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## MEROMORPHIC FUNCTION THAT SHARES ONE SMALL FUNCTION WITH ITS DERIVATIVE

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[Abstract](#)

[Contents](#)



[Home Page](#)

[Go Back](#)

[Close](#)

[Quit](#)

## Abstract

In this paper we study the problem of meromorphic function sharing one small function with its derivative and improve the results of K.-W. Yu and I. Lahiri and answer the open questions posed by K.-W. Yu.

*2000 Mathematics Subject Classification:* 30D35.

*Key words:* Meromorphic function; Shared value; Small function.

## Contents

|   |                                     |    |
|---|-------------------------------------|----|
| 1 | Introduction and Main Results ..... | 3  |
| 2 | Main Lemmas .....                   | 10 |
| 3 | Proof of Theorem 1.2 .....          | 11 |
| 4 | Proof of Theorem 1.3 .....          | 20 |

References



---

### Meromorphic Function That Shares One Small Function With Its Derivative

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 2 of 28

# 1. Introduction and Main Results

By a meromorphic function we shall always mean a function that is meromorphic in the open complex plane  $\mathbb{C}$ . It is assumed that the reader is familiar with the notations of Nevanlinna theory such as  $T(r, f)$ ,  $m(r, f)$ ,  $N(r, f)$ ,  $\overline{N}(r, f)$ ,  $S(r, f)$  and so on, that can be found, for instance, in [2], [5].

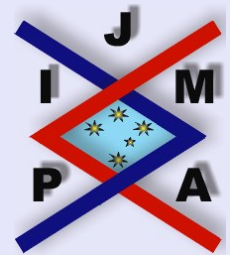
Let  $f$  and  $g$  be two non-constant meromorphic functions,  $a \in \mathbb{C} \cup \{\infty\}$ , we say that  $f$  and  $g$  share the value  $a$  **IM** (ignoring multiplicities) if  $f - a$  and  $g - a$  have the same zeros, they share the value  $a$  **CM** (counting multiplicities) if  $f - a$  and  $g - a$  have the same zeros with the same multiplicities. When  $a = \infty$  the zeros of  $f - a$  means the poles of  $f$  (see [5]).

Let  $l$  be a non-negative integer or infinite. For any  $a \in \mathbb{C} \cup \{\infty\}$ , we denote by  $E_l(a, f)$  the set of all  $a$ -points of  $f$  where an  $a$ -point of multiplicity  $m$  is counted  $m$  times if  $m \leq l$  and  $l + 1$  times if  $m > l$ . If  $E_l(a, f) = E_l(a, g)$ , we say  $f$  and  $g$  share the value  $a$  with weight  $l$  (see [3], [4]).

$f$  and  $g$  share a value  $a$  with weight  $l$  means that  $z_0$  is a zero of  $f - a$  with multiplicity  $m(\leq l)$  if and only if it is a zero of  $g - a$  with the multiplicity  $m(\leq l)$ , and  $z_0$  is a zero of  $f - a$  with multiplicity  $m(> l)$  if and only if it is a zero of  $g - a$  with the multiplicity  $n(> l)$ , where  $m$  is not necessarily equal to  $n$ .

We write  $f$  and  $g$  share  $(a, l)$  to mean that  $f$  and  $g$  share the value  $a$  with weight  $l$ . Clearly, if  $f$  and  $g$  share  $(a, l)$ , then  $f$  and  $g$  share  $(a, p)$  for all integers  $p$ ,  $0 \leq p \leq l$ . Also we note that  $f$  and  $g$  share a value  $a$  **IM** or **CM** if and only if  $f$  and  $g$  share  $(a, 0)$  or  $(a, \infty)$  respectively (see [3], [4]).

A function  $a(z)$  is said to be a small function of  $f$  if  $a(z)$  is a meromorphic function satisfying  $T(r, a) = S(r, f)$ , i.e.  $T(r, a) = o(T(r, f))$  as  $r \rightarrow +\infty$



---

**Meromorphic Function That  
Shares One Small Function  
With Its Derivative**

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 3 of 28

possibly outside a set of finite linear measure. Similarly, we define that  $f$  and  $g$  share a small function  $a$  IM or CM or with weight  $l$  by  $f - a$  and  $g - a$  sharing the value 0 IM or CM or with weight  $l$  respectively.

Brück [1] first considered the uniqueness problems of an entire function sharing one value with its derivative and proved the following result.

**Theorem A.** *Let  $f$  be an entire function which is not constant. If  $f$  and  $f'$  share the value 1 CM and if  $N\left(r, \frac{1}{f'}\right) = S(r, f)$ , then  $\frac{f'-1}{f-1} \equiv c$  for some constant  $c \in \mathbb{C} \setminus \{0\}$ .*

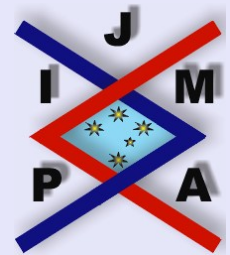
Brück [1] further posed the following conjecture.

**Conjecture 1.1.** *Let  $f$  be an entire function which is not constant,  $\rho_1(f)$  be the first iterated order of  $f$ . If  $\rho_1(f) < +\infty$  and  $\rho_1(f)$  is not a positive integer, and if  $f$  and  $f'$  share one value  $a$  CM, then  $\frac{f'-a}{f-a} \equiv c$  for some constant  $c \in \mathbb{C} \setminus \{0\}$ .*

Yang [7] proved that the conjecture is true if  $f$  is an entire function of finite order. Zhang [9] extended Theorem A to meromorphic functions. Yu [8] recently considered the problem of an entire or meromorphic function sharing one small function with its derivative and proved the following two theorems.

