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WEIGHTED MULTIPLICATIVE INTEGRAL INEQUALITIES

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Abstract

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Abstract

We give a generalization of a one-dimensional Carlson type inequality due to G.-S. Yang and J.-C. Fang and a generalization of a multidimensional type inequality due to L. Larsson. We point out the strong and weak parts of each result.

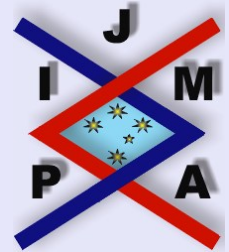
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1. Introduction

Let $(a_n)_{n \geq 1}$ be a non-zero sequence of non-negative numbers and f be a measurable function on $[0, \infty)$. In 1934, F. Carlson [2] proved that the following inequalities

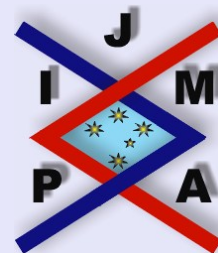
$$(1.1) \quad \left(\sum_{n=1}^{\infty} a_n \right)^4 < \pi^2 \sum_{n=1}^{\infty} a_n^2 \sum_{n=1}^{\infty} n^2 a_n^2,$$

$$(1.2) \quad \left(\int_0^{\infty} f(x) dx \right)^4 \leq \pi^2 \int_0^{\infty} f^2(x) dx \int_0^{\infty} x^2 f^2(x) dx$$

hold and $C = \pi^2$ is the best constant in both cases. Several generalizations and applications in different branches of mathematics have been given during the years. For a complete survey of the results and applications concerning the above inequalities and also interesting historical remarks see the book [5].

G.-S. Yang and J.-C. Fang in [6] proved the following generalization of inequality (1.1)

$$(1.3) \quad \left(\sum_{n=1}^{\infty} a_n \right)^{2p} < \left(\frac{\pi}{\alpha m} \right)^2 \sum_{n=1}^{\infty} a_n^{p(1+2r-rp)} g^{1-\alpha}(n) \\ \times \sum_{n=1}^{\infty} a_n^{p(1+2r-rp)} g^{1+\alpha}(n) \left(\sum_{n=1}^{\infty} a_n^{rp} \right)^{2(p-2)},$$



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when $(a_n)_{n \geq 1}$ is a sequence of nonnegative numbers and g is positive, continuously differentiable, $0 < m = \inf_{x>0} g'(x) < \infty$, $\lim_{x \rightarrow \infty} g(x) = \infty$, $p > 2$, $0 < \alpha \leq 1$, $r > 0$.

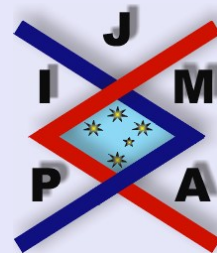
They also proved in [6] the analogue generalization of the integral inequality (1.2) as follows

$$(1.4) \quad \left(\int_0^\infty f(x) dx \right)^{2p} \leq \left(\frac{\pi}{\alpha m} \right)^2 \int_0^\infty f^{p(1+2r-rp)}(x) g^{1-\alpha}(x) dx \times \int_0^\infty f^{p(1+2r-rp)}(x) g^{1+\alpha}(x) dx \left(\int_0^\infty f^{rp}(x) dx \right)^{2(p-2)},$$

when f is a positive measurable function, g is positive, continuously differentiable and $0 < m = \inf_{x>0} g'(x) < \infty$, $\lim_{x \rightarrow \infty} g(x) = \infty$, $p > 2$, $0 < \alpha \leq 1$, $r > 0$.

