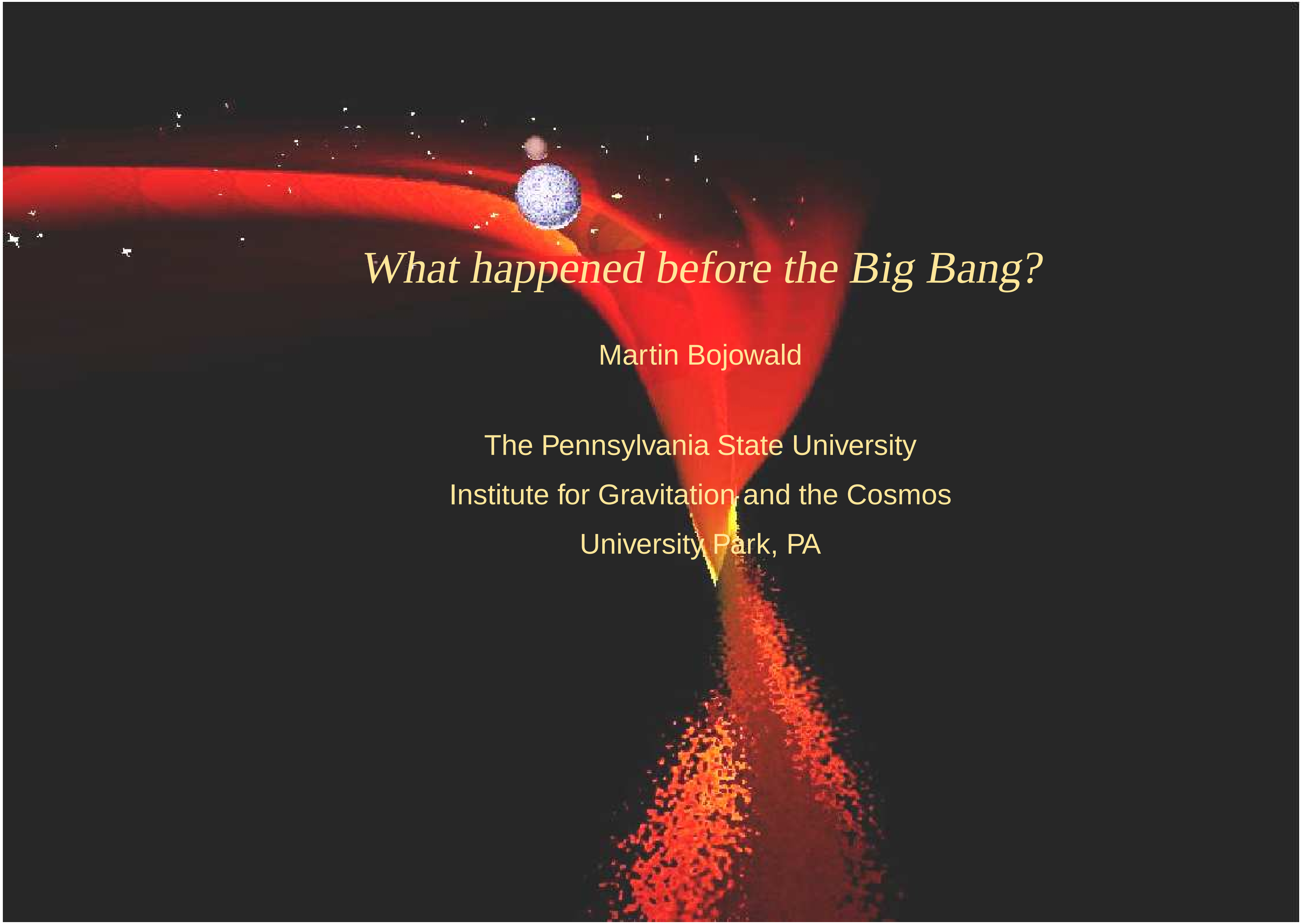


What happened before the Big Bang?

