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The Atiyah Extension of a Lie Algebra Deformation

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Abstract. We construct an analogue of the 'Atiyah class', or 'Atiyah extension' in the context of deformation theory of sheaves of Lie algebras. Given a (suitable) sheaf $\mathfrak{g}$ of Lie algebras on a space X, and a $\mathfrak{g}$-deformation ϕ over a formally smooth base, our Atiyah extension is a Lie algebra extension of the algebra of base vector fields by the ϕ-twist of $\mathfrak{g}$. It comes equipped with a representation of the ϕ-twist on any (admissible) $\mathfrak{g}$-module.
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The role of the Atiyah class (also known as Atiyah Chern class or Atiyah extension) in the study of vector bundles on manifolds is well known (see [3] for just one example and [6] for a discussion of that example from a viewpoint close to that of this note). The purpose of this note is to define an analogue (in fact, a generalization) of the Atiyah extension in the context of the deformation theory of Lie algebras. Given a sheaf of Lie algebras $\mathfrak{g}$ on a space X, a $\mathfrak{g}$-deformation ϕ (as reviewed below), say parametrized by a formally smooth local algebra $\mathcal{S}^\wedge$, simultaneously determines deformations E^ϕ for all $\mathfrak{g}$-modules E (including the adjoint module $\mathfrak{g}$ itself, which yields a Lie algebra $\mathfrak{g}^\phi$). Our Atiyah extension is a Lie algebra extension of the algebra of base vector fields by $\mathfrak{g}^\phi$, and comes equipped with a representation on E^ϕ for any $\mathfrak{g}$-module E. This yields an interesting class of Lie algebras and representations. For X a smooth curve, some related notions were studied by Beilinson and Schechtman [2] in connection with Virasoro algebras. Even in the classical context of vector bundles, the realization of the Atiyah extension as a Lie algebra is apparently new.

To proceed with the basic definitions, let

$$f : X_B \to B$$

be a continuous mapping of Hausdorff spaces with fibres $X_b = f^{-1}(b)$ and base B, which we assume endowed with a sheaf of local $\mathbb{C}$-algebras $\mathcal{O}_B$. A *Lie*

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pair $(\mathfrak{g}_B, E_B)$ on X_B/S consists of a sheaf $\mathfrak{g}_B$ of $f^{-1}\mathcal{O}_B$-Lie algebras (i.e. with $f^{-1}\mathcal{O}_B$- linear bracket), a sheaf E_B of $f^{-1}\mathcal{O}_B$-modules with $f^{-1}\mathcal{O}_B$-linear $\mathfrak{g}_B$-action. This pair is said to be *admissible* if it admits compatible soft resolutions $(\mathfrak{g}^{\cdot}_B, E^{\cdot}_B)$ such that $\mathfrak{g}^{\cdot}_B$ is a differential graded Lie algebra and $E^{\cdot}_B$ is a differential graded representation of $\mathfrak{g}^{\cdot}_B$, and moreover, $\Gamma(\mathfrak{g}^{\cdot}_B), \Gamma(E^{\cdot}_B)$ may be linearly topologized so that coboundaries (and cocycles) are closed, and the cohomology is of finite type as $\mathcal{O}_B$-module (and in particular vanishes in almost all degrees). Let's call such resolutions *good*. Note that if $(\mathfrak{g}^{\cdot}_B, E^{\cdot}_B)$ is an admissible pair then for any $b \in B$ the 'fibre'

$$(\mathfrak{g}_b, E_b) := (\mathfrak{g}_B, E_B) \otimes (\mathcal{O}_{B,b}/\mathfrak{m}_{B,b})$$

is an admissible pair on X_b.

