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ZERO AND COEFFICIENT INEQUALITIES FOR HYPERBOLIC POLYNOMIALS

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[Abstract](#)

[Contents](#)



[Home Page](#)

[Go Back](#)

[Close](#)

[Quit](#)

Abstract

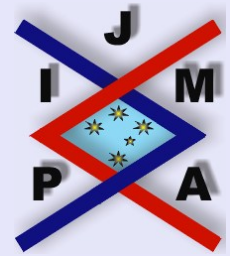
In this paper using classical inequalities and Cardan-Viète formulae some inequalities involving zeroes and coefficients of hyperbolic polynomials are given. Furthermore, considering real polynomials whose zeros lie in $\operatorname{Re}(z) > 0$, the previous results have been extended and new inequalities are obtained.

2000 Mathematics Subject Classification: 12D10, 26C10, 26D15.

Key words: Zeroes and coefficients, Inequalities in the complex plane, Inequalities for polynomials with real zeros, Hyperbolic polynomials.

Contents

1	Introduction	3
2	The Inequalities	4
	References	



Zero and Coefficient Inequalities for Hyperbolic Polynomials

J. Rubi -Masseg ,
J.L. D -Barrero and
P. Rubi -D 

Title Page

Contents



Go Back

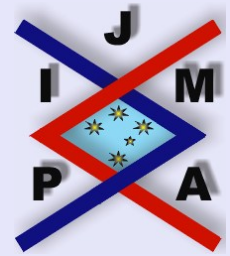
Close

Quit

Page 2 of 14

1. Introduction

The problem of finding relations between the zeroes and coefficients of a polynomial occupies a central role in the theory of equations. The most well known of such relations are Cardan-Viète's formulae [1]. Many papers devoted to obtaining inequalities between the zeros and coefficient have been written giving new bounds or improving the classical known ones ([2], [3], [4]). Furthermore, inequalities for polynomials with all zeros real also called hyperbolic polynomials, have been fully documented in [5]. In this paper, using some classical inequalities, several inequalities involving zeros and coefficients of polynomials with real zeros have been obtained and the main result has been extended to polynomials whose zeros lie in the right half plane.



Zero and Coefficient Inequalities for Hyperbolic Polynomials

J. Rubió-Massegú,
J.L. Díaz-Barrero and
P. Rubió-Díaz

Title Page

Contents



Go Back

Close

Quit

Page 3 of 14

2. The Inequalities

In what follows some zero and coefficient inequalities involving polynomials whose zeros are strictly positive real numbers are obtained. We begin with

Theorem 2.1. *Let $A(x) = \sum_{k=0}^n a_k x^k$, $a_n \neq 0$, be a hyperbolic polynomial with all its zeroes x_1, x_2, \dots, x_n strictly positive. If α, p and b are strictly positive real numbers such that $\alpha < p$, then*

$$(2.1) \quad \sum_{k=1}^n \frac{1}{[x_k^p + b]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \left(\frac{p - \alpha}{b} \right)^{\frac{1}{\alpha} - \frac{1}{p}} \left| \frac{a_1}{a_0} \right|.$$

Equality holds when $A(x) = a_n \left(x - \left(\frac{b\alpha}{p-\alpha} \right)^{\frac{1}{p}} \right)^n$.

Proof. Let β and a be strictly positive real numbers defined by $\beta = 1 - \frac{\alpha}{p} > 0$ and $a = \frac{b\alpha}{p\beta} > 0$. Taking into account that $\frac{\alpha}{p} + \beta = 1$ and applying the powered AM-GM inequality, we have for all k , $1 \leq k \leq n$,

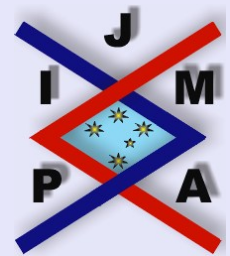
$$(2.2) \quad (x_k^p)^{\frac{\alpha}{p}} a^{\beta} \leq \frac{\alpha}{p} x_k^p + \beta a.$$

Inverting the terms in (2.2) yields

$$\frac{1}{\frac{\alpha}{p} x_k^p + \beta a} \leq \frac{1}{x_k^{\alpha} a^{\beta}}, \quad 1 \leq k \leq n,$$

or equivalently

$$\frac{1}{x_k^p + \frac{p}{\alpha} \beta a} \leq \frac{\alpha}{p} \cdot \frac{1}{x_k^{\alpha} a^{\beta}}.$$