On the other hand, using another technique, in [3], the following multidimensional extension of the inequality (1.4) was given

$$(1.5) \quad \left(\int_{\mathbb{R}^n} f(x) dx \right)^{2p} \leq C \left(\frac{1}{\alpha m^{n/\gamma}} \right)^2 \int_{\mathbb{R}^n} f^{p(1+2r-rp)}(x) g^{(n-\alpha)/\gamma}(x) dx \times \int_{\mathbb{R}^n} f^{q(1+2s-sq)}(x) g^{(n+\alpha)/\gamma}(x) dx \times \left(\int_{\mathbb{R}^n} f^{rp}(x) dx \right)^{p-2} \left(\int_{\mathbb{R}^n} f^{rq}(x) dx \right)^{q-2},$$



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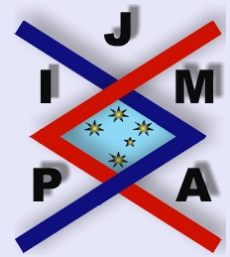
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for all positive and measurable functions f . Above, n is a positive integer, r, s are real numbers, $m, \gamma > 0, p, q > 2, 0 < \alpha < n, g : \mathbb{R}^n \rightarrow (0, \infty)$ with $g(x) \geq m |x|^\gamma$, and the constant C does not depend on m, α, γ . This inequality allows a more general setting of parameters and a much larger class of functions g . In [3] an example of admissible function g which is not even continuous was given. It is also shown that the condition $\lim_{x \rightarrow \infty} g(x) = \infty$ of (1.4) cannot be relaxed too much, in other words that g cannot be taken essentially bounded. The only weaker point of (1.5) is that it is not given an explicit value of the constant C . We also observe that the proof of (1.4) can be carried on for the value $\alpha = 1$ while this value is not allowed in the proof of (1.5) in the case $n = 1$, which means that Carlson's inequality (1.1) is only a limiting case of (1.5).

In Section 2 of this paper we give two-weight generalizations of the inequalities (1.4) and (1.5). In Section 3 we give a generalization of the discrete inequality (1.3) and some remarks.



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2. The Continuous Case

In the next theorem we prove a two-weight generalization of the inequality (1.4).

Theorem 2.1. *Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a positive measurable function, g_1 and g_2 be positive continuously differentiable and $0 < m = \inf_{x>0} (g_1'g_2 - g_2'g_1) < \infty$. Suppose that $p > 2$ and r is an arbitrary real number. Then the following inequality holds*

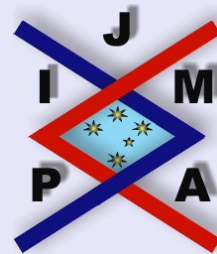
$$(2.1) \quad \left(\int_0^\infty f(x) dx \right)^{2p} \leq \left(\frac{\pi}{m} \right)^2 \int_0^\infty f^{p(1+2r-rp)}(x) g_1^2(x) dx \\ \times \int_0^\infty f^{p(1+2r-rp)}(x) g_2^2(x) dx \left(\int_0^\infty f^{rp}(x) dx \right)^{2(p-2)}.$$

Proof. Observe that the condition $0 < m = \inf_{x>0} (g_1'g_2 - g_2'g_1) < \infty$ implies that $\frac{g_1}{g_2}$ is strictly increasing. Let

$$A = \int_0^\infty f^{p(1+2r-rp)}(x) g_1^2(x) dx \quad \text{and} \quad B = \int_0^\infty f^{p(1+2r-rp)}(x) g_2^2(x) dx,$$

$\lambda > 0$ and q such that $\frac{1}{q} + \frac{1}{p} = 1$. By using Hölder's inequality once for the indices p and q and once for $\frac{p}{q}$ and $\frac{p}{p-q}$ we get

$$\int_0^\infty f(x) dx \\ \leq \left(\int_0^\infty f^q(x) \left(\lambda g_1^2(x) + \frac{1}{\lambda} g_2^2(x) \right)^{\frac{q}{p}} dx \right)^{\frac{1}{q}} \left(\int_0^\infty \frac{1}{\lambda g_1^2(x) + \frac{1}{\lambda} g_2^2(x)} dx \right)^{\frac{1}{p}}$$