Now let $\mathcal{S}$ be an augmented $\mathcal{O}_B$-algebra of finite type as $\mathcal{O}_B$-module, with maximal ideal $\mathfrak{m}_{\mathcal{S}}$ (below we shall also consider the case where $\mathcal{S}$ is an inverse limit of such algebras, hence is complete noetherian rather than finite type). By a *relative* $\mathfrak{g}_B$*-deformation of* E_B*, parametrized by* $\mathcal{S}$ we mean a sheaf E^{ϕ}_B of $\mathcal{S}$-modules on X_B, together with a maximal atlas of the following data

- An open covering (U_α) of X_B.
- $\mathcal{S}$-isomorphisms $\Phi_\alpha : E^{\phi}|_{U_\alpha} \xrightarrow{\sim} E|_{U_\alpha} \otimes_{\mathcal{O}_B} \mathcal{S}$.
- For each α, β, a lifting of $\Phi_\beta \circ \Phi_\alpha^{-1} \in \ Aut(E|_{U_\alpha \cap U_\beta} \otimes_{\mathcal{O}_B} \mathcal{S})$ to an element $\Psi_{\alpha,\beta} \in \exp(\mathfrak{g}_B \otimes \mathfrak{m}_{\mathcal{S}}(U_\alpha \cap U_\beta))$. If $\mathfrak{g}_B$ acts faithfully on E_B then the $\Psi_{\alpha,\beta}$ are uniquely determined by the Φ_α and form a cocycle; in general we require additionally that the $\Psi_{\alpha,\beta}$ form a cocycle.

Note that if $(\mathfrak{g}_B, E_B)$ is admissible then, as in the absolute case, for any relative deformation ϕ there is a good resolution $(E^{\cdot}, \partial)$ of E and a resolution of E^{ϕ} of the form

$$E^0 \otimes_{\mathcal{O}_B} \mathcal{S} \xrightarrow{\partial+\phi} E^1 \otimes_{\mathcal{O}_B} \mathcal{S} \cdots \tag{1}$$

with

$$\phi \in \Gamma(\mathfrak{g}^1_B) \otimes \mathfrak{m}_{\mathcal{S}}.$$

We call such a resolution a *good resolution* of E^{ϕ}. Let $(\mathfrak{g}_B, E_B)$ be an admissible pair on X_B/B, $\mathcal{S}$ a finite-length $\mathcal{O}_B$-algebra, and E^{ϕ} an admissible $\mathfrak{g}_B$-deformation parametrized by $\mathcal{S}$. There is a corresponding deformation $\mathfrak{g}^{\phi}$, and clearly $\mathfrak{g}^{\phi}$ is a Lie algebra acting on E^{ϕ}. We ignore momentarily the status of $\mathcal{E}^{\phi}$ as a deformation and just view it as a $\mathfrak{g}^{\phi}$-module over

$$X_{\mathcal{S}} = X_B \times_B \mathrm{Spec}(\mathcal{S}).$$

Let $\mathrm{Spec}(\mathcal{S}_1)$ be the first infinitesimal neighborhood of the diagonal in

$$\mathrm{Spec}(\mathcal{S}) \times_{\mathcal{O}_B} \mathrm{Spec}(\mathcal{S}) = \mathrm{Spec}_{\mathcal{O}_B}(\mathcal{S} \otimes_{\mathcal{O}_B} \mathcal{S})$$

with projections

$$p, q : \mathrm{Spec}(\mathcal{S}_1) \to \mathrm{Spec}(\mathcal{S}).$$

Then $p_* q^* E^\phi$ may be viewed as a first-order $\mathfrak{g}^\phi$-deformation of E^ϕ and we let

$$AC(\phi) \in H^1(\mathfrak{g}^\phi \otimes \mathfrak{m}_{\mathcal{S}_1}) = H^1(\mathfrak{g}^\phi \otimes_{\mathcal{S}} \Omega_{\mathcal{S}/B}) \tag{2}$$

be the associated (first-order) Kodaira-Spencer class. We call $AC(\phi)$ the *Atiyah class* of the deformation ϕ.