**Theorem B ([8]).** *Let  $f$  be a non-constant entire function and  $a \equiv a(z)$  be a meromorphic function such that  $a \not\equiv 0, \infty$  and  $T(r, a) = o(T(r, f))$  as  $r \rightarrow +\infty$ . If  $f - a$  and  $f^{(k)} - a$  share the value 0 CM and  $\delta(0, f) > \frac{3}{4}$ , then  $f \equiv f^{(k)}$ .*

**Theorem C ([8]).** *Let  $f$  be a non-constant, non-entire meromorphic function and  $a \equiv a(z)$  be a meromorphic function such that  $a \not\equiv 0, \infty$  and  $T(r, a) = o(T(r, f))$  as  $r \rightarrow +\infty$ . If*




---

**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

---

Title Page

Contents

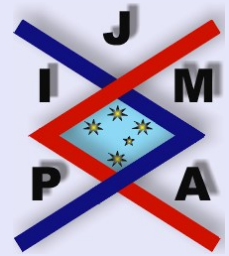


Go Back

Close

Quit

Page 4 of 28




---

**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 5 of 28

(i)  $f$  and  $a$  have no common poles,

(ii)  $f - a$  and  $f^{(k)} - a$  share the value 0 CM,

(iii)  $4\delta(0, f) + 2\Theta(\infty, f) > 19 + 2k$ ,

then  $f \equiv f^{(k)}$ , where  $k$  is a positive integer.

In the same paper Yu [8] further posed the following open questions:

1. Can a CM shared be replaced by an IM shared value?
2. Can the condition  $\delta(0, f) > \frac{3}{4}$  of Theorem B be further relaxed?
3. Can the condition (iii) of Theorem C be further relaxed?
4. Can, in general, the condition (i) of Theorem C be dropped?

Let  $p$  be a positive integer and  $a \in \mathbb{C} \cup \{\infty\}$ . We use  $N_p\left(r, \frac{1}{f}\right)$  to denote the counting function of the zeros of  $f - a$  (counted with proper multiplicities) whose multiplicities are not greater than  $p$ ,  $N_{(p+1)}\left(r, \frac{1}{f}\right)$  to denote the counting function of the zeros of  $f - a$  whose multiplicities are not less than  $p + 1$ . And  $\overline{N}_p\left(r, \frac{1}{f}\right)$  and  $\overline{N}_{(p+1)}\left(r, \frac{1}{f}\right)$  denote their corresponding reduced counting functions (ignoring multiplicities) respectively. We also use  $N_p\left(r, \frac{1}{f}\right)$  to denote the counting function of the zeros of  $f - a$  where a zero of multiplicity  $m$  is counted  $m$  times if  $m \leq p$  and  $p$  times if  $m > p$ . Clearly  $N_1\left(r, \frac{1}{f}\right) = \overline{N}\left(r, \frac{1}{f}\right)$ . Define

$$\delta_p(a, f) = 1 - \limsup_{r \rightarrow +\infty} \frac{N_p\left(r, \frac{1}{f-a}\right)}{T(r, f)}.$$

Obviously  $\delta_p(a, f) \geq \delta(a, f)$ .

Lahiri [4] improved the results of Zhang [9] with weighted shared value and obtained the following two theorems

**Theorem D ([4]).** *Let  $f$  be a non-constant meromorphic function and  $k$  be a positive integer. If  $f$  and  $f^{(k)}$  share  $(1, 2)$  and*

$$2\bar{N}(r, f) + N_2\left(r, \frac{1}{f^{(k)}}\right) + N_2\left(r, \frac{1}{f}\right) < (\lambda + o(1))T(r, f^{(k)})$$

for  $r \in I$ , where  $0 < \lambda < 1$  and  $I$  is a set of infinite linear measure, then  $\frac{f^{(k)}-1}{f-1} \equiv c$  for some constant  $c \in \mathbb{C} \setminus \{0\}$ .

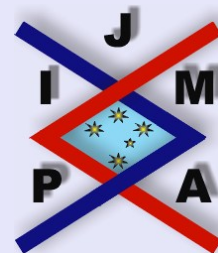
**Theorem E ([4]).** *Let  $f$  be a non-constant meromorphic function and  $k$  be a positive integer. If  $f$  and  $f^{(k)}$  share  $(1, 1)$  and*

$$2\bar{N}(r, f) + N_2\left(r, \frac{1}{f^{(k)}}\right) + 2\bar{N}\left(r, \frac{1}{f}\right) < (\lambda + o(1))T(r, f^{(k)})$$

for  $r \in I$ , where  $0 < \lambda < 1$  and  $I$  is a set of infinite linear measure, then  $\frac{f^{(k)}-1}{f-1} \equiv c$  for some constant  $c \in \mathbb{C} \setminus \{0\}$ .

In the same paper Lahiri [4] also obtained the following result which is an improvement of Theorem C.

**Theorem F ([4]).** *Let  $f$  be a non-constant meromorphic function and  $k$  be a positive integer. Also, let  $a \equiv a(z) (\neq 0, \infty)$  be a meromorphic function such that  $T(r, a) = S(r, f)$ . If*



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents

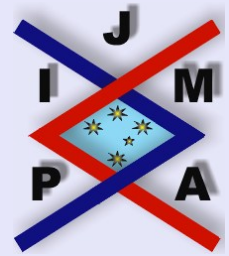


Go Back

Close

Quit

Page 6 of 28



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 7 of 28

(i)  $a$  has no zero (pole) which is also a zero (pole) of  $f$  or  $f^{(k)}$  with the same multiplicity.

(ii)  $f - a$  and  $f^{(k)} - a$  share  $(0, 2)$  CM,

(iii)  $2\delta_{2+k}(0, f) + (4 + k)\Theta(\infty, f) > 5 + k$ ,

then  $f \equiv f^{(k)}$ .

In this paper, we still study the problem of a meromorphic or entire function sharing one small function with its derivative and obtain the following two results which are the improvement and complement of the results of Yu [8] and Lahiri [4] and answer the four open questions of Yu in [8].