Zero and Coefficient
Inequalities for Hyperbolic
Polynomials

J. Rubió-Massequé,
J.L. Díaz-Barrero and
P. Rubió-Díaz

Title Page

Contents



Go Back

Close

Quit

Page 4 of 14

Taking into account that $\frac{p}{\alpha}\beta a = b$ and $\beta = \frac{p-\alpha}{p}$, we have

$$\begin{aligned} \frac{1}{x_k^p + b} &\leq \frac{\alpha}{p} \cdot \frac{1}{\left(\frac{b\alpha}{p\beta}\right)^\beta} \frac{1}{x_k^\alpha} \\ &= \frac{\alpha}{p} \cdot \frac{p^\beta \beta^\beta}{b^\beta \alpha^\beta} \frac{1}{x_k^\alpha} \\ &= \frac{\alpha}{p} \cdot \frac{p^{1-\frac{\alpha}{p}} \left(\frac{p-\alpha}{p}\right)^{1-\frac{\alpha}{p}}}{b^{1-\frac{\alpha}{p}} \alpha^{1-\frac{\alpha}{p}}} \cdot \frac{1}{x_k^\alpha} \\ &= \frac{\alpha^{\frac{\alpha}{p}}}{p} \left(\frac{p-\alpha}{b}\right)^{1-\frac{\alpha}{p}} \frac{1}{x_k^\alpha}. \end{aligned}$$

Raising to $\frac{1}{\alpha}$ both sides of the preceding inequality, yields

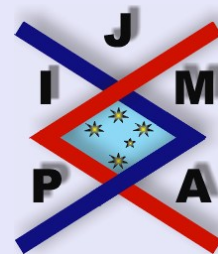
$$\frac{1}{[x_k^p + b]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \left(\frac{p-\alpha}{b}\right)^{\frac{1}{\alpha}-\frac{1}{p}} \frac{1}{x_k}, \quad 1 \leq k \leq n.$$

Finally, adding up the preceding inequalities, we obtain

$$\sum_{k=1}^n \frac{1}{[x_k^p + b]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \left(\frac{p-\alpha}{b}\right)^{\frac{1}{\alpha}-\frac{1}{p}} \sum_{k=1}^n \frac{1}{x_k} = \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \left(\frac{p-\alpha}{b}\right)^{\frac{1}{\alpha}-\frac{1}{p}} \left| \frac{a_1}{a_0} \right|$$

and (2.1) is proved.

Notice that equality holds in (2.1) if and only if equality holds in (2.2) for $1 \leq k \leq n$. Namely, equality holds when $x_k^p = a$, $1 \leq k \leq n$ or $x_k = a^{\frac{1}{p}} = \left(\frac{b\alpha}{p\beta}\right)^{\frac{1}{p}} = \left(\frac{b\alpha}{p-\alpha}\right)^{\frac{1}{p}}$. That is, when $A(x) = a_n \left(x - \left(\frac{b\alpha}{p-\alpha}\right)^{\frac{1}{p}}\right)^n$, $a_n \neq 0$. \square



Zero and Coefficient Inequalities for Hyperbolic Polynomials

J. Rubi -Masseg ,
J.L. D -Barrero and
P. Rubi -D 

Title Page

Contents



Go Back

Close

Quit

Page 5 of 14

When $\alpha > p$ changing α by $\frac{1}{\alpha}$ and p by $\frac{1}{p}$ into (2.1), we have the following:

Corollary 2.2. *If α, p and b are strictly positive real numbers such that $\alpha > p$, then*

$$\sum_{k=1}^n \frac{1}{[x_k^{\frac{1}{p}} + b]^{\alpha}} \leq \frac{p^p}{\alpha^{\alpha}} \left(\frac{\alpha - p}{b} \right)^{\alpha - p} \left| \frac{a_1}{a_0} \right|.$$

Multiplying both sides of (2.2) by $\frac{p}{\alpha}$ and raising to $\frac{1}{\alpha}$, we obtain for $1 \leq k \leq n$,

$$\left(x_k^p + \frac{p}{\alpha} \beta a \right)^{\frac{1}{\alpha}} \geq \left(\frac{p}{\alpha} \right)^{\frac{1}{\alpha}} a^{\frac{\beta}{\alpha}} x_k.$$

Setting $\beta = 1 - \frac{\alpha}{p}$, $a = \frac{b\alpha}{p\beta}$ into the preceding expression and, after adding up the resulting inequalities, we get