Martin Bojowald

The Pennsylvania State University

Institute for Gravitation and the Cosmos

University Park, PA

Cosmic expansion

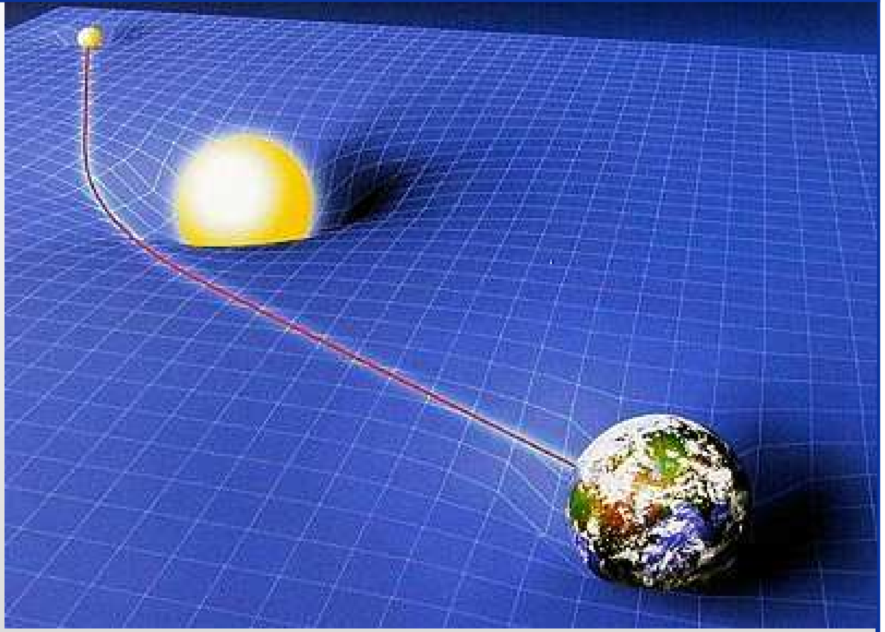
1929: Edwin Hubble demonstrates linear relation between redshift of most galaxies and their distance from Earth.

Explanation: Relative escape velocity proportional to distance.
Space in between is *expanding.*

Not possible in traditional Newtonian physics, where space is absolute and does not take part in physical interrelations.

Can only be explained by *general relativity,* whose homogeneous and isotropic solutions had already been analyzed (Einstein, Friedmann, Lemaitre, de Sitter).

General relativity



Curved space-time: distances

$$\mathrm{d}s^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu$$

depend on place in space and time, not constant as in Minkowski form $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$.
Gravitational force from local change of metric coefficients $g_{\mu\nu}(x)$ subject to Einstein's equation.

$\longrightarrow$ Space-time determines motion of bodies, but its own dynamical equations are sourced by matter: *non-linear theory.*

$\longrightarrow$ Structure of *space and time* not absolute but subject to physical laws. Arises from *solutions,* not presupposed.

Consequences

Dynamical metric subject to physical laws. Implies new effects, e.g. propagation of small perturbations: *Gravitational waves.*

But: Mathematical equations determine space-time metric, solutions can become *singular* and inextendable. Space and time stop where equations no longer allow finite solutions.

Happens in *black holes,* and at the *big bang* (if we rewind the expansion of the universe).

Hawking–Penrose theorems: For matter as we know it, a space-time which expanded at one time can only have existed for a *finite amount of time* after a singularity.

The description of the universe according to general relativity cannot be complete.

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Singularity problem

Isotropic cosmology: universe expansion by increasing scale factor $a(t)$, $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$.

