

Effective bounds for the zeros of linear recurrences in function fields

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RÉSUMÉ. Dans cet article, on utilise la généralisation de l'inégalité de Mason (due à Brownawell et Masser [8]) afin d'exhiber des bornes supérieures effectives pour les zéros d'une suite linéaire récurrente définie sur un corps de fonctions à une variable.

De plus, on étudie de problèmes similaires dans ce contexte, comme l'équation $G_n(x) = G_m(P(x))$, $(m, n) \in \mathbb{N}^2$, où $(G_n(x))$ est une suite récurrente de polynômes et $P(x)$ un polynôme fixé. Ce problème a été étudié auparavant dans [14,15,16,17,32].

ABSTRACT. In this paper, we use the generalisation of Mason's inequality due to Brownawell and Masser (cf. [8]) to prove effective upper bounds for the zeros of a linear recurring sequence defined over a field of functions in one variable.

Moreover, we study similar problems in this context as the equation $G_n(x) = G_m(P(x))$, $(m, n) \in \mathbb{N}^2$, where $(G_n(x))$ is a linear recurring sequence of polynomials and $P(x)$ is a fixed polynomial. This problem was studied earlier in [14,15,16,17,32].

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