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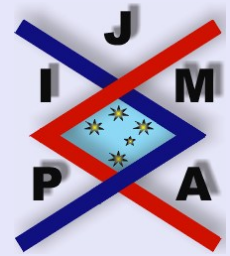
$$\begin{aligned}
&\leq \frac{1}{m^{\frac{1}{p}}} \left(\int_0^\infty \frac{\left(\frac{g_1(x)}{g_2(x)} \right)'}{\lambda \left(\frac{g_1(x)}{g_2(x)} \right)^2 + \frac{1}{\lambda}} dx \right)^{\frac{1}{p}} \left(\int_0^\infty f^q(x) \left(\lambda g_1^2(x) + \frac{1}{\lambda} g_2^2(x) \right)^{\frac{q}{p}} dx \right)^{\frac{1}{q}} \\
&= \frac{1}{m^{\frac{1}{p}}} \left(\arctan \frac{g_1(x)}{\lambda g_2(x)} \Big|_0^\infty \right)^{\frac{1}{p}} \\
&\quad \times \left(\int_0^\infty f^{q-r(p-q)}(x) f^{r(p-q)} \left(\lambda g_1^2(x) + \frac{1}{\lambda} g_2^2(x) \right)^{\frac{q}{p}} dx \right)^{\frac{1}{q}} \\
&\leq \left(\frac{\pi}{2m} \right)^{\frac{1}{p}} \left(\int_0^\infty f^{p(1+2r-rp)}(x) \left(\lambda g_1^2(x) + \frac{1}{\lambda} g_2^2(x) \right) dx \right)^{\frac{1}{p}} \left(\int_0^\infty f^{rp}(x) dx \right)^{\frac{p-2}{p}} \\
&= \left(\frac{\pi}{2m} \right)^{\frac{1}{p}} \left(\lambda A + \frac{1}{\lambda} B \right)^{\frac{1}{p}} \left(\int_0^\infty f^{rp}(x) dx \right)^{\frac{p-2}{p}}.
\end{aligned}$$

Taking now $\lambda = \sqrt{\frac{B}{A}}$ we get the desired inequality and this completes the proof. \square

Remark 1. If $rp = 1$ the inequality (2.1) reduces to

$$\left(\int_0^\infty f(x) dx \right)^4 \leq \left(\frac{\pi}{m} \right)^2 \int_0^\infty f^2(x) g_1^2(x) dx \int_0^\infty f^2(x) g_2^2(x) dx$$

which becomes (1.2) for $g_1(x) = x$, $g_2(x) = 1$, $x > 0$. The same happens if we let $p \rightarrow 2$ in (2.1). If we let $g_1(x) = g^{\frac{1+\alpha}{2}}(x)$, $g_2(x) = g^{\frac{1-\alpha}{2}}(x)$ in (2.1) we get (1.4) which means that (2.1) generalizes also the inequality (4) of [1]. The same



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inequalities can be given if we replace the interval $[0, \infty)$ by bounded intervals $[a, b]$ or by $(-\infty, \infty)$. On the other hand we can see that it is not necessary to suppose $\inf_{x>0} g_2(x) \geq k > 0$, in other words, the weights $g_2(x) = e^{-x}$ and $g_2(x) = e^x$ are allowed. An interesting case is when $g_2(x) = 1$, $g_1(x) = A_n(x; a) = x(x + na)^{n-1}$, $a > 0$, $n \in \mathbb{N}$, $n \geq 1$ (Abel polynomials). The inequality (2.1) becomes

$$\left(\int_0^\infty f(x) dx \right)^4 \leq \left(\frac{\pi}{(na)^{n-1}} \right)^2 \int_0^\infty f^2(x) dx \int_0^\infty f^2(x) A_n^2(x; a) dx.$$

To prove a multidimensional extension of the above inequality we need the following lemma which is a special case of Theorem 2 in [4].

Lemma 2.2. *Let $(Z, d\zeta)$ be a measure space on which weights $\beta \geq 0$, $\beta_0 > 0$ and $\beta_1 > 0$ are defined. Suppose that $p_0, p_1 \in (1, 2)$ and $\theta \in (0, 1)$. Suppose also that there is a constant C such that*

$$(2.2) \quad \zeta \left(\left\{ z : 2^m \leq \frac{\beta_0(z)}{\beta_1(z)} < 2^{m+1} \right\} \right) \leq C, \quad m \in \mathbb{Z}$$

