sin(\pi/p)$ is best possible (see [3, Chapter 9]). Inequality (1.1) is important in analysis and its applications (see [4, Chapter 5]). In recent years, Yang and Gao [2, 7], have given a strengthened version of (1.1) for $\lambda = 0$ as

$$(1.2) \quad \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_m b_n}{m+n} < \left\{ \sum_{n=1}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{1-\gamma}{n^{\frac{1}{p}}} \right] a_n^p \right\}^{\frac{1}{p}} \\ \times \left\{ \sum_{n=1}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{1-\gamma}{n^{\frac{1}{q}}} \right] b_n^q \right\}^{\frac{1}{q}},$$

where $\gamma = 0.5772^+$, is Euler's constant. Later, Yang and Debnath [6] proved a



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distinctly strengthened version of (1.1) for $\lambda = 0$ as

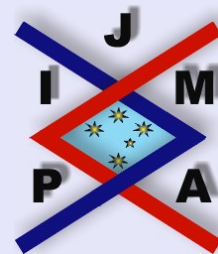
$$(1.3) \quad \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_m b_n}{m+n} < \left\{ \sum_{n=1}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{1}{2n^{\frac{1}{p}} + n^{-\frac{1}{q}}} \right] a_n^p \right\}^{\frac{1}{p}} \\ \times \left\{ \sum_{n=1}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{1}{2n^{\frac{1}{q}} + n^{-\frac{1}{p}}} \right] b_n^q \right\}^{\frac{1}{q}},$$

which is not comparable with (1.2).

Inequality (1.1) for $\lambda = 1$ is called the more accurate Hardy-Hilbert's inequality, which has been strengthened as

$$(1.4) \quad \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{a_m b_n}{m+n+1} < \left\{ \sum_{n=0}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{\ln 2 - \gamma}{(2n+1)^{1+\frac{1}{p}}} \right] a_n^p \right\}^{\frac{1}{p}} \\ \times \left\{ \sum_{n=0}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{\ln 2 - \gamma}{(2n+1)^{1+\frac{1}{q}}} \right] b_n^q \right\}^{\frac{1}{q}},$$

by introducing an inequality of the weight coefficient $W(n, r)$ in the form (see



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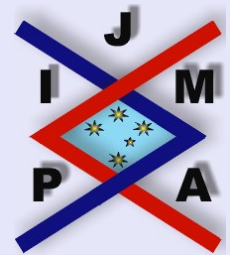
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[5, equation (2.9)]:

$$(1.5) \quad W(n, r) = \sum_{m=0}^{\infty} \frac{1}{m+n+1} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{r}} < \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{\ln 2 - \gamma}{(2n+1)^{2-\frac{1}{r}}} \quad (r > 1, n \in N_0).$$

In this paper we will give another strengthened version of (1.1) for $\lambda = 1$, which is not comparable with (1.4). We need some preparatory works.



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2. Some Lemmas

Lemma 2.1. *If $f \in C^6[0, \infty)$, $f^{(2q)}(x) > 0$ ($q = 0, 1, 2, 3$), $f^{(i-1)}(\infty) = 0$ ($i = 1, 2, \dots, 6$), and $\sum_{n=0}^{\infty} f(n) < \infty$, then we have*

$$(2.1) \quad \sum_{k=2}^{\infty} (-1)^k f(k) > \frac{1}{2} f(2) \quad (\text{see [1, equation (4.4)]}),$$

$$(2.2) \quad \sum_{m=0}^{\infty} f(m) = \int_0^{\infty} f(t) dt + \frac{1}{2} f(0) + \int_0^{\infty} \bar{B}_1 f'(t) dt,$$

$$(2.3) \quad \int_0^{\infty} \bar{B}_1 f'(t) dt = -\frac{1}{12} f'(0) + \delta_2, \quad \delta_2 = \frac{1}{6} \int_0^{\infty} \bar{B}_3 f'''(t) dt < 0,$$

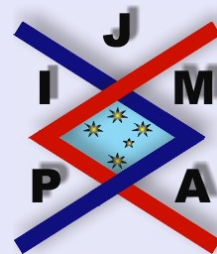
where $\bar{B}_i(t)$ ($i = 1, 3$) are Bernoulli functions (see [5, equations (1.7)-(1.9)]).

