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Abstract

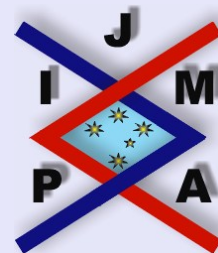
We consider the convolution $P(A, B) \star P(C, D)$ of the classes of analytic functions subordinated to the homographies $\frac{1+Az}{1-Bz}$ and $\frac{1+Cz}{1-Dz}$ respectively, where A, B, C, D are some complex numbers. In 1988 J. Stankiewicz and Z. Stankiewicz [11] showed that for certain A, B, C, D there exist X, Y such that $P(A, B) \star P(C, D) \subset P(X, Y)$. In this paper we verify the conjecture that $P(X, Y) \subset (A, B) \star P(C, D)$ for some A, B, C, D, X, Y .

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1. Introduction

Let $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ denote the open unit disc and let \mathcal{H} be the class of functions regular in Δ . We will denote by \mathcal{N} the class of functions $f \in \mathcal{H}$ normalized by $f(0) = 1$. The class of Schwarz functions Ω is the class of functions $\omega \in \mathcal{H}$, such that $\omega(0) = 0$ and $|\omega(z)| < 1$ for $z \in \Delta$. We say that a function f is subordinate to a function g in Δ (and write $f \prec g$ or $f(z) \prec g(z)$) if there exists a function $\omega \in \Omega$ such that $f(z) = g(\omega(z)); z \in \Delta$. If the function g is univalent in Δ , then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$. In this case we have $\omega(z) = g^{-1}(f(z))$.

Let the functions f and g be of the form

$$f(z) = \sum_{n=0}^{\infty} a_n z^n, \quad g(z) = \sum_{n=0}^{\infty} b_n z^n, \quad z \in \Delta.$$

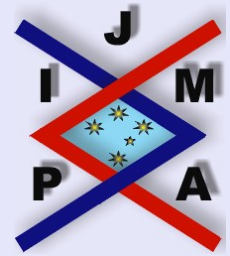
We say that the Hadamard product of f and g is the function $f \star g$ if

$$(f \star g)(z) = f(z) \star g(z) = \sum_{n=0}^{\infty} a_n b_n z^n.$$

J. Hadamard [2] proved, that the radius of convergence of $f \star g$ is the product of the radii of convergence of the corresponding series f and g . The function $f \star g$ is also called the *convolution* of the functions f and g .

For the classes $Q_1 \subset \mathcal{H}$ and $Q_2 \subset \mathcal{H}$ the convolution $Q_1 \star Q_2$ is defined as

$$Q_1 \star Q_2 = \{h \in \mathcal{H}; h = f \star g, f \in Q_1, g \in Q_2\}.$$



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The problem of connections between functions f, g and their convolution $f \star g$ or between Q_1, Q_2 and $Q_1 \star Q_2$ has often been investigated. Many conjectures have been given, however, many of them have still not been verified.

In 1958, G. Pólya and I.J. Schoenberg [7] conjectured that the Hadamard product of two convex mappings is a convex mapping. In 1961 H.S. Wilf [12] gave the more general supposition that if F and G are convex mappings in Δ and f is subordinate to F , then the convolution $f \star G$ is subordinate to $F \star G$.

In 1973, S. Ruscheweyh and T. Sheil-Small [9] proved both conjectures and more results of this type. Their very important results we may write as:

Theorem A. *If $f \in S^c$ and $g \in S^c$, then $f \star g \in S^c$, where S^c is the class of univalent and convex functions. Moreover $S^c \star S^c = S^c$.*

Theorem B. *If $F \in S^c, G \in S^c$ and $f \prec F$, then $f \star G \prec F \star G$.*

In 1985, S. Ruscheweyh and J. Stankiewicz [10] proved some generalizations of Theorems A and B:

Theorem C. *If the functions F and G are univalent and convex in Δ , then for all functions f and g , if $f \prec F$ and $g \prec G$ then $f \star g \prec F \star G$.*

For the given complex numbers A, B such that $A + B \neq 0$ and $|B| \leq 1$ let us denote

$$P(A, B) = \left\{ f \in \mathcal{N} : f(z) \prec \frac{1 + Az}{1 - Bz} \right\}.$$