Corollary 2.3. *If α, p and b are strictly positive real numbers such that $\alpha < p$, then*

$$\sum_{k=1}^n (x_k^p + b)^{\frac{1}{\alpha}} \geq \frac{p^{\frac{1}{\alpha}}}{\alpha^{\frac{1}{p}}} \left(\frac{b}{p - \alpha} \right)^{\frac{1}{\alpha} - \frac{1}{p}} \left| \frac{a_{n-1}}{a_n} \right|$$

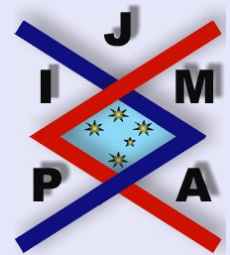
holds.

Another, immediate consequence of (2.1) is the following.

Corollary 2.4. *Let $A(x) = \sum_{k=0}^n a_k x^k$, $a_n \neq 0$, be a hyperbolic polynomial with all its zeroes x_1, x_2, \dots, x_n strictly positive. Then,*

$$\sum_{k=1}^n \frac{1}{[x_k^n + 2n - 1]^2} \leq \frac{1}{4n^2} \left| \frac{a_1}{a_0} \right|$$

holds.



Zero and Coefficient Inequalities for Hyperbolic Polynomials

J. Rubi -Masseg ,
J.L. D -Barrero and
P. Rubi -D 

Title Page

Contents



Go Back

Close

Quit

Page 6 of 14

Proof. Setting $\alpha = \frac{1}{2}$, $p = n$ and $b = 2n - 1$ into (2.1), we have

$$\begin{aligned} \sum_{k=1}^n \frac{1}{[x_k^n + 2n - 1]^2} &\leq \frac{\left(\frac{1}{2}\right)^{\frac{1}{n}}}{n^2} \left(\frac{n - \frac{1}{2}}{2n - 1}\right)^{2 - \frac{1}{n}} \left|\frac{a_1}{a_0}\right| \\ &= \frac{\left(\frac{1}{2}\right)^{\frac{1}{n}}}{n^2} \left(\frac{1}{2}\right)^{2 - \frac{1}{n}} \left|\frac{a_1}{a_0}\right| \\ &= \frac{1}{4n^2} \left|\frac{a_1}{a_0}\right|. \end{aligned}$$

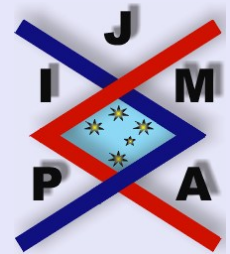
Note that equality holds when $x_k = 1$, $1 \leq k \leq n$. That is, when $A(x) = a_n(x - 1)^n$. This completes the proof. \square

Considering the reverse polynomial $A^*(x) = x^n \overline{A(1/\bar{x})} = \sum_{k=0}^n a_{n-k} x^k$, we have the following

Theorem 2.5. *If α, p and b are strictly positive real numbers such that $\alpha < p$, then*

$$(2.3) \quad \sum_{k=1}^n \left(\frac{x_k^p}{x_k^p + b}\right)^{\frac{1}{\alpha}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}} b^{\frac{1}{p}}} \cdot (p - \alpha)^{\frac{1}{\alpha} - \frac{1}{p}} \left|\frac{a_{n-1}}{a_n}\right|.$$

Equality holds when $A(x) = a_n \left(x - \left(\frac{b(p-\alpha)}{\alpha}\right)^{\frac{1}{p}}\right)^n$, $a_n \neq 0$.



Zero and Coefficient Inequalities for Hyperbolic Polynomials

J. Rubi -Masseg ,
J.L. D -Barrero and
P. Rubi -D 

Title Page

Contents



Go Back

Close

Quit

Page 7 of 14

Proof. Since $A^*(x)$ has zeros $\frac{1}{x_1}, \dots, \frac{1}{x_n}$, then applying (2.1) to it, we get

$$\sum_{k=1}^n \frac{1}{\left[\left(\frac{1}{x_k}\right)^p + b\right]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \cdot \left(\frac{p-\alpha}{b}\right)^{\frac{1}{\alpha}-\frac{1}{p}} \left|\frac{a_{n-1}}{a_n}\right|.$$