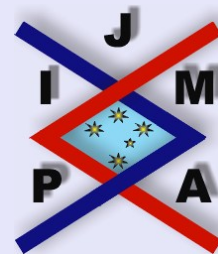
and that

$$\frac{\beta}{\beta_0^\theta \beta_1^{1-\theta}} \in L^\infty(Z, d\zeta).$$

Then there is a constant A such that

$$(2.3) \quad \|f\beta\|_{L^1(Z, d\zeta)} \leq A \|f\beta_0\|_{L^{p_0}(Z, d\zeta)}^\theta \|f\beta_1\|_{L^{p_1}(Z, d\zeta)}^{1-\theta}.$$

The constant A can be chosen of the form $A = A_0 C^{1-\theta/p_0 - (1-\theta)/p_1}$, where A_0 does not depend on C .



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We are now ready to prove our next multidimensional result which is also a generalization of Theorem 2 of [3]. The technique is similar to that used in the last mentioned theorem. We suppose for simplicity that f is a nonnegative function.

Theorem 2.3. *Let n be a positive integer and $p, q > 2, a < 1$ and $r, s \in \mathbb{R}$. Suppose that for some positive constants m, k , the functions $g_1, g_2 : \mathbb{R}^n \rightarrow (0, \infty)$ satisfy*

$$(2.4) \quad g_2(x) \geq m |x|^{(nap)/2} \quad \text{and} \quad g_1(x) \geq k |x|^{n(p+q-ap)/2}.$$

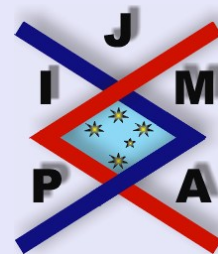
Then there is a constant B independent of m, k, a such that

$$(2.5) \quad \left(\int_{\mathbb{R}^n} f(x) dx \right)^{p+q} \leq \frac{B}{(1-a)^2 m^2 k^2} \int_{\mathbb{R}^n} f^{p(1+2r-rp)}(x) g_2^2(x) dx \\ \times \int_{\mathbb{R}^n} f^{q(1+2r-rp)}(x) g_1^2(x) dx \\ \times \left(\int_{\mathbb{R}^n} f^{rp}(x) dx \right)^{p-2} \left(\int_{\mathbb{R}^n} f^{rq}(x) dx \right)^{q-2}.$$

Proof. In Lemma 2.2 put $Z = \mathbb{R}^n$, $d\zeta(x) = \frac{dx}{|x|^n}$, where dx is the Lebesgue measure in \mathbb{R}^n , $p_0 = p'$, $p_1 = q'$, $\frac{1}{p} + \frac{1}{p'} = 1$, $\frac{1}{q} + \frac{1}{q'} = 1$. Let $\beta(x) = |x|^n$, $\beta_0(x) = |x|^{na}$ and $\beta_1(x) = |x|^{n \frac{1-a\theta}{1-\theta}} = |x|^{n \frac{p+q-ap}{q}}$, where $\theta = \frac{p}{p+q}$.

We observe that

$$\frac{\beta}{\beta_0^\theta \beta_1^{1-\theta}} \equiv 1 \in L^\infty(\mathbb{Z}, d\zeta).$$



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Also, easy computations give

$$\frac{\beta_0(x)}{\beta_1(x)} = |x|^{\frac{n(a-1)}{1-\theta}} = |x|^{\frac{n(a-1)(p+q)}{q}}.$$

Let

$$\tau = \frac{n(1-a)(p+q)}{q} > 0.$$

Thus $\frac{\beta_0(x)}{\beta_1(x)} \in [2^m, 2^{m+1})$ if and only if $2^{-(m+1)/\tau} \leq |x| \leq 2^{-m/\tau}$. Using polar coordinates we get