W. Janowski introduced the class $P(A, B)$ in [3] and considered it for some real A and B . If $A = B = 1$ then the class $P(1, 1)$ is the class of functions with positive real part (Carathéodory functions). Note, that for $|B| < 1$ the



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class $P(A, B)$ is the class of bounded functions. In 1988 J. Stankiewicz and Z. Stankiewicz [11] investigated the convolution properties of the class $P(A, B)$ and proved the following theorem:

Theorem D. *If A, B, C, D are some complex numbers such that $A + B \neq 0$, $C + D \neq 0$, $|B| \leq 1$, $|D| \leq 1$, then*

$$P(A, B) \star P(C, D) \subset P(AC + AD + BC, BD),$$

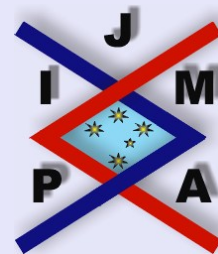
moreover, if $|B| = 1$ or $|D| = 1$, then

$$P(A, B) \star P(C, D) = P(AC + AD + BC, BD).$$

The equality between the class $P(A, B) \star P(C, D)$ and the class $P(AC + AD + BC, BD)$ for $|B| < 1$ and $|D| < 1$ was an open problem. In [5] K. Piejko, J. Sokół and J. Stankiewicz proved that the above mentioned classes are different. In this paper we give an extension of this result. First we need two theorems.

Theorem E (G. Eneström [1], S. Kakeya [4]). *If $a_0 > a_1 > \dots > a_n > 0$, where $n \in \mathbb{N}$, then the polynomial $p(z) = a_0 + a_1z + a_2z^2 + \dots + a_nz^n$ has no roots in $\bar{\Delta} = \{z \in \mathbb{C} : |z| \leq 1\}$.*

Theorem F (W. Rogosinski [8]). *If the function $f(z) = \sum_{n=0}^{\infty} \alpha_n z^n$ is subordinated to the function $F(z) = \sum_{n=0}^{\infty} \beta_n z^n$ in Δ , then $\sum_{n=0}^{\infty} |\alpha_n|^2 \leq \sum_{n=0}^{\infty} |\beta_n|^2$.*



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2. Main Result

We prove the following theorem.

Theorem 2.1. *Let A, B, C, D be some complex numbers such that $B + A \neq 0$, $C + D \neq 0$, $|B| < 1$, $|D| < 1$, then there are not complex numbers X, Y , $X + Y \neq 0$, $|Y| \leq 1$ such that $P(X, Y) \subset P(A, B) \star P(C, D)$.*

Proof. As in [5], the proof will be divided into three steps. First we give a family of bounded functions ω^ν ; $\nu = 1, 2, 3, \dots$ having special properties of coefficients. Afterwards we construct, using ω^ν , a function belonging to the class $P(X, Y)$ and finally we will show that such a function is not in the class $P(A, B) \star P(C, D)$.

Now we use a method of E. Landau [6] and find some functions ω^ν , $\nu = 1, 2, 3, \dots$, which are basic in this proof. We observe that

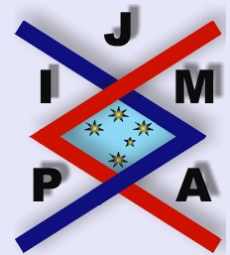
$$\frac{1}{1-z} = \left(\frac{1}{\sqrt{1-z}} \right)^2 = \left(\sum_{k=0}^{\infty} p_k z^k \right)^2 = 1 + z + z^2 + z^3 + \dots,$$

where

$$(2.1) \quad p_0 = 1 \quad \text{and} \quad p_k = \frac{1 \cdot 3 \cdot \dots \cdot (2k-1)}{2 \cdot 4 \cdot \dots \cdot 2k}; \quad k = 1, 2, 3, \dots$$

For some $\nu \in \mathbb{N}$ we set

$$K_\nu(z) = \sum_{k=0}^{\nu} p_k z^k = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \dots + p_\nu z^\nu$$