Setting the weight coefficient $W(n, r)$ in the form:

$$(2.4) \quad \begin{aligned} W(n, r) &= \sum_{m=0}^{\infty} \frac{1}{m+n+1} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{r}} \\ &= \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{\theta(n, r)}{(2n+1)^{2-\frac{1}{r}}} \quad (r > 1, n \in N_0), \end{aligned}$$

then we find

$$(2.5) \quad \theta(n, r) = \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} (2n+1)^{2-\frac{1}{r}} - (2n+1)^2 \sum_{m=0}^{\infty} \frac{1}{m+n+1} \left(\frac{1}{2m+1} \right)^{\frac{1}{r}}.$$



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If we define the function $f(x)$ as $f(x) = \frac{1}{(x+n+1)} \left(\frac{1}{2x+1}\right)^{\frac{1}{r}}$, $x \in [0, \infty)$, then we have $f(0) = \frac{1}{(n+1)}$,

$$f'(x) = -\frac{1}{(x+n+1)^2} \left(\frac{1}{2x+1}\right)^{\frac{1}{r}} - \frac{2}{r(x+n+1)} \left(\frac{1}{2x+1}\right)^{1+\frac{1}{r}},$$

$$f'(0) = -\frac{1}{(n+1)^2} - \frac{2}{r(n+1)},$$

and

$$\int_0^\infty f(x) dx = \frac{1}{(2n+1)^{\frac{1}{r}}} \int_{\frac{1}{2n+1}}^\infty \frac{1}{(y+1)} \left(\frac{1}{y}\right)^{\frac{1}{r}} dy$$

$$= \frac{1}{(2n+1)^{\frac{1}{r}}} \left[\frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{\left(1 - \frac{1}{r} + \nu\right) (2n+1)^{\nu+1-\frac{1}{r}}} \right].$$

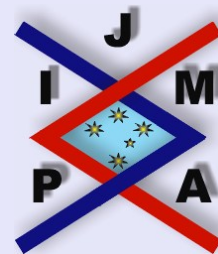
By (2.2) and (2.5), we have

$$(2.6)$$

$$\theta(n, r) = -\frac{(2n+1)^2}{2(n+1)} + \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{\left(1 - \frac{1}{r} + \nu\right) (2n+1)^{\nu-1}}$$

$$+ \int_0^\infty \bar{B}_1(t) \left[\frac{(2n+1)^2}{(t+n+1)^2 (2t+1)^{\frac{1}{r}}} + \frac{2(2n+1)^2}{r(t+n+1)(2t+1)^{1+\frac{1}{r}}} \right] dt.$$

By Lemma 4 of [5, p. 1106] and (2.4), we have



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Lemma 2.2. For $r > 1$, $n \in N_0$, we have $\theta(n, r) > \theta(n, \infty)$, and

$$(2.7) \quad W(n, r) < \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{\theta(n, \infty)}{(2n+1)^{2-\frac{1}{r}}} \quad (r > 1, n \in N_0),$$

where

$$(2.8) \quad \theta(n, \infty) = -\frac{(2n+1)^2}{2(n+1)} + \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{(1+\nu)(2n+1)^{\nu-1}} + \int_0^\infty \bar{B}_1(t) \left[\frac{(2n+1)^2}{(t+n+1)^2} \right] dt.$$

Since by (2.3) and (2.1), we have

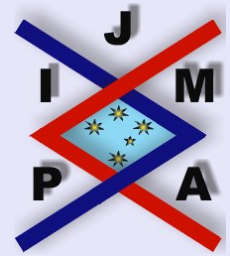
$$\begin{aligned} \int_0^\infty \bar{B}_1(t) \frac{1}{(t+n+1)^2} dt &= -\frac{1}{12(n+1)^2} - \frac{1}{3!} \int_0^\infty \bar{B}_3(t) \left[\frac{1}{(t+n+1)} \right]''' dt \\ &> -\frac{1}{12(n+1)^2} \end{aligned}$$

and

$$\begin{aligned} \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{(1+\nu)(2n+1)^{\nu-1}} &= (2n+1) - \frac{1}{2} + \sum_{\nu=2}^{\infty} \frac{(-1)^\nu}{(1+\nu)(2n+1)^{\nu-1}} \\ &> (2n+1) - \frac{1}{2} + \frac{1}{6(2n+1)}. \end{aligned}$$

Then by (2.8), we find

$$(2.9) \quad \theta(n, \infty) > \frac{1}{6} - \frac{1}{6(n+1)} - \frac{1}{12(n+1)^2} + \frac{1}{6(2n+1)}.$$