Friedmann equation for non-relativistic matter (mass M, "dust"):

$$\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 - \frac{2GM}{a} = 0$$

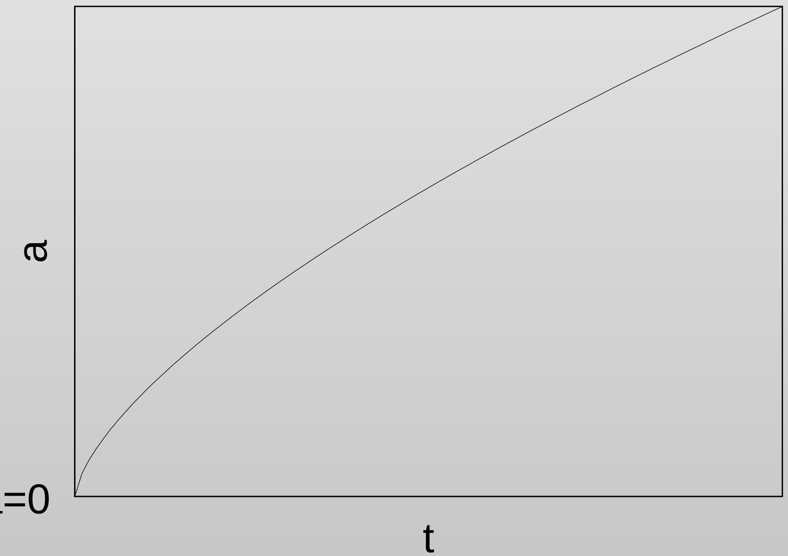
(kinetic plus potential Newtonian energy)

Solution: $a(t) \propto (t - t_0)^{2/3}$.

Properties:

(i) *Smaller volume* in the past.

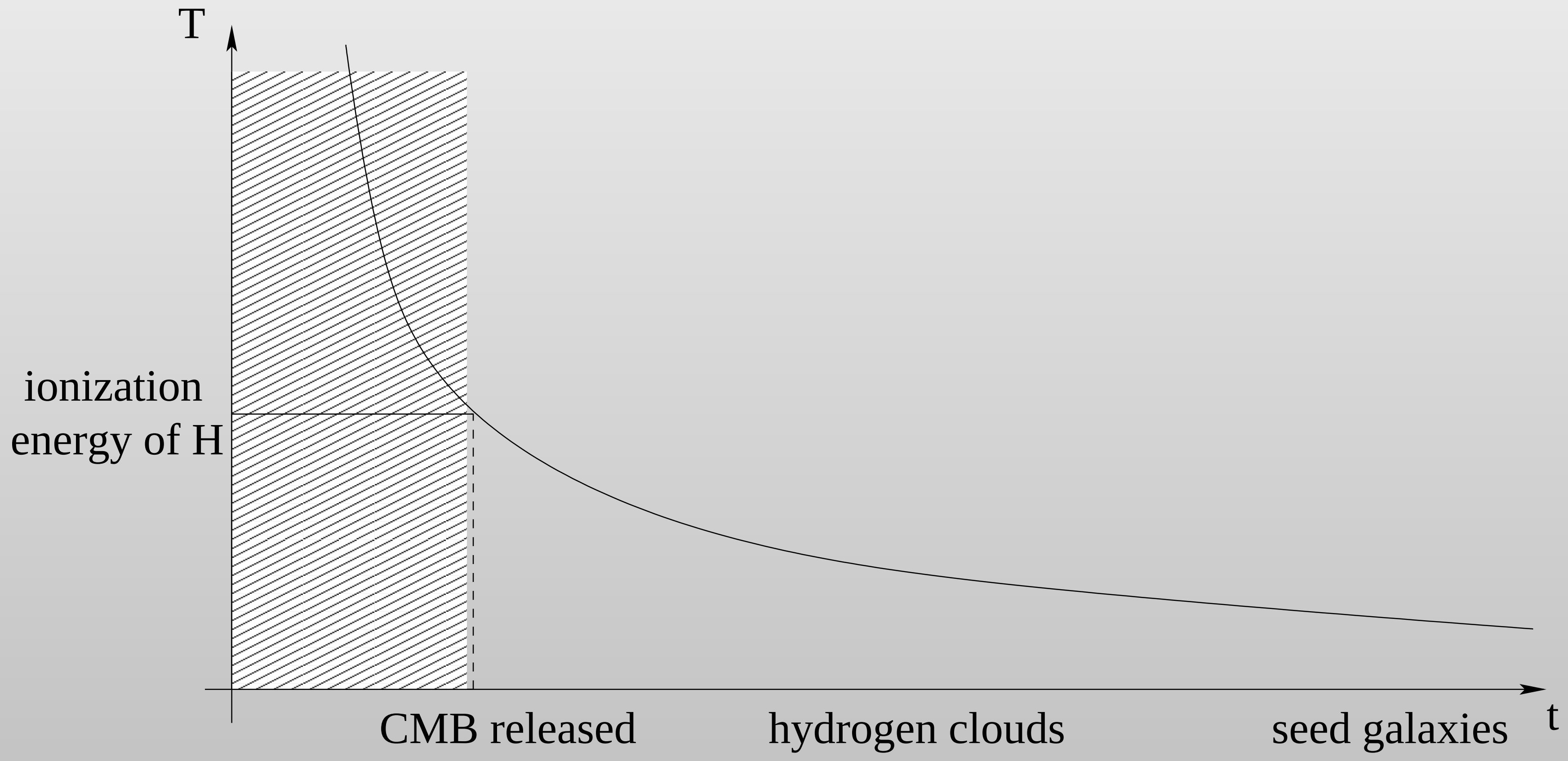
(ii) Energy density M/a^3 behaves as $1/t^2$, universe *hotter* in the past.