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and note that

$$K_\nu^2(z) = 1 + z + z^2 + \cdots + z^\nu + b_{\nu+1}z^{\nu+1} + \cdots + b_{2\nu}z^{2\nu},$$

where $b_{\nu+1}, \dots, b_{2\nu} \in \mathbb{C}$. Let ω^ν be given by

$$(2.2) \quad \omega^\nu(z) = \frac{z^{\nu+1}K_\nu\left(\frac{1}{z}\right)}{K_\nu(z)} = z \frac{z^\nu + p_1z^{\nu-1} + p_2z^{\nu-2} + \cdots + p_\nu}{1 + p_1z + p_2z^2 + \cdots + p_\nu z^\nu}.$$

Since

$$p_k > p_k \frac{2k+1}{2k+2} = p_{k+1} \quad \text{where } k = 0, 1, 2, 3, \dots,$$

then for $\nu \in \mathbb{N}$ we have $1 > p_1 > p_2 > p_3 > \cdots > p_\nu > 0$.

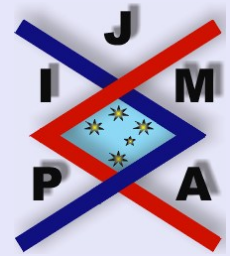
Applying Theorem **E** to the polynomial K_ν we obtain $K_\nu(z) \neq 0$ for $|z| \leq 1$, hence the function ω^ν is regular in Δ . Moreover $\omega^\nu(0) = 0$ and on the circle $|z| = 1$ we have

$$|\omega^\nu(e^{it})| = \left| \frac{e^{(\nu+1)it}K_\nu(e^{-it})}{K_\nu(e^{it})} \right| = \frac{|K_\nu(e^{-it})|}{|K_\nu(e^{it})|} = 1; \quad t \in \mathbb{R}.$$

In this way we conclude that for $\nu \in \{1, 2, 3, \dots\}$

$$(2.3) \quad \omega^\nu \in \Omega.$$

Let, for a certain $\nu \in \mathbb{N}$, the function ω^ν be represented by following power series expansions: $\omega^\nu(z) = \gamma_1^\nu z + \gamma_2^\nu z^2 + \gamma_3^\nu z^3 + \cdots$ and let $s_n^\nu(z)$ denote the



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partial sum

$$s_n^\nu(z) = \sum_{k=1}^n \gamma_k^\nu z^k = \gamma_1^\nu z + \gamma_2^\nu z^2 + \gamma_3^\nu z^3 + \cdots + \gamma_n^\nu z^n$$

for all $n \in \{1, 2, 3, \dots, \nu + 1\}$.

Now we will estimate $s_n^\nu(1) = \gamma_1^\nu + \gamma_2^\nu + \gamma_3^\nu + \cdots + \gamma_n^\nu$. If we integrate on a circle $C : |z| = r$ in a counterclockwise direction with $0 < r < 1$, then we obtain

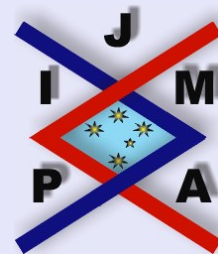
$$(2.4) \quad \int_C az^m dz = 0 \quad \text{and} \quad \int_C \frac{a}{z} dz = 2\pi ai,$$

for all integers $m \neq -1$ and $a \in \mathbb{C}$. Hence

$$\begin{aligned} s_n^\nu(1) &= \gamma_1^\nu + \gamma_2^\nu + \gamma_3^\nu + \cdots + \gamma_n^\nu \\ &= \frac{1}{2\pi i} \int_C \omega^\nu(z) \left(\frac{1}{z^2} + \frac{1}{z^3} + \frac{1}{z^4} + \cdots + \frac{1}{z^{n+1}} \right) dz \\ &= \frac{1}{2\pi i} \int_C \frac{\omega^\nu(z)}{z^{n+1}} (1 + z + z^2 + z^3 + \cdots + z^{n-1}) dz \\ &= \frac{1}{2\pi i} \int_C \frac{\omega^\nu(z)}{z^{n+1}} Q(z) dz, \end{aligned}$$

where

$$Q(z) = 1 + z + z^2 + \cdots + z^{n-1} + d_n z_n + d_{n+1} z^{n+1} + \cdots + d_s z^s$$



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is any polynomial whose first n terms are equal to $1 + z + z^2 + \dots + z^{n-1}$.