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Lemma 2.3. For $r > 1$, $n \in N_0$, we have

$$(2.10) \quad W(n, r) < \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{1}{13(n+1)(2n+1)^{1-\frac{1}{r}}}.$$

Proof. Define the function $g(x)$ as

$$g(x) = \frac{1}{12} - \frac{1}{6(2x+1)} + \frac{1}{12(x+1)} + \frac{1}{12(2x+1)^2}, \quad x \in [0, \infty).$$

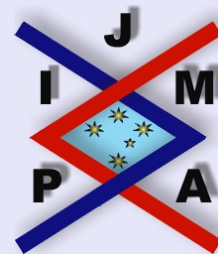
Then by (2.8), we have $\theta(n, \infty) > \frac{2n+1}{n+1}g(n)$. Since $g(1) > 0.0787 > \frac{1}{13}$, and for $x \in [1, \infty)$,

$$g'(x) = \frac{1}{3(2x+1)^2} - \frac{1}{12(x+1)^2} - \frac{1}{3(2x+1)^3} = \frac{4x^2 + 2x - 1}{12(x+1)^2(2x+1)^3} > 0,$$

then for $n \geq 1$, we have $\theta(n, \infty) > \frac{2n+1}{(n+1)}g(1) > \frac{2n+1}{13(n+1)}$. Hence by (2.7), inequality (2.10) is valid for $n \geq 1$. Since $\ln 2 - \gamma = 0.1159^+ > \frac{1}{13}$, then by (1.5), we find

$$(2.11) \quad W(0, r) < \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{\ln 2 - \gamma}{(2 \times 0 + 1)^{2-\frac{1}{r}}} < \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{1}{13(0+1)(2 \times 0 + 1)^{1-\frac{1}{r}}}.$$

It follows that (2.10) is valid for $r > 1$, and $n \in N_0$. This proves the lemma. \square



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3. Theorem and Remarks

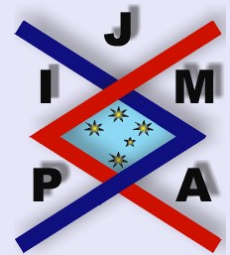
Theorem 3.1. If $p > 1, \frac{1}{p} + \frac{1}{q} = 1, a_n, b_n \geq 0, 0 < \sum_{n=0}^{\infty} a_n^p < \infty,$ and $0 < \sum_{n=0}^{\infty} b_n^q < \infty,$ then

(3.1)

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{a_m b_n}{m+n+1} < \left\{ \sum_{n=0}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{1}{13(n+1)(2n+1)^{\frac{1}{p}}} \right] a_n^p \right\}^{\frac{1}{p}} \\ \times \left\{ \sum_{n=0}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{1}{13(n+1)(2n+1)^{\frac{1}{q}}} \right] b_n^q \right\}^{\frac{1}{q}},$$

$$(3.2) \quad \sum_{m=0}^{\infty} \left(\sum_{n=0}^{\infty} \frac{a_n}{m+n+1} \right)^p \\ < \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} \right]^{p-1} \sum_{n=0}^{\infty} \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} - \frac{1}{13(n+1)(2n+1)^{\frac{1}{p}}} \right] a_n^p.$$

Proof. By Hölder's inequality, we have



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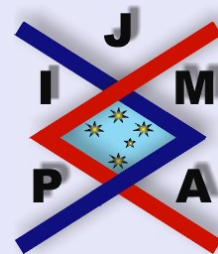
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$$\begin{aligned}
& \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{a_m b_n}{m+n+1} \\
&= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{a_m}{(m+n+1)^{\frac{1}{p}}} \left(\frac{m+\frac{1}{2}}{n+\frac{1}{2}} \right)^{\frac{1}{pq}} \right] \left[\frac{b_n}{(m+n+1)^{\frac{1}{q}}} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{pq}} \right] \\
&\leq \left\{ \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{m+\frac{1}{2}}{n+\frac{1}{2}} \right)^{\frac{1}{q}} \right] a_m^p \right\}^{\frac{1}{p}} \\
&\quad \times \left\{ \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{p}} \right] b_n^q \right\}^{\frac{1}{q}} \\
&= \left\{ \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{m+\frac{1}{2}}{n+\frac{1}{2}} \right)^{\frac{1}{q}} \right] a_m^p \right\}^{\frac{1}{p}} \\
&\quad \times \left\{ \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{p}} \right] b_n^q \right\}^{\frac{1}{q}} \\
&= \left\{ \sum_{m=0}^{\infty} W(m, q) a_m^p \right\}^{\frac{1}{p}} \left\{ \sum_{n=0}^{\infty} W(n, p) b_n^q \right\}^{\frac{1}{q}}.
\end{aligned}$$

Since $\sin(\pi/q) = \sin(\pi/p)$, by (2.10) for $r = p, q$, we have (3.1).