A cochain representative for $AC(\phi)$ may be constructed as follows. Let

$$\phi \in \Gamma(\mathfrak{g}^1) \otimes \mathfrak{m}_{\mathcal{S}}$$

be a Kodaira-Spencer cochain corresponding to E^ϕ, satisfying the integrability condition

$$\partial\phi = -\frac{1}{2}[\phi, \phi].$$

Let

$$d_{\mathcal{S}} : \Gamma(\mathfrak{g}^1) \otimes \mathfrak{m}_{\mathcal{S}} \to \Gamma(\mathfrak{g}^1) \otimes \Omega_{\mathcal{S}/B}$$

be the map induced by exterior derivative on $\mathfrak{m}_{\mathcal{S}}$. Set

$$\psi = d_{\mathcal{S}}(\phi). \tag{3}$$

Then

$$AC(\phi) = [\psi].$$

Note that differentiating the integrability condition for ϕ yields

$$\partial\psi = -[\phi, \psi].$$

Since $(\mathfrak{g}^\cdot, \partial + \mathrm{ad}(\phi))$ is a resolution of $\mathfrak{g}^\phi$, this means that ψ is a cocycle for $\mathfrak{g}^\phi$.

Example 1. Let X_B/B be a family of complex manifolds and let E be a vector bundle on X_B with a $\mathfrak{g}$-structure. To recall what that means, let $G(E) = ISO(\mathbb{C}^r, E)$, $r = \mathrm{rk}(E)$ be the associated principal bundle, i.e. the open subset of the geometric vector bundle $\mathfrak{hom}(\mathbb{C}^r, E)$ consisting of fibrewise isomorphisms, with the obvious action of GL_r. Let $\mathfrak{D}(E)$ be the sheaf of GL_r-invariant vector fields on $G(E)$, which may also be identified as the sheaf of relative derivations of $(E, \mathcal{O}_X)$, consisting of pairs $(v, a), v \in T_{X_B/B}, a \in Hom_{\mathbb{C}}(E, E)$ such that

$$a(fe) = fa(e) + v(f)e, \ \forall f \in \mathcal{O}_X, e \in E.$$

Note that $\mathfrak{D}(E)$ is an extension of Lie algebras

$$0 \to \mathfrak{gl}(E) \to \mathfrak{D}(E) \to T_{X_B/B} \to 0 \tag{4}$$

Then a $\mathfrak{g}$-structure on E is a Lie subalgebra $\hat{\mathfrak{g}} \subseteq \mathfrak{D}(E)$ which fits in an exact sequence

$$\begin{array}{ccccccccc} 0 & \to & \mathfrak{g} & \to & \hat{\mathfrak{g}} & \to & T_{X_B/B} & \to & 0 \\ & & \cap & & \cap & & \| & & \\ 0 & \to & \mathfrak{gl}(E) & \to & \mathfrak{D}(E) & \to & T_{X_B/B} & \to & 0. \end{array}$$

Note that in this case a maximal integral submanifold $\hat{G}$ of $\hat{\mathfrak{g}}$ yields a principal subbundle of $G(E)$ with structure group $G = \exp(\mathfrak{g})$ and conversely such a principal subbundle with Lie algebra $\mathfrak{g}$ yields a $\mathfrak{g}$-structure. Now let P^m be the m-th neighborhood of the diagonal in $X_B \times_B X_B$, viewed as an $\mathcal{O}_X$-algebra via the 1st projection p_1. Then clearly a $\mathfrak{g}$-structure on E yields a structure of $\mathfrak{g}$-deformation on $P^m(E) = P^m \otimes p_2^*(E)$, parametrized by P^m, for any m, and as above this admits a good (Dolbeault) resolution. We denote this deformation by $P^m(E, \mathfrak{g})$.

In particular, taking

$$\mathcal{S} = P^1_X = \mathcal{O}_{X\times_B X}/\mathcal{I}^2_{\Delta_X} = \mathcal{O}_X \oplus \Omega_{X_B/B}$$