**Theorem 1.2.** *Let  $f$  be a non-constant meromorphic function and  $k(\geq 1)$ ,  $l(\geq 0)$  be integers. Also, let  $a \equiv a(z) (\not\equiv 0, \infty)$  be a meromorphic function such that  $T(r, a) = S(r, f)$ . Suppose that  $f - a$  and  $f^{(k)} - a$  share  $(0, l)$ . If  $l \geq 2$  and*

$$(1.1) \quad 2\bar{N}(r, f) + N_2\left(r, \frac{1}{f^{(k)}}\right) + N_2\left(r, \frac{1}{(f/a)'}\right) < (\lambda + o(1))T(r, f^{(k)}),$$

or  $l = 1$  and

$$(1.2) \quad 2\bar{N}(r, f) + N_2\left(r, \frac{1}{f^{(k)}}\right) + 2\bar{N}\left(r, \frac{1}{(f/a)'}\right) < (\lambda + o(1))T(r, f^{(k)}),$$

or  $l = 0$ , i.e.  $f - a$  and  $f^{(k)} - a$  share the value 0 IM and

$$(1.3) \quad 4\bar{N}(r, f) + 3N_2\left(r, \frac{1}{f^{(k)}}\right) + 2\bar{N}\left(r, \frac{1}{(f/a)'}\right) < (\lambda + o(1))T(r, f^{(k)}),$$

for  $r \in I$ , where  $0 < \lambda < 1$  and  $I$  is a set of infinite linear measure, then  $\frac{f^{(k)}-a}{f-a} \equiv c$  for some constant  $c \in \mathbb{C} \setminus \{0\}$ .

**Theorem 1.3.** Let  $f$  be a non-constant meromorphic function and  $k(\geq 1)$ ,  $l(\geq 0)$  be integers. Also, let  $a \equiv a(z) (\not\equiv 0, \infty)$  be a meromorphic function such that  $T(r, a) = S(r, f)$ . Suppose that  $f - a$  and  $f^{(k)} - a$  share  $(0, l)$ . If  $l \geq 2$  and

$$(1.4) \quad (3 + k)\Theta(\infty, f) + 2\delta_{2+k}(0, f) > k + 4,$$

or  $l = 1$  and

$$(1.5) \quad (4 + k)\Theta(\infty, f) + 3\delta_{2+k}(0, f) > k + 6,$$

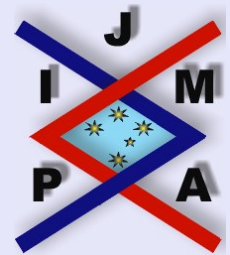
or  $l = 0$ , i.e.  $f - a$  and  $f^{(k)} - a$  share the value 0 IM and

$$(1.6) \quad (6 + 2k)\Theta(\infty, f) + 5\delta_{2+k}(0, f) > 2k + 10,$$

then  $f \equiv f^{(k)}$ .

Clearly Theorem 1.2 extends the results of Lahiri (Theorem D and E) to small functions. Theorem 1.3 gives the improvements of Theorem C and F, which removes the restrictions on the zeros (poles) of  $a(z)$  and  $f(z)$  and relaxes other conditions, which also includes a result of meromorphic function sharing one value or small function IM with its derivative, so it answers the four open questions of Yu [8].

From Theorem 1.2 we have the following corollary which is the improvement of Theorem A.




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**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 8 of 28

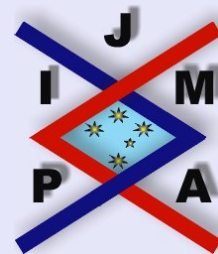


**Corollary 1.4.** *Let  $f$  be an entire function which is not constant. If  $f$  and  $f'$  share the value 1 IM and if  $N\left(r, \frac{1}{f}\right) = S(r, f)$ , then  $\frac{f'-1}{f-1} \equiv c$  for some constant  $c \in \mathbb{C} \setminus \{0\}$ .*

From Theorem 1.3 we have

**Corollary 1.5.** *Let  $f$  be a non-constant entire function and  $a \equiv a(z) (\neq 0, \infty)$  be a meromorphic function such that  $T(r, a) = S(r, f)$ . If  $f - a$  and  $f^{(k)} - a$  share the value 0 CM and  $\delta(0, f) > \frac{1}{2}$ , or if  $f - a$  and  $f^{(k)} - a$  share the value 0 IM and  $\delta(0, f) > \frac{4}{5}$ , then  $f \equiv f^{(k)}$ .*

Clearly Corollary 1.5 is an improvement and complement of Theorem B.




---

**Meromorphic Function That  
Shares One Small Function  
With Its Derivative**

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 9 of 28

## 2. Main Lemmas

**Lemma 2.1** (see [4]). *Let  $f$  be a non-constant meromorphic function,  $k$  be a positive integer, then*

$$N_p \left( r, \frac{1}{f^{(k)}} \right) \leq N_{p+k} \left( r, \frac{1}{f} \right) + k\bar{N}(r, f) + S(r, f).$$

This lemma can be obtained immediately from the proof of Lemma 2.3 in [4] which is the special case  $p = 2$ .

**Lemma 2.2** (see [5]). *Let  $f$  be a non-constant meromorphic function,  $n$  be a positive integer.  $P(f) = a_n f^n + a_{n-1} f^{n-1} + \dots + a_1 f$  where  $a_i$  is a meromorphic function such that  $T(r, a_i) = S(r, f)$  ( $i = 1, 2, \dots, n$ ). Then*

$$T(r, P(f)) = nT(r, f) + S(r, f).$$



---

**Meromorphic Function That  
Shares One Small Function  
With Its Derivative**

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 10 of 28

### 3. Proof of Theorem 1.2

Let  $F = \frac{f}{a}$ ,  $G = \frac{f^{(k)}}{a}$ , then  $F - 1 = \frac{f-a}{a}$ ,  $G - 1 = \frac{f^{(k)}-a}{a}$ . Since  $f - a$  and  $f^{(k)} - a$  share  $(0, l)$ ,  $F$  and  $G$  share  $(1, l)$  except the zeros and poles of  $a(z)$ .

Define

$$(3.1) \quad H = \left( \frac{F''}{F'} - 2 \frac{F'}{F-1} \right) - \left( \frac{G''}{G'} - 2 \frac{G'}{G-1} \right),$$

We have the following two cases to investigate.