Developing the LHS of the preceding inequality, we have

$$\frac{1}{b^{\frac{1}{\alpha}}} \sum_{k=1}^n \left(\frac{x_k^p}{\frac{1}{b} + x_k^p}\right)^{\frac{1}{\alpha}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \cdot \left(\frac{p-\alpha}{b}\right)^{\frac{1}{\alpha}-\frac{1}{p}} \left|\frac{a_{n-1}}{a_n}\right|,$$

and rearranging terms, yields

$$\begin{aligned} \sum_{k=1}^n \left(\frac{x_k^p}{\frac{1}{b} + x_k^p}\right)^{\frac{1}{\alpha}} &\leq b^{\frac{1}{\alpha}} \cdot \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \cdot \left(\frac{p-\alpha}{b}\right)^{\frac{1}{\alpha}-\frac{1}{p}} \left|\frac{a_{n-1}}{a_n}\right| \\ &= \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \cdot b^{\frac{1}{p}} \cdot (p-\alpha)^{\frac{1}{\alpha}-\frac{1}{p}} \left|\frac{a_{n-1}}{a_n}\right|. \end{aligned}$$

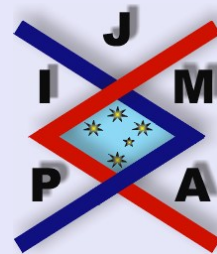
Finally, replacing b by $1/b$ in the preceding inequality we get (2.3) as claimed.

Applying Theorem 2.1, equality in (2.3) holds when

$$A^*(x) = a_n \left(x - \left(\frac{\alpha}{b(p-\alpha)}\right)^{\frac{1}{p}}\right)^n.$$

Taking into account that we have changed b by $1/b$, equality will hold if and only if

$$A(x) = a_n \left(x - \left(\frac{b(p-\alpha)}{\alpha}\right)^{\frac{1}{p}}\right)^n, \quad a_n \neq 0$$



**Zero and Coefficient
Inequalities for Hyperbolic
Polynomials**

J. Rubió-Massequé,
J.L. Díaz-Barrero and
P. Rubió-Díaz

Title Page

Contents



Go Back

Close

Quit

Page 8 of 14

and the proof is completed. □

Next, we state and prove the following:

Theorem 2.6. *Let $A(x)$ be a hyperbolic polynomial with zeros x_1, x_2, \dots, x_n such that $x_1 \leq x_2 \leq \dots \leq x_n$. Let α, p and b be strictly positive real numbers such that $\alpha < p$. If $a < x_1$ or $a > x_n$, then*

$$(2.4) \quad \sum_{k=1}^n \frac{1}{[|x_k - a|^p + b]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \left(\frac{p - \alpha}{b} \right)^{\frac{1}{\alpha} - \frac{1}{p}} \left| \frac{P'(a)}{P(a)} \right|.$$

Equality holds when

$$A(x) = a_n \left(x - \left[a + \left(\frac{b\alpha}{p - \alpha} \right)^{\frac{1}{p}} \right] \right)^n \quad \text{or}$$

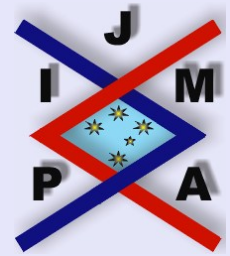
$$A(x) = a_n \left(x - \left[a - \left(\frac{b\alpha}{p - \alpha} \right)^{\frac{1}{p}} \right] \right)^n.$$

Proof. First, we observe that (2.1) applied to polynomial $P(-t)$ where $P(t)$ has all its zeros t_1, t_2, \dots, t_n negative, yields

$$(2.5) \quad \sum_{k=1}^n \frac{1}{[|t_k|^p + b]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \cdot \left(\frac{p - \alpha}{b} \right)^{\frac{1}{\alpha} - \frac{1}{p}} \left| \frac{a_1}{a_0} \right|,$$

where equality holds when

$$P(t) = a_n \left(t + \left(\frac{b\alpha}{p - \alpha} \right)^{\frac{1}{p}} \right)^n, \quad a_n \neq 0.$$



**Zero and Coefficient
Inequalities for Hyperbolic
Polynomials**

J. Rubi -Masseg ,
J.L. D az-Barrero and
P. Rubi -D az

Title Page

Contents



Go Back

Close

Quit

Page 9 of 14

Now, we consider the hyperbolic polynomial of the statement and assume that (i) $a < x_1$ or (ii) $a > x_n$. Let $B(x) = A(x + a)$, the zeros of which are $x_1 - a, x_2 - a, \dots, x_n - a$. Observe that, they are positive for case (i), and negative for case (ii). On the other hand, coefficients a_0 and a_1 of $B(x)$ are $B(0) = A(a)$ and $B'(0) = A'(a)$ respectively. Applying (2.1) to $B(x)$ in case (i) or (2.5) in case (ii) we get (2.4).