(the standard 1st order deformation of $\mathcal{O}_X$), we get a first-order relative $\mathfrak{g}$-deformation $P^1(E, \mathfrak{g})$ parametrized by $\mathcal{S}$. Note that in this case $\Omega_{\mathcal{S}/B} = \Omega_{X_B/B}$ and its $\mathcal{S}$-module structure factors through $\mathcal{O}_X$. Thus the Atiyah-Chern class

$$AC(P^1(E,\mathfrak{g})) \in H^1(\mathfrak{g} \otimes \Omega_{X_B/B})$$

and it is easy to see that it coincides with the usual Atiyah-Chern class of the $\mathfrak{g}$-structure E which may be defined, e.g. differential-geometrically in terms of a $\mathfrak{g}$-connection (and which reduces to the usual Atiyah-Chern class if $\mathfrak{g} = \mathfrak{gl}(E)$, cf. [1]). Indeed our good resolution in this case takes the form

$$E^0 \otimes (\mathcal{O}_X \oplus \Omega_{X_B/B}) \to E^1 \otimes (\mathcal{O}_X \otimes \Omega_{X_B/B}) \dots$$

with differential

$$\begin{pmatrix} \bar\partial & \phi \\ 0 & \bar\partial \end{pmatrix}$$

and note that in this case $\phi = \psi$ since $\mathfrak{m}_{\mathcal{S}} = \Omega_{\mathcal{S}}$. Assuming E is endowed with a $\bar\partial$- connection, the parallel lift of a section e of E to $E^0 \otimes (\mathcal{O}_X \oplus \Omega_{X_B/B})$ is given by $(e, \nabla e)$ and consequently we have

$$\phi(e) = [\bar\partial, \nabla](e).$$

Thus

$$\psi = [\bar\partial, \nabla] \tag{5}$$

In other words, for any section v of $T_{X_B/B}$, holomorphic or not, we have

$$\psi\neg v = [\bar\partial, \nabla_v].$$ ■

Example 2. Consider an ordinary first-order deformation ϕ of a complex manifold X, corresponding to an algebra S of exponent 1. Suppose this deformation comes from a geometric family

$$\pi : \mathcal{X} \to Y$$

with $\mathcal{X}, Y$ smooth, $S = \mathcal{O}_{Y,0}/\mathfrak{m}^2_{Y,0}$. Then it is easy to see that $AC(\phi)$ corresponds to the extension

$$0 \to T_X \to \mathfrak{D}_\pi \to T_0Y \otimes \mathbb{C}_X \to 0$$

where $\mathfrak{D}_\pi$ is the subsheaf of $T_{\mathcal{X}} \otimes \mathcal{O}_X$ consisting of 'vector fields locally constant in the normal direction', i.e. those derivations $\mathcal{O}_{\mathcal{X}} \to \mathcal{O}_X$ that preserve the subsheaf $\pi^{-1}\mathcal{O}_Y \subset \mathcal{O}_X$. ■

The last example suggests an interpretation of the Atiyah class as an extension also in the general case. To state this, let ϕ be a relative deformation parametrized by $\mathcal{S}$ as above, and set

$$I = \mathrm{Ann}(\Omega_{\mathcal{S}/B}) \subset \mathcal{S}, \mathcal{S}' = \mathcal{S}/I, \phi' = \phi \otimes_{\mathcal{S}} \mathcal{S}'$$

and let $\Omega^{vv}_{\mathcal{S}/B}$ denote the double dual as $\mathcal{S}'$-module. Note that

$$\Omega^{vv}_{\mathcal{S}/B} = \mathrm{Der}_{\mathcal{O}_B}(\mathcal{S}, \mathcal{S}')^v$$

(dual as left $\mathcal{S}'$-module).

We will also consider the analogous situation over a formally smooth, complete noetherian augmented local $\mathcal{O}_B$-algebra $\mathcal{S}^\wedge$ (which is thus locally a power series algebra over $\mathcal{O}_B$), where of course dual means as (left) $\mathcal{S}^\wedge$-module.