If for $n \leq \nu + 1$ we set

$$Q(z) = K_\nu^2(z) = 1 + z + z^2 + \dots + z^{n-1} + z^n + \dots + z^\nu + b_{\nu+1}z^{\nu+1} + \dots + b_{2\nu}z^{2\nu},$$

then we obtain

$$s_n^\nu(1) = \frac{1}{2\pi i} \int_C \frac{\omega^\nu(z)}{z^{n+1}} K_\nu^2(z) dz.$$

From (2.2) it follows that

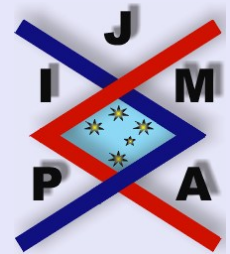
$$\begin{aligned} s_n^\nu(1) &= \frac{1}{2\pi i} \int_C \frac{z^{\nu+1} K_\nu\left(\frac{1}{z}\right)}{z^{n+1} K_\nu(z)} K_\nu^2(z) dz \\ &= \frac{1}{2\pi i} \int_C z^{\nu-n} K_\nu\left(\frac{1}{z}\right) K_\nu(z) dz \\ &= \frac{1}{2\pi i} \int_C z^{\nu-n} \left(1 + p_1 \frac{1}{z} + p_2 \frac{1}{z^2} + \dots + p_\nu \frac{1}{z^\nu}\right) \\ &\quad \times (1 + p_1 z + p_2 z^2 + \dots + p_\nu z^\nu) dz. \end{aligned}$$

Using (2.4) we get

$$s_n^\nu(1) = p_{\nu-n+1} + p_{\nu-n+2}p_1 + p_{\nu-n+3}p_2 + \dots + p_\nu p_{n-1},$$

where (p_k) are given by (2.1), and so

$$(2.5) \quad s_n^\nu(1) = \sum_{k=1}^n \gamma_k^\nu = \sum_{k=1}^n p_{\nu-n+k} p_{k-1}.$$



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By the above we have $\gamma_1^\nu = s_1^\nu(1) = p_\nu$ and for all $\nu \in \{1, 2, 3, \dots\}$ and $n \in \{2, 3, 4, \dots, \nu + 1\}$

$$\gamma_n^\nu = s_n^\nu(1) - s_{n-1}^\nu(1) = \sum_{k=1}^n p_{\nu-n+k} p_{k-1} - \sum_{k=1}^{n-1} p_{\nu-n+1+k} p_{k-1}.$$

Therefore

$$(2.6) \quad \gamma_n^\nu = \sum_{k=1}^{n-1} (p_{\nu-n+k} - p_{\nu-n+1+k}) p_{k-1} + p_\nu p_{n-1}.$$

The sequence (p_n) is positive and decreasing, then from (2.6) it follows that

$$(2.7) \quad \gamma_n^\nu > 0 \quad \text{for all } \nu \in \mathbb{N} \quad \text{and} \quad n \in \{1, 2, 3, \dots, \nu + 1\}.$$

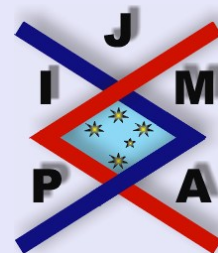
We conclude from (2.5) that for $n = \nu + 1$

$$s_{\nu+1}^\nu(1) = \gamma_1^\nu + \gamma_2^\nu + \dots + \gamma_\nu^\nu + \gamma_{\nu+1}^\nu = \sum_{k=1}^{\nu+1} p_{k-1} p_{k-1} = 1 + \sum_{k=1}^{\nu} p_k^2.$$

Since $\sum_{k=1}^{\infty} p_k^2 = \infty$ then

$$(2.8) \quad \lim_{\nu \rightarrow \infty} s_{\nu+1}^\nu(1) = \lim_{\nu \rightarrow \infty} \left(1 + \sum_{k=1}^{\nu} p_k^2 \right) = +\infty.$$

Now using the properties of ω^ν we construct the function belonging to the class $P(X, Y)$ which is not in the class $P(A, B) \star P(C, D)$, where $|B| < 1$ and $|D| < 1$.