(iii) Zero volume, infinite energy density a finite time ago: *Mathematical singularity: big bang.* Density, temperature infinite.

Big bang

Dust assumption not valid at very early times (radiation, quark-gluon plasma, ???) but singularity remains for classical space-times under very general conditions. Theory incomplete.



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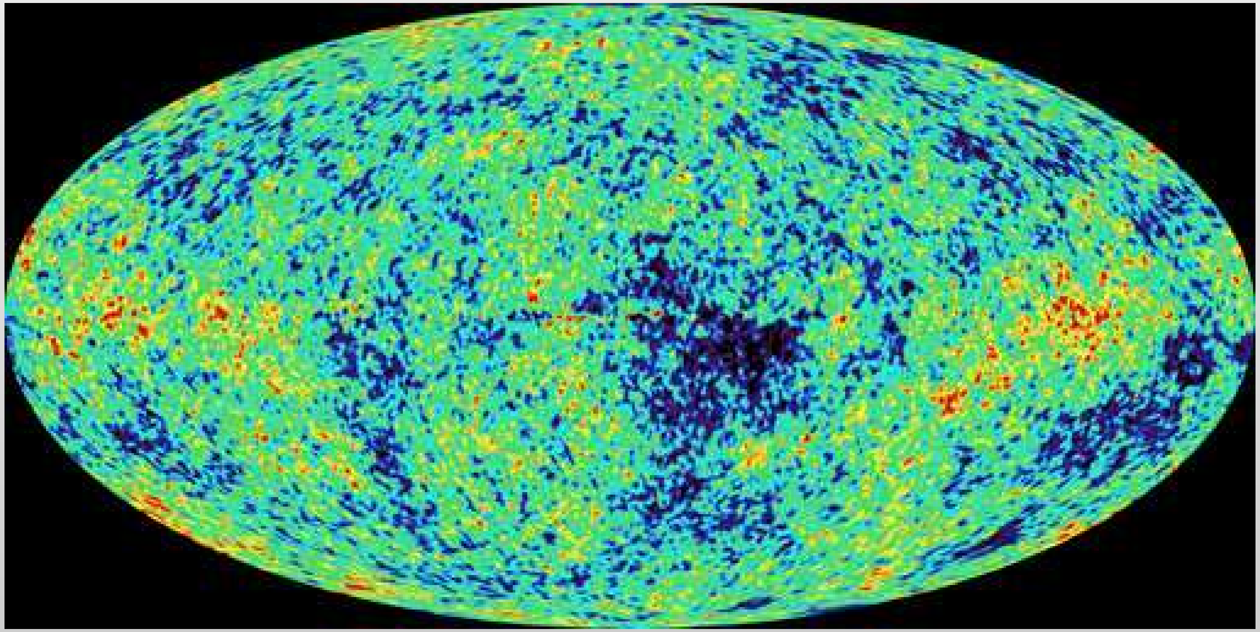
Cosmic microwave background

Observations: last electromagnetic image from cosmic microwave background after plasma ceased to be opaque. Properties of pre-history may show up in details.