Theorem . *(a) Let $\mathcal{S}$ be an augmented $\mathcal{O}_B$-algebra of finite type as $\mathcal{O}_B$-module, and let ϕ be a relative $\mathfrak{g}_B$-deformation parametrized by $\mathcal{S}$. Then the image of $AC(\phi)$ in $H^1(\mathfrak{g}_B^{\phi'} \otimes \Omega^{vv}_{\mathcal{S}/B})$ corresponds to an extension of $\mathcal{S}'$ modules*

$$0 \to \mathfrak{g}_B^{\phi'} \to \mathfrak{D}(\phi) \to f^{-1}\mathrm{Der}_{\mathcal{O}_B}(\mathcal{S}, \mathcal{S}') \to 0 \tag{6}$$

and for any admissible $\mathfrak{g}_B$-module E_B there is a natural action pairing

$$\mathfrak{D}(\phi) \times E_B^{\phi} \to E_B^{\phi'}.$$

(b) If $\phi^\wedge$ is a relative $\mathfrak{g}_B$-deformation parametrized by a formally smooth noetherian $\mathcal{O}_B$-algebra $\mathcal{S}^\wedge$, then the image of $AC(\phi^\wedge)$ in $H^1(\mathfrak{g}_B^{\phi^\wedge} \otimes \Omega^{vv}_{\mathcal{S}^\wedge/B})$ corresponds to an extention of $\mathcal{S}^\wedge$-Lie algebras

$$0 \to \mathfrak{g}_B^{\phi^\wedge} \to \mathfrak{D}(\phi^\wedge) \xrightarrow{\nu} f^{-1}T_{\mathcal{S}^\wedge} \to 0 \tag{7}$$

where $T_{\mathcal{S}^\wedge} = \mathrm{Der}_{\mathcal{O}_B}(\mathcal{S}^\wedge, \mathcal{S}^\wedge)$, and for any admissible $\mathfrak{g}_B$-module E_B, $\mathfrak{D}(\phi^\wedge)$ acts on $E_B^{\phi^\wedge}$ satisfying the rule

$$d(f.v) = f.d(v) + \nu(d)(f).v, \forall d \in \mathfrak{D}(\phi^\wedge), f \in \mathcal{S}^\wedge, v \in E_B^{\phi^\wedge} \tag{8}$$

Proof. For brevity we shall work out the formal case, the artinian case being similar. For convenience, we will drop the B substript. We let $(\mathfrak{g}^\cdot, E^\cdot)$ be a pair (differential graded Lie algebra, differential graded module) forming a soft resolution of $(\mathfrak{g}, E)$; also let $(C^\cdot, \partial)$ be a soft resolution of $f^{-1}\mathcal{O}_B$, and note that $\mathfrak{g}^\cdot$ is a $C^\cdot$-module. Then clearly $\mathfrak{D}(\phi^\wedge)$, i.e. the extension corresponding to $AC(\phi^\wedge)$ is resolved by the complex

$$\mathfrak{D}^\cdot(\phi^\wedge) = \mathfrak{g}^\cdot \otimes \mathcal{S}^\wedge \oplus C^\cdot \otimes T_{\mathcal{S}^\wedge}$$

with differential given by the matrix

$$\begin{pmatrix} \partial + \phi^\wedge & \psi^\wedge \\ 0 & \partial \end{pmatrix}$$

where $\psi^\wedge = d_{\mathcal{S}^\wedge}(\phi^\wedge)$ as in (3), which defines in an obvious way a map $C^i \otimes T_{\mathcal{S}^\wedge} \to \mathfrak{g}^{i+1} \otimes \mathcal{S}^\wedge$.

Now we claim that $\mathfrak{D}^\cdot(\phi^\wedge)$ is a differential graded Lie algebra: indeed since $\mathfrak{g}^\cdot \otimes \mathcal{S}^\wedge$ and $T_{\mathcal{S}^\wedge} \otimes C^\cdot$ with the induced differentials are clearly differential graded Lie algebra's (in the latter case, the bracket is induced by that of $T_{\mathcal{S}^\wedge}$), and $T_{\mathcal{S}^\wedge} \otimes C^\cdot$ acts on $\mathfrak{g}^\cdot \otimes \mathcal{S}^\wedge$ via the action of $T_{\mathcal{S}^\wedge}$ on $\mathcal{S}^\wedge$ and the $C^\cdot$-module structure of $\mathfrak{g}^\cdot$, it suffices to show that $\psi^\wedge$ is a derivation, which comes from the following calculation:

$$\psi^\wedge([v_1, v_2]) = [v_1, v_2](\phi^\wedge) = v_1(v_2(\phi^\wedge)) - v_2(v_1(\phi^\wedge))$$