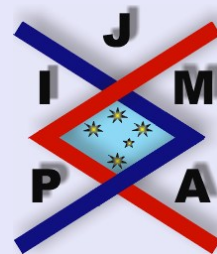
$$\zeta \left(\left\{ \frac{\beta_0(x)}{\beta_1(x)} \in [2^m, 2^{m+1}) \right\} \right) = \omega_n \int_{2^{-(m+1)/\tau}}^{2^{-m/\tau}} \frac{dr}{r} = \frac{\omega_n \log 2}{\tau},$$

where ω_n denotes the surface area of the unit sphere in \mathbb{R}^n . Hence (2.2) holds with $C = \frac{\omega_n \log 2}{\tau}$. Since the conditions of Lemma 2.2 are satisfied, using (2.3) we get

$$\begin{aligned} \int_{\mathbb{R}^n} f(x) dx &= \int_Z f(x) \beta(x) d\zeta(x) \\ &\leq A \left(\int_Z (f(x) \beta_0(x))^{p_0} d\zeta(x) \right)^{\frac{\theta}{p_0}} \left(\int_Z (f(x) \beta_1(x))^{p_1} d\zeta(x) \right)^{\frac{1-\theta}{p_1}} \\ &= A \left(\int_{\mathbb{R}^n} |x|^{nap'} f^{p'}(x) dx \right)^{\frac{p-1}{p+q}} \left(\int_{\mathbb{R}^n} |x|^{n\frac{p+q-ap}{q} q'} f^{q'}(x) dx \right)^{\frac{q-1}{p+q}}. \end{aligned}$$

If we write

$$|x|^{nap'} f^{p'}(x) = \left(|x|^{nap'} f^{p'(1+2r-rp)}(x) \right) f^{p'r(p-2)}(x),$$



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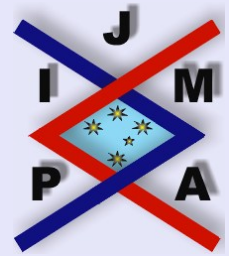


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$$|x|^{n\frac{p+q-\alpha p}{q}q'} f^{q'}(x) = \left(|x|^{n\frac{p+q-\alpha p}{q}q'} f^{q'(1+2s-sq)}(x) \right) f^{p's(q-2)}(x)$$

and apply Hölder's inequality with $(p - 1)$ and $(p - 1)/(p - 2)$ in the first integral and $(q - 1)$ and $(q - 1)/(q - 2)$ in the second integral we get

$$\begin{aligned} & \left(\int_{\mathbb{R}^n} f(x) dx \right)^{p+q} \\ & \leq A^{p+q} \int_{\mathbb{R}^n} |x|^{n\alpha p} f^{p(1+2r-rp)}(x) dx \int_{\mathbb{R}^n} |x|^{n(p+q-\alpha p)} f^{q(1+2s-sq)}(x) \\ & \quad \times \left(\int_{\mathbb{R}^n} f^{rp}(x) dx \right)^{p-2} \left(\int_{\mathbb{R}^n} f^{sq}(x) dx \right)^{q-2}. \end{aligned}$$

By Lemma 2.2 we can choose $A = A_0 \left(\frac{\omega_n \log 2}{\tau} \right)^{2/(p+q)}$, i.e. $A^{p+q} = \frac{B}{(1-a)^2}$, where B does not depend on a . Using (2.4) in estimating the integrals we get the inequality (2.5) and the proof is complete. \square

Corollary 2.4. *Let n be a positive integer and $p, q > 2$, $0 < \alpha < n$ and $r, s \in \mathbb{R}$. Suppose that for some positive constants m, γ , the function $g : \mathbb{R}^n \rightarrow (0, \infty)$ satisfies*

$$(2.6) \quad g(x) \geq m |x|^\gamma.$$

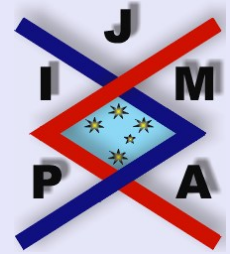
Then there is a constant C independent of m, γ, α such that

$$\left(\int_{\mathbb{R}^n} f(x) dx \right)^{p+q} \leq \frac{C}{\alpha^2 m^{2n/\gamma}} \int_{\mathbb{R}^n} f^{p(1+2r-rp)}(x) g^{(n-\alpha)/\gamma}(x) dx$$

$$\begin{aligned} & \times \int_{\mathbb{R}^n} f^{q(1+2r-rp)}(x) g^{(n+\alpha)/\gamma}(x) dx \\ & \times \left(\int_{\mathbb{R}^n} f^{rp}(x) dx \right)^{p-2} \left(\int_{\mathbb{R}^n} f^{rq}(x) dx \right)^{q-2}. \end{aligned}$$