**Case 1.**  $H \equiv 0$ . *Integration yields*

$$(3.2) \quad \frac{1}{F-1} \equiv C \frac{1}{G-1} + D,$$

where  $C$  and  $D$  are constants and  $C \neq 0$ . If there exists a pole  $z_0$  of  $f$  with multiplicity  $p$  which is not the pole and zero of  $a(z)$ , then  $z_0$  is the pole of  $F$  with multiplicity  $p$  and the pole of  $G$  with multiplicity  $p + k$ . This contradicts with (3.2). So

$$(3.3) \quad \overline{N}(r, f) \leq \overline{N}(r, a) + \overline{N}\left(r, \frac{1}{a}\right) = S(r, f),$$

$$\overline{N}(r, F) = S(r, f), \quad \overline{N}(r, G) = S(r, f).$$

(3.2) also shows  $F$  and  $G$  share the value 1 CM. Next we prove  $D = 0$ . We first assume that  $D \neq 0$ , then

$$(3.4) \quad \frac{1}{F-1} \equiv \frac{D(G-1 + \frac{C}{D})}{G-1}.$$



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 11 of 28

So

$$(3.5) \quad \bar{N} \left( r, \frac{1}{G - 1 + \frac{C}{D}} \right) = \bar{N}(r, F) = S(r, f).$$

If  $\frac{C}{D} \neq 1$ , by the second fundamental theorem and (3.3), (3.5) and  $S(r, G) = S(r, f)$ , we have

$$\begin{aligned} T(r, G) &\leq \bar{N}(r, G) + \bar{N} \left( r, \frac{1}{G} \right) + \bar{N} \left( r, \frac{1}{G - 1 + \frac{C}{D}} \right) + S(r, G) \\ &\leq \bar{N} \left( r, \frac{1}{G} \right) + S(r, f) \leq T(r, G) + S(r, f). \end{aligned}$$

So

$$(3.6) \quad T(r, G) = \bar{N} \left( r, \frac{1}{G} \right) + S(r, f),$$

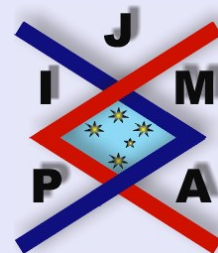
i.e.

$$T(r, f^{(k)}) = \bar{N} \left( r, \frac{1}{f^{(k)}} \right) + S(r, f),$$

this contradicts with conditions (1.1), (1.2) and (1.3) of this theorem.

If  $\frac{C}{D} = 1$ , from (3.4) we know

$$\frac{1}{F - 1} \equiv C \frac{G}{G - 1},$$



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 12 of 28

then

$$\left(F - 1 - \frac{1}{C}\right) G \equiv -\frac{1}{C}.$$

Noticing that  $F = \frac{f}{a}$ ,  $G = \frac{f^{(k)}}{a}$ , we have

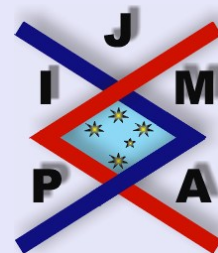
$$(3.7) \quad \frac{1}{f(f - (1 + \frac{1}{C})a)} \equiv -\frac{C}{a^2} \cdot \frac{f^{(k)}}{f}.$$

By Lemma 2.2 and (3.3) and (3.7), then

$$(3.8) \quad \begin{aligned} 2T(r, f) &= T\left(r, f\left(f - \left(1 + \frac{1}{C}\right)a\right)\right) + S(r, f) \\ &= T\left(r, \frac{1}{f(f - (1 + \frac{1}{C})a)}\right) + S(r, f) \\ &= T\left(r, \frac{f^{(k)}}{f}\right) + S(r, f) \\ &\leq N\left(r, \frac{1}{f}\right) + k\bar{N}(r, f) + S(r, f) \\ &\leq T(r, f) + S(r, f). \end{aligned}$$

So  $T(r, f) = S(r, f)$ , this is impossible. Hence  $D = 0$ , and  $\frac{G-1}{F-1} \equiv C$ , i.e.  $\frac{f^{(k)}-a}{f-a} \equiv C$ . This is just the conclusion of this theorem.

**Case 2.**  $H \neq 0$ . From (3.1) it is easy to see that  $m(r, H) = S(r, f)$ .



Meromorphic Function That  
Shares One Small Function  
With Its Derivative

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 13 of 28

**Subcase 2.1**  $l \geq 1$ . From (3.1) we have

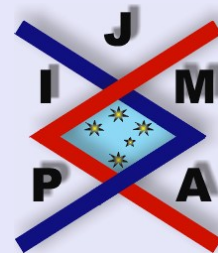
$$(3.9) \quad N(r, H) \leq \bar{N}(r, F) + \bar{N}_{(l+1)}\left(r, \frac{1}{F-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) + \bar{N}_0\left(r, \frac{1}{F'}\right) + \bar{N}_0\left(r, \frac{1}{G'}\right) + \bar{N}(r, a) + \bar{N}\left(r, \frac{1}{a}\right),$$

where  $N_0\left(r, \frac{1}{F'}\right)$  denotes the counting function of the zeros of  $F'$  which are not the zeros of  $F$  and  $F-1$ , and  $\bar{N}_0\left(r, \frac{1}{F'}\right)$  denotes its reduced form. In the same way, we can define  $N_0\left(r, \frac{1}{G'}\right)$  and  $\bar{N}_0\left(r, \frac{1}{G'}\right)$ . Let  $z_0$  be a simple zero of  $F-1$  but  $a(z_0) \neq 0, \infty$ , then  $z_0$  is also the simple zero of  $G-1$ . By calculating  $z_0$  is the zero of  $H$ , so

$$(3.10) \quad N_1\left(r, \frac{1}{F-1}\right) \leq N\left(r, \frac{1}{H}\right) + N(r, a) + N\left(r, \frac{1}{a}\right) \leq N(r, H) + S(r, f).$$

Noticing that  $N_1\left(r, \frac{1}{G}\right) = N_1\left(r, \frac{1}{F}\right) + S(r, f)$ , we have

$$(3.11) \quad \bar{N}\left(r, \frac{1}{G-1}\right) = N_1\left(r, \frac{1}{F-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F-1}\right) \leq \bar{N}(r, F) + \bar{N}_{(l+1)}\left(r, \frac{1}{F-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F-1}\right)$$



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 14 of 28

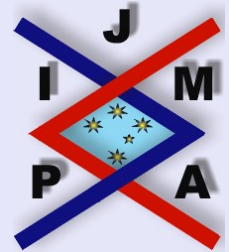
$$\begin{aligned}
& + \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) \\
& + \bar{N}_0\left(r, \frac{1}{F'}\right) + \bar{N}_0\left(r, \frac{1}{G'}\right) + S(r, f).
\end{aligned}$$