Finally, we see that equality in (2.4) holds in the case (i) when

$$B(x) = a_n \left(x - \left(\frac{b\alpha}{p - \alpha} \right)^{\frac{1}{p}} \right)^n,$$

or equivalently when

$$A(x) = a_n \left(x - \left[a + \left(\frac{b\alpha}{p - \alpha} \right)^{\frac{1}{p}} \right] \right)^n.$$

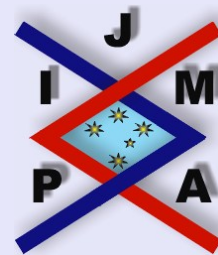
In case (ii) we will get equality when

$$B(x) = a_n \left(x + \left(\frac{b\alpha}{p - \alpha} \right)^{\frac{1}{p}} \right)^n, \quad a_n \neq 0.$$

That is, when

$$A(x) = a_n \left(x - \left[a - \left(\frac{b\alpha}{p - \alpha} \right)^{\frac{1}{p}} \right] \right)^n$$

and the proof is completed. □



Zero and Coefficient Inequalities for Hyperbolic Polynomials

J. Rubi -Masseg ,
J.L. D -Barrero and
P. Rubi -D 

Title Page

Contents



Go Back

Close

Quit

Page 10 of 14

Finally, in the sequel we will extend the result obtained in Theorem 2.1 to real polynomials whose zeros lie in the half plane $\text{Re}(z) > 0$ and they have an imaginary part “sufficiently small”. This is stated and proved in the following.

Theorem 2.7. *Let $A(z) = \sum_{k=0}^n a_k z^k$ be a polynomial with real coefficients whose zeros z_1, z_2, \dots, z_n lie in $\text{Re}(z) > 0$ and suppose that $|\text{Im}(z)| \leq r \text{Re}(z_k)$, $1 \leq k \leq n$ for some real $r \geq 0$. Let α, p and b be strictly positive real numbers such that $\alpha < p$, then*

$$(2.6) \quad \sum_{k=1}^n \frac{1}{[|z_k|^p + b]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \cdot \left(\frac{p - \alpha}{b}\right)^{\frac{1}{\alpha} - \frac{1}{p}} \cdot \sqrt{1 + r^2} \left| \frac{a_1}{a_0} \right|.$$

For $r > 0$, equality holds when n is even and

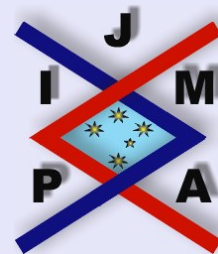
$$A(z) = \left(z^2 - \frac{2}{\sqrt{1 + r^2}} \cdot \left(\frac{b\alpha}{p - \alpha}\right)^{\frac{1}{p}} z + \left(\frac{b\alpha}{p - \alpha}\right)^{\frac{2}{p}} \right)^{\frac{n}{2}}.$$

Note that when $r = 0$ the preceding result reduces to (2.1).

Proof. Setting $x_k = |z_k|$ and repeating the procedure followed in proving (2.1), we get

$$\sum_{k=1}^n \frac{1}{[|z_k|^p + b]^{\frac{1}{\alpha}}} \leq \frac{\alpha^{\frac{1}{p}}}{p^{\frac{1}{\alpha}}} \cdot \left(\frac{p - \alpha}{b}\right)^{\frac{1}{\alpha} - \frac{1}{p}} \sum_{k=1}^n \frac{1}{|z_k|}.$$

Next, we will find an upper bound for the sum $S = \sum_{k=1}^n \frac{1}{|z_k|}$. Reordering the zeros of $A(z)$ in the way $z_1, \bar{z}_1, z_2, \bar{z}_2, \dots, z_s, \bar{z}_s, x_1, \dots, x_t$, where x_1, \dots, x_t are



Zero and Coefficient Inequalities for Hyperbolic Polynomials

J. Rubió-Massequé,
J.L. Díaz-Barrero and
P. Rubió-Díaz

Title Page

Contents



Go Back

Close

Quit

Page 11 of 14

the real zeros (if any), then the preceding sum becomes

$$S = 2 \sum_{k=1}^s \frac{1}{|z_k|} + \sum_{k=1}^t \frac{1}{|x_k|} = 2 \sum_{k=1}^s \frac{|z_k|}{|z_k|^2} + \sum_{k=1}^t \frac{1}{|x_k|}.$$