Proof. The condition (2.4) of Theorem 2.3 implies (2.6) if $a = 1 - \frac{\alpha}{np}$, $g_1(x) = g^{(n+\alpha)/2\gamma}(x)$, $g_2(x) = g^{(n-\alpha)/2\gamma}(x)$. \square

Remark 2. *The above corollary is just Theorem 2 of [3]. On the other hand, our Theorem 2.3 is more general than Theorem 2 of [3] since the value $a = 0$ is allowed. This means that g_2 can be taken equivalent with a constant. Thus our inequality can be considered a generalization of Carlson's inequality. In the same way as in [3] one can prove that g_1 cannot be taken essentially bounded. It is also obvious that the condition (2.4) is to some extent weaker than (2.1) although g_2 has to be bounded from below.*



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3. The Discrete Case

For completeness we also formulate the discrete case which is a generalization of (1.3).

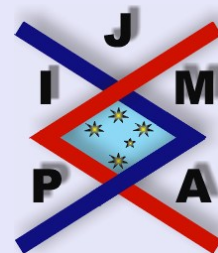
Theorem 3.1. *Let $(a_n)_{n \geq 1}$ be a sequence of nonnegative numbers and g_1 and g_2 be positive, continuously differentiable functions such that $0 < m = \inf_{x>0} (g'_1 g_2 - g'_2 g_1) < \infty$, and suppose that g_2 is an increasing function*

$$(3.1) \quad \left(\sum_{n=1}^{\infty} a_n \right)^{2p} < \left(\frac{\pi}{m} \right)^2 \sum_{n=1}^{\infty} a_n^{p(1+2r-rp)} g_1^2(n) \\ \times \sum_{n=1}^{\infty} a_n^{p(1+2r-rp)} g_2^2(n) \left(\sum_{n=1}^{\infty} a_n^{rp} \right)^{2(p-2)}.$$

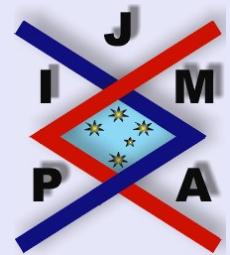
Proof. The proof carries on in the same manner as Theorem 2.1. We also use the fact that in the conditions of the hypothesis the function $\frac{1}{\lambda g_1^2(\cdot) + \frac{1}{\lambda} g_2^2(\cdot)}$, $\lambda > 0$ is decreasing and in this case the sum $\sum_{n=1}^{\infty} \left(\lambda g_1^2(n) + \frac{1}{\lambda} g_2^2(n) \right)^{-1}$ can be estimated by the integral $\int_0^{\infty} \frac{1}{\lambda g_1^2(x) + \frac{1}{\lambda} g_2^2(x)} dx$. \square

Remark 3. *Observe the fact that g_2 is an increasing function implies that g_1 is also increasing. If $rp = 1$ then the inequality (3.1) reduces to*

$$\left(\sum_{n=1}^{\infty} a_n \right)^4 \leq \left(\frac{\pi}{m} \right)^2 \sum_{n=1}^{\infty} a_n^2 g_1^2(n) \sum_{n=1}^{\infty} a_n^2 g_2^2(n)$$



which becomes (1.1) for $g_1(n) = n$, $g_2(n) = 1$, $n \in \mathbb{N}$. The same is true if we let $p \rightarrow 2$ in (3.1). If we let $g_1(x) = g^{\frac{1-\alpha}{2}}(x)$, $g_2(x) = g^{\frac{1+\alpha}{2}}(x)$ in (3.1) we get (1.4) which means that (2.1) generalizes inequality (6) of [6].



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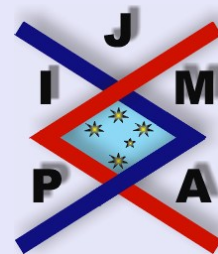
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