By the second fundamental theorem and (3.11) and noting  $\bar{N}(r, F) = \bar{N}(r, G) + S(r, f)$ , then

$$\begin{aligned}
(3.12) \quad T(r, G) & \leq \bar{N}(r, G) + \bar{N}\left(r, \frac{1}{G}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) \\
& \quad - N_0\left(r, \frac{1}{G'}\right) + S(r, G) \\
& \leq 2\bar{N}(r, F) + \bar{N}\left(r, \frac{1}{G}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F}\right) \\
& \quad + \bar{N}_{(l+1)}\left(r, \frac{1}{F-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F-1}\right) \\
& \quad + \bar{N}_0\left(r, \frac{1}{F'}\right) + S(r, f).
\end{aligned}$$

While  $l \geq 2$ ,

$$\begin{aligned}
(3.13) \quad \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(l+1)}\left(r, \frac{1}{F-1}\right) \\
+ \bar{N}_{(2)}\left(r, \frac{1}{F-1}\right) + \bar{N}_0\left(r, \frac{1}{F'}\right) \leq N_2\left(r, \frac{1}{F'}\right),
\end{aligned}$$



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 15 of 28

so

$$T(r, G) \leq 2\bar{N}(r, F) + N_2\left(r, \frac{1}{G}\right) + N_2\left(r, \frac{1}{F'}\right) + S(r, f),$$

i.e.

$$T(r, f^{(k)}) \leq 2\bar{N}(r, f) + N_2\left(r, \frac{1}{f^{(k)}}\right) + N_2\left(r, \frac{1}{(f/a)'}\right) + S(r, f).$$

This contradicts with (1.1).

While  $l = 1$ , (3.13) turns into

$$\begin{aligned} \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(l+1)}\left(r, \frac{1}{F-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F-1}\right) + \bar{N}_0\left(r, \frac{1}{F'}\right) \\ \leq 2\bar{N}\left(r, \frac{1}{F'}\right). \end{aligned}$$

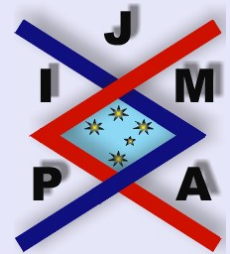
Similarly as above, we have

$$T(r, f^{(k)}) \leq 2\bar{N}(r, f) + N_2\left(r, \frac{1}{f^{(k)}}\right) + 2\bar{N}\left(r, \frac{1}{(f/a)'}\right) + S(r, f).$$

This contradicts with (1.2).

**Subcase 2.2**  $l = 0$ . In this case,  $F$  and  $G$  share 1 IM except the zeros and poles of  $a(z)$ .

Let  $z_0$  be the zero of  $F - 1$  with multiplicity  $p$  and the zero of  $G - 1$  with multiplicity  $q$ . We denote by  $N_E^1\left(r, \frac{1}{F}\right)$  the counting function of the zeros of



Meromorphic Function That  
Shares One Small Function  
With Its Derivative

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 16 of 28



$F - 1$  where  $p = q = 1$ ; by  $\overline{N}_E^{(2)}\left(r, \frac{1}{F}\right)$  the counting function of the zeros of  $F - 1$  where  $p = q \geq 2$ ; by  $\overline{N}_L\left(r, \frac{1}{F}\right)$  the counting function of the zeros of  $F - 1$  where  $p > q \geq 1$ , each point in these counting functions is counted only once. In the same way, we can define  $N_E^{(1)}\left(r, \frac{1}{G}\right)$ ,  $\overline{N}_E^{(2)}\left(r, \frac{1}{G}\right)$  and  $\overline{N}_L\left(r, \frac{1}{G}\right)$ . It is easy to see that

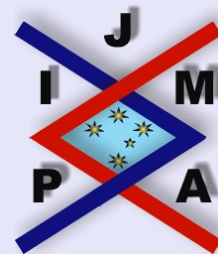
$$N_E^{(1)}\left(r, \frac{1}{F-1}\right) = N_E^{(1)}\left(r, \frac{1}{G-1}\right) + S(r, f),$$

$$\overline{N}_E^{(2)}\left(r, \frac{1}{F-1}\right) = \overline{N}_E^{(2)}\left(r, \frac{1}{G-1}\right) + S(r, f),$$

$$\begin{aligned} (3.14) \quad \overline{N}\left(r, \frac{1}{F-1}\right) &= \overline{N}\left(r, \frac{1}{G-1}\right) + S(r, f) \\ &= N_E^{(1)}\left(r, \frac{1}{F-1}\right) + \overline{N}_E^{(2)}\left(r, \frac{1}{F-1}\right) \\ &\quad + \overline{N}_L\left(r, \frac{1}{F-1}\right) + \overline{N}_L\left(r, \frac{1}{G-1}\right) + S(r, f). \end{aligned}$$

From (3.1) we have now

$$\begin{aligned} (3.15) \quad N(r, H) &\leq \overline{N}(r, F) + \overline{N}_{(2)}\left(r, \frac{1}{F}\right) + \overline{N}_{(2)}\left(r, \frac{1}{G}\right) + \overline{N}_L\left(r, \frac{1}{F-1}\right) \\ &\quad + \overline{N}_L\left(r, \frac{1}{G-1}\right) + \overline{N}_0\left(r, \frac{1}{F'}\right) + \overline{N}_0\left(r, \frac{1}{G'}\right) + S(r, f). \end{aligned}$$



**Meromorphic Function That  
Shares One Small Function  
With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 17 of 28

In this case, (3.10) is replaced by

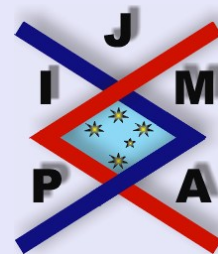
$$(3.16) \quad N_E^{(1)} \left( r, \frac{1}{F-1} \right) \leq N(r, H) + S(r, f).$$