On the other hand, by Cardan-Viète formulae, we have

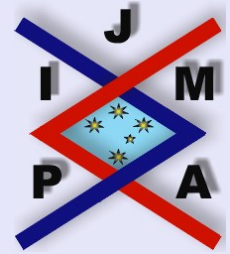
$$-\frac{a_1}{a_0} = \sum_{k=1}^s \left[\frac{1}{z_k} + \frac{1}{\bar{z}_k} \right] + \sum_{k=1}^t \frac{1}{x_k} = 2 \sum_{k=1}^s \frac{\operatorname{Re} z_k}{|z_k|^2} + \sum_{k=1}^t \frac{1}{x_k}.$$

Taking into account that $|z_k| = \sqrt{(\operatorname{Re} z_k)^2 + (\operatorname{Im} z_k)^2} \leq \sqrt{1+r^2} |\operatorname{Re} z_k|$ and the fact that the zeros of $A(z)$ lie in $\operatorname{Re}(z) > 0$, yields

$$\begin{aligned} (2.7) \quad S &= 2 \sum_{k=1}^s \frac{|z_k|}{|z_k|^2} + \sum_{k=1}^t \frac{1}{|x_k|} \\ &\leq 2\sqrt{1+r^2} \sum_{k=1}^s \frac{|\operatorname{Re} z_k|}{|z_k|^2} + \sum_{k=1}^t \frac{1}{|x_k|} \\ &\leq \sqrt{1+r^2} \left(2 \sum_{k=1}^s \frac{|\operatorname{Re} z_k|}{|z_k|^2} + \sum_{k=1}^t \frac{1}{|x_k|} \right) \\ &= \sqrt{1+r^2} \left| \frac{a_1}{a_0} \right|, \end{aligned}$$

from which (2.6) immediately follows.

Next, we will see when equality holds in (2.6). If $r > 0$, to get equality in (2.6) we require that (i) all the zeros of $A(z)$ have modulus $|z_k| = \left(\frac{b\alpha}{p-\alpha} \right)^{\frac{1}{p}}$,



**Zero and Coefficient
Inequalities for Hyperbolic
Polynomials**

J. Rubi -Masseg ,
J.L. D -Barrero and
P. Rubi -D 

Title Page

Contents



Go Back

Close

Quit

Page 12 of 14

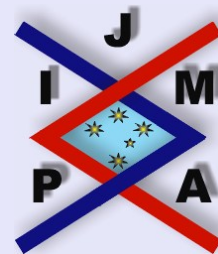
because when $x_k = |z_k|$ the powered GM-AM inequality (2.2) must become equality, (ii) $|\operatorname{Im} z_k| = r \operatorname{Re} z_k$, $1 \leq k \leq s$, due to the fact that the inequality in (2.7) must become equality, and (iii) all the zeros of $A(z)$ must be complex because the second inequality in (2.7) also must be an equality. Now it is easy to see that the previous conditions are equivalent to say that n is even and

$$z_k = \frac{1}{\sqrt{1+r^2}} \left(\frac{b\alpha}{p-\alpha} \right)^{\frac{1}{p}} [1+ri], \quad 1 \leq k \leq \frac{n}{2}.$$

Multiplying the preceding zeros we get that inequality in (2.6) holds when n is even and

$$A(z) = \left(z^2 - \frac{2}{\sqrt{1+r^2}} \left(\frac{b\alpha}{p-\alpha} \right)^{\frac{1}{p}} z + \left(\frac{b\alpha}{p-\alpha} \right)^{\frac{2}{p}} \right)^{\frac{n}{2}}.$$

This completes the proof. □



**Zero and Coefficient
Inequalities for Hyperbolic
Polynomials**

J. Rubi -Masseg ,
J.L. D -Barrero and
P. Rubi -D 

Title Page

Contents



Go Back

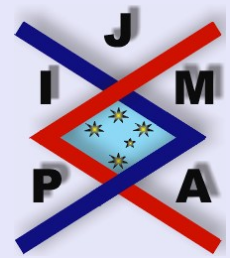
Close

Quit

Page 13 of 14

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Title Page

Contents



Go Back

Close

Quit

Page 14 of 14