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Since for $|x| = 1$ we have $P(A, B) = P(Ax, Bx)$, we can assume without loss of generality that $B \in [0; 1)$, $D \in [0; 1)$ and $Y \in [0; 1]$.

For fixed $\nu \in \mathbb{N}$ let h^ν be given by

$$(2.9) \quad h^\nu(z) = \frac{1 + X\omega^\nu(z)}{1 - Y\omega^\nu(z)},$$

where $w^\nu \in \Omega$ is given by (2.2). It is clearly that $h^\nu \in P(X, Y)$. Suppose that there exist the functions $f \in P(A, B)$, $g \in P(C, D)$ such that

$$(2.10) \quad f(z) \star g(z) = h^\nu(z).$$

Let the functions f and g have the following form:

$$f(z) = \frac{1 + A\omega_1(z)}{1 - B\omega_1(z)} = 1 + (A + B) \frac{\omega_1(z)}{1 - B\omega_1(z)}$$

and

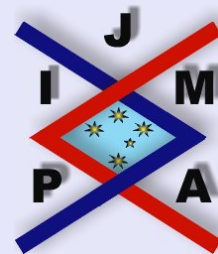
$$g(z) = \frac{1 + C\omega_2(z)}{1 - D\omega_2(z)} = 1 + (C + D) \frac{\omega_2(z)}{1 - D\omega_2(z)},$$

where $\omega_1, \omega_2 \in \Omega$. For simplicity of notation we write

$$f(z) = 1 + (A + B)\tilde{f}(z), \quad g(z) = 1 + (C + D)\tilde{g}(z),$$

where

$$(2.11) \quad \tilde{f}(z) = \frac{\omega_1(z)}{1 - B\omega_1(z)} \quad \text{and} \quad \tilde{g}(z) = \frac{\omega_2(z)}{1 - D\omega_2(z)}.$$



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Using these notations we can rewrite (2.10) as

$$\left[1 + (A + B)\tilde{f}(z)\right] \star \left[1 + (C + D)\tilde{g}(z)\right] = \frac{1 + X\omega^\nu(z)}{1 - Y\omega^\nu(z)}$$

and so

$$(2.12) \quad \tilde{f}(z) \star \tilde{g}(z) = \frac{X + Y}{(A + B)(C + D)} \tilde{h}^\nu(z),$$

where $\tilde{h}^\nu(z) = \frac{\omega^\nu(z)}{1 - BD\omega^\nu(z)}$.

Let the functions \tilde{h}^ν , \tilde{f} and \tilde{g} have the following expansions in Δ :

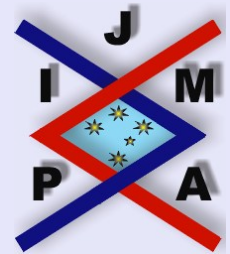
$$(2.13) \quad \tilde{h}^\nu(z) = \sum_{n=1}^{\infty} c_n^\nu z^n, \quad \tilde{f}(z) = \sum_{n=1}^{\infty} a_n z^n, \quad \tilde{g}(z) = \sum_{n=1}^{\infty} b_n z^n.$$

From (2.11) it follows, that $\tilde{f}(z) \prec \frac{z}{1 - Bz}$ and $\tilde{g}(z) \prec \frac{z}{1 - Dz}$, and hence by Theorem F (and since $0 \leq B < 1$ and $0 \leq D < 1$) we obtain

$$\sum_{n=1}^{\infty} |a_n|^2 \leq \frac{1}{1 - B^2} \quad \text{and} \quad \sum_{n=1}^{\infty} |b_n|^2 \leq \frac{1}{1 - D^2}.$$