From (3.14), (3.15) and (3.16), we have

$$\begin{aligned} \bar{N} \left( r, \frac{1}{G-1} \right) &\leq \bar{N}(r, F) + \bar{N}_{(2)} \left( r, \frac{1}{F} \right) + \bar{N}_{(2)} \left( r, \frac{1}{G} \right) + \bar{N}_E^{(2)} \left( r, \frac{1}{F-1} \right) \\ &\quad + 2\bar{N}_L \left( r, \frac{1}{F-1} \right) + 2\bar{N}_L \left( r, \frac{1}{G-1} \right) \\ &\quad + \bar{N}_0 \left( r, \frac{1}{F'} \right) + \bar{N}_0 \left( r, \frac{1}{G'} \right) + S(r, f) \\ &\leq \bar{N}(r, F) + 2\bar{N} \left( r, \frac{1}{F'} \right) + 2\bar{N}_L \left( r, \frac{1}{G-1} \right) \\ &\quad + \bar{N}_{(2)} \left( r, \frac{1}{G} \right) + \bar{N}_0 \left( r, \frac{1}{G'} \right) + S(r, f). \end{aligned}$$

By the second fundamental theorem, then

$$\begin{aligned} T(r, G) &\leq \bar{N}(r, G) + \bar{N} \left( r, \frac{1}{G} \right) + \bar{N} \left( r, \frac{1}{G-1} \right) - N_0 \left( r, \frac{1}{G'} \right) + S(r, G) \\ &\leq 2\bar{N}(r, G) + 2\bar{N} \left( r, \frac{1}{F'} \right) + \bar{N} \left( r, \frac{1}{G} \right) \\ &\quad + \bar{N}_{(2)} \left( r, \frac{1}{G} \right) + 2\bar{N}_L \left( r, \frac{1}{G-1} \right) + S(r, f) \end{aligned}$$



**Meromorphic Function That  
Shares One Small Function  
With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 18 of 28

$$\leq 2\bar{N}(r, G) + 2\bar{N}\left(r, \frac{1}{F'}\right) + \bar{N}\left(r, \frac{1}{G}\right) + 2\bar{N}\left(r, \frac{1}{G'}\right) + S(r, f).$$

From Lemma 2.1 for  $p = 1, k = 1$  we know

$$\bar{N}\left(r, \frac{1}{G'}\right) \leq N_2\left(r, \frac{1}{G}\right) + \bar{N}(r, G) + S(r, G).$$

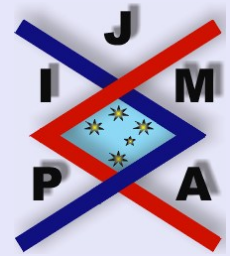
So

$$T(r, G) \leq 4\bar{N}(r, F) + 3N_2\left(r, \frac{1}{G}\right) + 2\bar{N}\left(r, \frac{1}{F'}\right) + S(r, f),$$

i.e.

$$T(r, f^{(k)}) \leq 4\bar{N}(r, f) + 3N_2\left(r, \frac{1}{f^{(k)}}\right) + 2\bar{N}\left(r, \frac{1}{(f/a)'}\right) + S(r, f).$$

This contradicts with (1.3). The proof is complete.




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**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 19 of 28

## 4. Proof of Theorem 1.3

The proof is similar to that of Theorem 1.2. We define  $F$  and  $G$  and (3.1) as above, and we also distinguish two cases to discuss.

**Case 3.**  $H \equiv 0$ . We also have (3.2). From (3.3) we know that  $\Theta(\infty, f) = 1$ , and from (1.4), (1.5) and (1.6), we further know  $\delta_{2+k}(0, f) > \frac{1}{2}$ . Assume that  $D \neq 0$ , then

$$-\frac{D(F-1-\frac{1}{D})}{F-1} \equiv C \frac{1}{G-1},$$

so

$$\bar{N}\left(r, \frac{1}{F-1-\frac{1}{D}}\right) = \bar{N}(r, G) = S(r, f).$$

If  $D \neq -1$ , using the second fundamental theorem for  $F$ , similarly as (3.6) we have

$$T(r, F) = \bar{N}\left(r, \frac{1}{F}\right) + S(r, f),$$

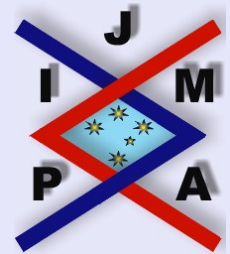
i.e.

$$T(r, f) = \bar{N}\left(r, \frac{1}{f}\right) + S(r, f).$$

Hence  $\Theta(0, f) = 0$ , this contradicts with  $\Theta(0, f) \geq \delta_{2+k}(0, f) > \frac{1}{2}$ .

If  $D = -1$ , then  $\bar{N}\left(r, \frac{1}{F}\right) = S(r, f)$ , i.e.  $\bar{N}\left(r, \frac{1}{f}\right) = S(r, f)$ , and

$$\frac{F}{F-1} \equiv C \frac{1}{G-1}.$$



Meromorphic Function That  
Shares One Small Function  
With Its Derivative

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 20 of 28

Then

$$F(G - 1 - C) \equiv -C$$

and thus,

$$(4.1) \quad f^{(k)} \left( f^{(k)} - (1 + C)a \right) \equiv -Ca^2 \frac{f^{(k)}}{f}.$$

As same as (3.8), by Lemma 2.2 and (3.3) and  $\bar{N} \left( r, \frac{1}{f} \right) = S(r, f)$ , from (4.1) we have

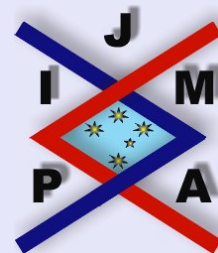
$$\begin{aligned} 2T(r, f^{(k)}) &= T \left( r, \frac{f^{(k)}}{f} \right) + S(r, f) \\ &= N \left( r, \frac{f^{(k)}}{f} \right) + S(r, f) \\ &\leq k\bar{N}(r, f) + k\bar{N} \left( r, \frac{1}{f} \right) + S(r, f) = S(r, f). \end{aligned}$$