$$= v_1(\psi^\wedge(v_2)) - v_2(\psi^\wedge(v_1)).$$

Now since $\mathfrak{D}^\cdot(\phi^\wedge)$ is a differential graded Lie algebra, the fact that it acts on $E^{\phi^\wedge}$ follows from the fact that the differential of $\mathfrak{D}^\cdot(\phi^\wedge)$ is just commutator with the differential on the resolution of $E^{\phi^\wedge}$, i.e. $\partial + \phi^\wedge$. To check the latter, it is firstly clear on the $\mathfrak{g}^\cdot \otimes \mathcal{S}^\wedge$ summand; for the other summand, take $v \in T_{\mathcal{S}^\wedge} \otimes C^\cdot$. Then

$$[v, \partial + \phi^\wedge] = [v, \partial] + [v, \phi^\wedge] = \partial(v) + \psi^\wedge(v).$$

This shows that the obvious term-by-term pairing induces a pairing of complexes

$$\mathfrak{D}^\cdot(\phi^\wedge) \times (E^\cdot, \partial + \phi^\wedge) \to (E^\cdot, \partial + \phi^\wedge),$$

whence a pairing $\mathfrak{D}(\phi^\wedge) \times E^{\phi^\wedge} \to E^{\phi^\wedge}$; that this is in fact a Lie action is clear from the fact that the corresponding assertion holds term-by-term. This completes the proof. ■

Example 2 bis. If X_B/B is a family of complex manifolds and $\mathfrak{g}_B = T_{X/B}$ is the vertical tangent algebra acting on $\mathcal{O}_X$, $\phi^\wedge$ is a deformation of complex structure corresponding geometrically to $\mathcal{X} \to \mathrm{Spec}(\mathcal{S})$, $\mathfrak{g}^{\phi^\wedge}$ is just the vertical tangent algebra $T_{\mathcal{X}/\mathcal{S}}$, i.e. the derivations of $\mathcal{O}_{\mathcal{X}}$ killing $\mathcal{S}$ (cf. [6]), and $\mathfrak{D}(\phi^\wedge)$ is the algebra of derivations of $\mathcal{O}_{\mathcal{X}}$ leaving $\mathcal{S}$ invariant.

As a specific example, we consider a so-called *Schiffer variation*, where X is a compact Riemann surface, $\mathcal{S} = \mathbb{C}[[s]]$, and for some point $p \in X$, we pick a local coordinate z centered at p and set

$$\phi = s\frac{\partial}{\partial z} d\bar{z}$$

(the corresponding 1st order deformation corresponds to the bicanonical image of p under the identification $H^1(T_X)^* = H^0(K_X^{\otimes 2})$). This yields a formal deformation $\mathcal{X}/\mathcal{S}$, where a holomorphic function near p locally has the form $A(z - s\bar{z})$ where A is a power series with coefficients in $\mathcal{S}$. A local holomorphic generator of the relative tangent algebra $T_{\mathcal{X}/\mathcal{S}}$ is

$$v = \frac{1}{1 - s\bar{s}}\Big(\frac{\partial}{\partial z} + \bar{s}\frac{\partial}{\partial \bar{z}}\Big).$$

We seek a lift of $\frac{\partial}{\partial s}$ that is holomorphic and horizontal, i.e. kills both $z - s\bar{z}$ and its conjugate. By a direct computation, such a horizontal lift is given by

$$w = \frac{\partial}{\partial s} + \bar{z}\frac{\partial}{\partial z} + s\bar{s}v.$$

Then locally, the Lie algebra $\mathfrak{D}(\phi)$ is determined as $\mathcal{O}_{\mathcal{X}}v \oplus \mathcal{S}w$ with $[v, w] = 0$, v annihilating $\mathcal{S}$ and w acting on $\mathcal{O}_{\mathcal{X}}$ in the natural way. Globally, one could either glue these local data with a Čech twist corresponding to ϕ, or take the standard differential graded Lie algebra Dolbeault resolution of $\mathfrak{D}(\phi)$ and twist the differential by adding ϕ.

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