By (2.10), we have $W(n, p) < \pi/\sin(\pi/p)$. Then by Hölder's inequality,



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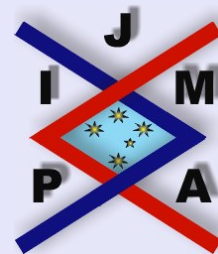
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we obtain

$$\begin{aligned}
 & \sum_{n=0}^{\infty} \frac{a_n}{m+n+1} \\
 &= \sum_{n=0}^{\infty} \left[\frac{a_n}{(m+n+1)^{\frac{1}{p}}} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{pq}} \right] \left[\frac{1}{(m+n+1)^{\frac{1}{q}}} \left(\frac{m+\frac{1}{2}}{n+\frac{1}{2}} \right)^{\frac{1}{pq}} \right] \\
 &\leq \left\{ \sum_{n=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{q}} \right] a_n^p \right\}^{\frac{1}{p}} \left\{ \sum_{n=0}^{\infty} \frac{1}{(m+n+1)} \left(\frac{m+\frac{1}{2}}{n+\frac{1}{2}} \right)^{\frac{1}{p}} \right\}^{\frac{1}{q}} \\
 &= \left\{ \sum_{n=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{q}} \right] a_n^p \right\}^{\frac{1}{p}} \{W(m, p)\}^{\frac{1}{q}} \\
 &< \left\{ \sum_{n=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{q}} \right] a_n^p \right\}^{\frac{1}{p}} \left\{ \frac{\pi}{\sin\left(\frac{\pi}{p}\right)} \right\}^{\frac{1}{q}}.
 \end{aligned}$$

Then we find

$$\begin{aligned}
 & \sum_{m=0}^{\infty} \left(\sum_{n=0}^{\infty} \frac{a_n}{m+n+1} \right)^p \\
 &< \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{q}} \right] a_n^p \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} \right]^{\frac{p}{q}}
 \end{aligned}$$



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$$\begin{aligned}
&= \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} \right]^{p-1} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left[\frac{1}{(m+n+1)} \left(\frac{n+\frac{1}{2}}{m+\frac{1}{2}} \right)^{\frac{1}{q}} \right] a_n^p \\
&= \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} \right]^{p-1} \sum_{n=0}^{\infty} W(n, q) a_n^p.
\end{aligned}$$

Hence by (2.10) for $r = q$, we have (3.2). The theorem is proved. \square

Remark 3.1. Inequality (3.1) is a definite improvement over (1.1) for $\lambda = 1$.

Remark 3.2. Since for $n \geq 2$, $13(2 - \gamma) < 2 - \frac{1}{n+1}$, then

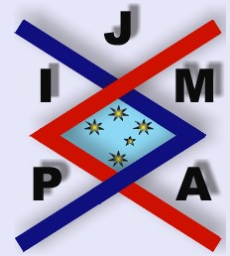
$$(3.3) \quad \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{\ln 2 - \gamma}{(2n+1)^{2-\frac{1}{r}}} > \frac{\pi}{\sin\left(\frac{\pi}{r}\right)} - \frac{1}{13(n+1)(2n+1)^{1-\frac{1}{r}}} \quad (r > 1, n \geq 2).$$

In view of (2.11) and (3.3), it follows that (3.1) and (1.4) represent two distinct versions of strengthened inequalities. However, they are not comparable.

Remark 3.3. Inequality (3.2) reduces to

$$(3.4) \quad \sum_{m=0}^{\infty} \left(\sum_{n=0}^{\infty} \frac{a_n}{m+n+1} \right)^p < \left[\frac{\pi}{\sin\left(\frac{\pi}{p}\right)} \right]^p \sum_{n=0}^{\infty} a_n^p.$$

This is an equivalent form of the more accurate Hardy-Hilbert's inequality (see [3, Chapter 9]).



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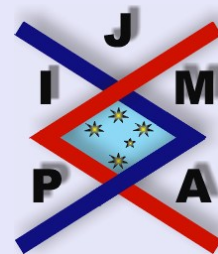
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