So  $T(r, f^{(k)}) = S(r, f)$  and  $T \left( r, \frac{f^{(k)}}{f} \right) = S(r, f)$ . Hence

$$\begin{aligned} T(r, f) &\leq T \left( r, \frac{f}{f^{(k)}} \right) + T(r, f^{(k)}) + O(1) \\ &= T \left( r, \frac{f^{(k)}}{f} \right) + T(r, f^{(k)}) + O(1) = S(r, f), \end{aligned}$$

this is impossible. Therefore  $D = 0$ , and from (3.2) then

$$G - 1 \equiv \frac{1}{C}(F - 1).$$



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 21 of 28

If  $C \neq 1$ , then

$$G \equiv \frac{1}{C}(F - 1 + C),$$

and

$$N\left(r, \frac{1}{G}\right) = N\left(r, \frac{1}{F - 1 + C}\right).$$

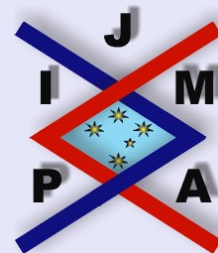
By the second fundamental theorem and (3.3) we have

$$\begin{aligned} T(r, F) &\leq \bar{N}(r, F) + \bar{N}\left(r, \frac{1}{F}\right) + \bar{N}\left(r, \frac{1}{F - 1 + C}\right) + S(r, G) \\ &\leq \bar{N}\left(r, \frac{1}{F}\right) + \bar{N}\left(r, \frac{1}{G}\right) + S(r, f). \end{aligned}$$

By Lemma 2.1 for  $p = 1$  and (3.3), we have

$$\begin{aligned} T(r, f) &\leq \bar{N}\left(r, \frac{1}{f}\right) + \bar{N}\left(r, \frac{1}{f^{(k)}}\right) + S(r, f) \\ &\leq \bar{N}\left(r, \frac{1}{f}\right) + N_{1+k}\left(r, \frac{1}{f}\right) + \bar{N}(r, f) + S(r, f) \\ &\leq 2N_{1+k}\left(r, \frac{1}{f}\right) + S(r, f). \end{aligned}$$

Hence  $\delta_{1+k}(0, f) \leq \frac{1}{2}$ . This is a contradiction with  $\delta_{1+k}(0, f) \geq \delta_{2+k}(0, f) > \frac{1}{2}$ . So  $C = 1$  and  $F \equiv G$ , i.e.  $f \equiv f^{(k)}$ . This is just the conclusion of this theorem.



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 22 of 28

**Case 4.**  $H \neq 0$ .

**Subcase 4.1**  $l \geq 1$ . As similar as Subcase 2.1, From (3.9) and (3.10) we have

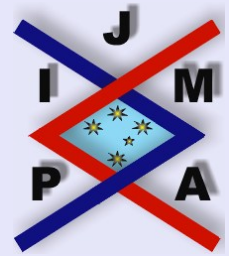
$$\begin{aligned}
 & \bar{N}\left(r, \frac{1}{F-1}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) \\
 &= N_1\left(r, \frac{1}{F-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F-1}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) \\
 &\leq \bar{N}(r, F) + \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) \\
 &\quad + \bar{N}_{(l+1)}\left(r, \frac{1}{G-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G-1}\right) \\
 &\quad + \bar{N}\left(r, \frac{1}{G-1}\right) + \bar{N}_0\left(r, \frac{1}{F'}\right) + \bar{N}_0\left(r, \frac{1}{G'}\right) + S(r, f).
 \end{aligned}$$

While  $l \geq 2$ ,

$$\begin{aligned}
 \bar{N}_{(l+1)}\left(r, \frac{1}{G-1}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G-1}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) &\leq N\left(r, \frac{1}{G-1}\right) \\
 &\leq T(r, G) + O(1),
 \end{aligned}$$

so

$$\begin{aligned}
 & \bar{N}\left(r, \frac{1}{F-1}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) \\
 &\leq \bar{N}(r, F) + \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right)
 \end{aligned}$$



**Meromorphic Function That  
Shares One Small Function  
With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 23 of 28

$$+ \bar{N}_0 \left( r, \frac{1}{F'} \right) + \bar{N}_0 \left( r, \frac{1}{G'} \right) + T(r, G) + S(r, f).$$

By the second fundamental theorem, we have

$$\begin{aligned} & T(r, F) + T(r, G) \\ & \leq \bar{N}(r, F) + \bar{N}(r, G) + \bar{N} \left( r, \frac{1}{F} \right) + \bar{N} \left( r, \frac{1}{G} \right) + \bar{N} \left( r, \frac{1}{F-1} \right) \\ & \quad + \bar{N} \left( r, \frac{1}{G-1} \right) - N_0 \left( r, \frac{1}{F'} \right) - N_0 \left( r, \frac{1}{G'} \right) + S(r, F) + S(r, G) \\ & \leq 3\bar{N}(r, F) + N_2 \left( r, \frac{1}{F} \right) + N_2 \left( r, \frac{1}{G} \right) + T(r, G) + S(r, f), \end{aligned}$$

so

$$T(r, F) \leq 3\bar{N}(r, F) + N_2 \left( r, \frac{1}{F} \right) + N_2 \left( r, \frac{1}{G} \right) + S(r, f),$$

i.e.

$$T(r, f) \leq 3\bar{N}(r, f) + N_2 \left( r, \frac{1}{f} \right) + N_2 \left( r, \frac{1}{f^{(k)}} \right) + S(r, f).$$

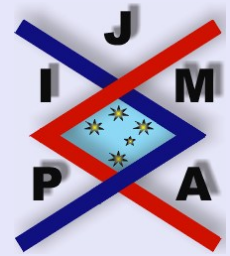
By Lemma 2.1 for  $p = 2$  we have

$$T(r, f) \leq (3+k)\bar{N}(r, f) + 2N_{2+k} \left( r, \frac{1}{f} \right) + S(r, f),$$

so

$$(3+k)\Theta(\infty, f) + 2\delta_{2+k}(0, f) \leq k + 4.$$

This contradicts with (1.4).