Let us note, that

$$\begin{aligned} \sum_{n=1}^{\infty} (|a_n|^2 + |b_n|^2) &= \sum_{n=1}^{\infty} [(|a_n| - |b_n|)^2 + 2|a_n||b_n|] \\ &\leq \frac{1}{1 - |B|^2} + \frac{1}{1 - |D|^2}. \end{aligned}$$



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From (2.12) and (2.13) we obtain

$$a_n b_n = \frac{X + Y}{(A + B)(C + D)} c_n^\nu, \quad \text{for } n = 1, 2, 3, \dots,$$

therefore

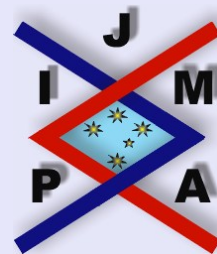
$$(2.14) \quad \sum_{n=1}^{\infty} (|a_n| - |b_n|)^2 \leq \frac{1}{1 - B^2} + \frac{1}{1 - D^2} - \left| \frac{2(X + Y)}{(A + B)(C + D)} \right| \sum_{n=1}^{\infty} |c_n^\nu|.$$

Now we observe that

$$\begin{aligned} \sum_{n=1}^{\infty} c_n^\nu z^n &= \tilde{h}^\nu(z) = \frac{\omega^\nu(z)}{1 - Y\omega^\nu(z)} \\ &= \omega^\nu(z) + Y [\omega^\nu(z)]^2 + Y^2 [\omega^\nu(z)]^3 + \dots \\ &= (\gamma_1^\nu z + \gamma_2^\nu z^2 + \dots) + Y (\gamma_1^\nu z + \gamma_2^\nu z^2 + \dots)^2 \\ &\quad + Y^2 (\gamma_1^\nu z + \gamma_2^\nu z^2 + \dots)^3 + \dots \\ &= \gamma_1^\nu z + (\gamma_2^\nu + Y(\gamma_1^\nu)^2) z^2 + (\gamma_3^\nu + 2Y\gamma_1^\nu\gamma_2^\nu + Y^2(\gamma_1^\nu)^3) z^3 + \dots \end{aligned}$$

Since $Y \in [0; 1]$ and since (2.7) we have

$$\begin{aligned} \sum_{n=1}^{\infty} |c_n^\nu| &= |\gamma_1^\nu| + |\gamma_2^\nu + Y(\gamma_1^\nu)^2| + |\gamma_3^\nu + 2Y\gamma_1^\nu\gamma_2^\nu + Y^2(\gamma_1^\nu)^3| + \dots \\ &\geq \sum_{n=1}^{\nu+1} |c_n^\nu| \geq \gamma_1^\nu + \gamma_2^\nu + \gamma_3^\nu + \dots + \gamma_{\nu+1}^\nu = s_{\nu+1}^\nu(1). \end{aligned}$$



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From the above we have for all $\nu \in \{1, 2, 3, \dots\}$

$$(2.15) \quad \sum_{n=1}^{\infty} |c_n^\nu| \geq s_{\nu+1}^\nu(1).$$

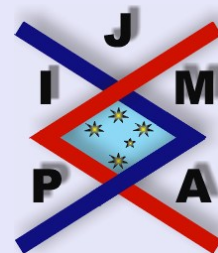
Combining (2.14) and (2.15) we obtain

$$(2.16) \quad \sum_{n=1}^{\infty} (|a_n| - |b_n|)^2 \leq \frac{1}{1-B^2} + \frac{1}{1-D^2} - \left| \frac{2(X+Y)}{(A+B)(C+D)} \right| s_{\nu+1}^\nu(1).$$

From (2.8) it follows that we are able to choose a suitable ν such that the right side of (2.16) is negative. In this way (2.16) follows the contradiction and the proof is complete. \square

From Theorem 2.1 and Theorem D we immediately have the following

Corollary 2.2. *The classes $P(A, B) \star P(C, D)$ and $P(AD + AC + BC, BD)$ are equal if and only if $|B| = 1$ or $|D| = 1$.*



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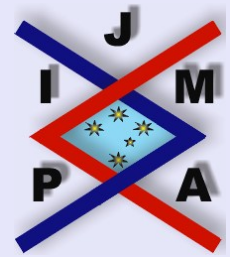
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