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 24 of 28



While  $l = 1$ ,

$$\bar{N}_{(l+1)}\left(r, \frac{1}{G-1}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) \leq N\left(r, \frac{1}{G-1}\right) \leq T(r, G) + O(1),$$

so by Lemma 2.1 for  $p = 1, k = 1$ , we have

$$\begin{aligned} & \bar{N}\left(r, \frac{1}{F-1}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) \\ & \leq \bar{N}(r, F) + \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F-1}\right) \\ & \quad + \bar{N}_0\left(r, \frac{1}{F'}\right) + \bar{N}_0\left(r, \frac{1}{G'}\right) + T(r, G) + S(r, f) \\ & \leq \bar{N}(r, F) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) + \bar{N}\left(r, \frac{1}{F'}\right) + \bar{N}_0\left(r, \frac{1}{G'}\right) + T(r, G) + S(r, f) \\ & \leq 2\bar{N}(r, F) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) + N_2\left(r, \frac{1}{F}\right) + \bar{N}_0\left(r, \frac{1}{G'}\right) + T(r, G) + S(r, f) \end{aligned}$$

As same as above, by the second fundamental theorem we have

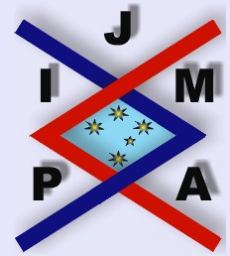
$$T(r, F) + T(r, G) \leq 4\bar{N}(r, F) + 2N_2\left(r, \frac{1}{F}\right) + N_2\left(r, \frac{1}{G}\right) + T(r, G) + S(r, f),$$

so

$$T(r, F) \leq 4\bar{N}(r, F) + 2N_2\left(r, \frac{1}{F}\right) + N_2\left(r, \frac{1}{G}\right) + S(r, f),$$

i.e.

$$T(r, f) \leq 4\bar{N}(r, f) + 2N_2\left(r, \frac{1}{f}\right) + N_2\left(r, \frac{1}{f^{(k)}}\right) + S(r, f).$$



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 25 of 28

By Lemma 2.1 for  $p = 2$  we have

$$T(r, f) \leq (4 + k)\overline{N}(r, f) + 3N_{2+k}\left(r, \frac{1}{f}\right) + S(r, f),$$

so

$$(4 + k)\Theta(\infty, f) + 3\delta_{2+k}(0, f) \leq k + 6.$$

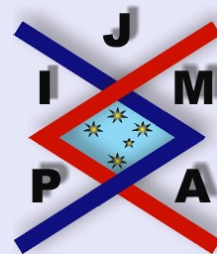
This contradicts with (1.5).

**Subcase 4.2**  $l = 0$ . From (3.14), (3.15) and (3.16) and Lemma 2.1 for  $p = 1, k = 1$ , noticing

$$\begin{aligned} \overline{N}_E^{(2)}\left(r, \frac{1}{G-1}\right) + \overline{N}_L\left(r, \frac{1}{G-1}\right) + \overline{N}\left(r, \frac{1}{G-1}\right) &\leq N\left(r, \frac{1}{G-1}\right) \\ &\leq T(r, G) + S(r, f), \end{aligned}$$

then

$$\begin{aligned} &\overline{N}\left(r, \frac{1}{F-1}\right) + \overline{N}\left(r, \frac{1}{G-1}\right) \\ &= N_E^{(1)}\left(r, \frac{1}{F-1}\right) + \overline{N}_E^{(2)}\left(r, \frac{1}{F-1}\right) + \overline{N}_L\left(r, \frac{1}{F-1}\right) \\ &\quad + \overline{N}_L\left(r, \frac{1}{G-1}\right) + \overline{N}\left(r, \frac{1}{G-1}\right) \\ &\leq \overline{N}(r, F) + \overline{N}_{(2)}\left(r, \frac{1}{F}\right) + \overline{N}_{(2)}\left(r, \frac{1}{G}\right) + 2\overline{N}_L\left(r, \frac{1}{F-1}\right) \\ &\quad + \overline{N}_L\left(r, \frac{1}{G-1}\right) + \overline{N}_E^{(2)}\left(r, \frac{1}{G-1}\right) + \overline{N}_L\left(r, \frac{1}{G-1}\right) \end{aligned}$$



Meromorphic Function That  
Shares One Small Function  
With Its Derivative

Qingcai Zhang

Title Page

Contents



Go Back

Close

Quit

Page 26 of 28

$$\begin{aligned}
& + \bar{N}\left(r, \frac{1}{G-1}\right) + \bar{N}_0\left(r, \frac{1}{F'}\right) + \bar{N}_0\left(r, \frac{1}{G'}\right) + S(r, f) \\
\leq & \bar{N}(r, F) + 2\bar{N}\left(r, \frac{1}{F'}\right) + \bar{N}\left(r, \frac{1}{G'}\right) + T(r, G) + S(r, f) \\
\leq & 4\bar{N}(r, F) + 2N_2\left(r, \frac{1}{F}\right) + N_2\left(r, \frac{1}{G}\right) + T(r, G) + S(r, f).
\end{aligned}$$

As same as above, by the second fundamental theorem, we can obtain

$$T(r, F) + T(r, G) \leq 6\bar{N}(r, F) + 3N_2\left(r, \frac{1}{F}\right) + 2N_2\left(r, \frac{1}{G}\right) + T(r, G) + S(r, f),$$

so

$$T(r, F) \leq 6\bar{N}(r, F) + 3N_2\left(r, \frac{1}{F}\right) + 2N_2\left(r, \frac{1}{G}\right) + S(r, f),$$

i.e.

$$T(r, f) \leq 6\bar{N}(r, f) + 3N_2\left(r, \frac{1}{f}\right) + 2N_2\left(r, \frac{1}{f^{(k)}}\right) + S(r, f).$$

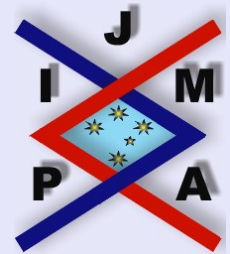
By Lemma 2.1 for  $p = 2$  we have

$$T(r, f) \leq (6 + 2k)\bar{N}(r, f) + 5N_{2+k}\left(r, \frac{1}{f}\right) + S(r, f),$$

so

$$(6 + 2k)\Theta(\infty, f) + 5\delta_{2+k}(0, f) \leq 2k + 10.$$

This contradicts with (1.6). Now the proof has been completed.



**Meromorphic Function That Shares One Small Function With Its Derivative**

Qingcai Zhang

Title Page

Contents



Go Back

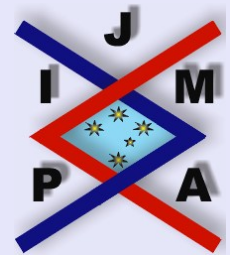
Close

Quit

Page 27 of 28

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**Meromorphic Function That  
Shares One Small Function  
With Its Derivative**

Qingcai Zhang

---

Title Page

Contents



Go Back

Close

Quit

Page 28 of 28