"*Gemalt hätt ich dich: nicht an die Wand,*
an den Himmel selber von Rand zu Rand."

Rainer Maria Rilke: Das Stundenbuch

Attraction

A singularity is a lawless place, which cannot be the beginning of a world understandable by physical laws.

Complete picture requires a framework without singularity.

The big bang may then be a true *beginning,* or a high energy transition from a universe which existed *before the big bang.*

Main problem: gravitational force according to general relativity is always *attractive,* and no other force can prevent the total collapse once matter is sufficiently dense.

"*Wir sahen dies an der einfachsten aller Naturerscheinungen, der Schwere, die nicht aufhört zu streben und nach einem ausdehnungslosen Mittelpunkt, dessen Erscheinung ihre und der Materie Vernichtung wäre, zu drängen, wenn auch schon das ganze Weltall zusammengeballt wäre.*"

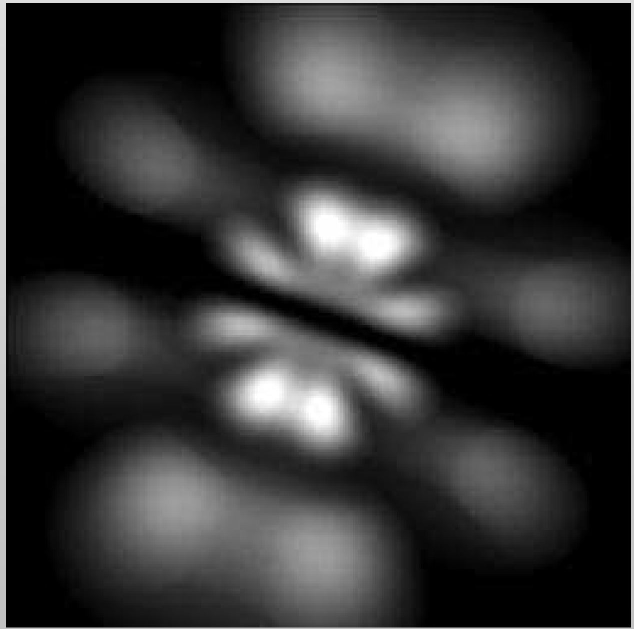
Arthur Schopenhauer: Die Welt als Wille und Vorstellung

Gravity and quantum physics

General relativity: universe as a whole as well as massive objects and their aggregates.

Quantum physics: microscopic world and its elementary structure at small distances.

Usually separate, but both relevant when large energy densities are involved (e.g., neutron stars, big bang nucleosynthesis).



Gravity and quantum physics

General relativity: universe as a whole as well as massive objects and their aggregates.

Quantum physics: microscopic world and its elementary structure at small distances.

Usually separate, but both relevant when large energy densities are involved (e.g., neutron stars, big bang nucleosynthesis).

But these are quantum effects of matter in a classical gravitational field and space-time.

What about *quantum effects of gravity* itself?

Dimensional argument:
tiny *Planck length* $\ell_{\rm P} = \sqrt{G\hbar/c^3} \approx 10^{-35}{\rm m}$
huge *Planck mass* $M_{\rm P} = \sqrt{\hbar c/G} \approx 10^{18}{\rm GeV} \approx 10^{-6}{\rm g}$.

Quantum gravity

These are the scales of the *elementary world of quantum space-time.*
To be taken into account for microscopes with $10^{-35}\mathrm{m}$ resolution (10 atto-attometers), or for $10^{18}\mathrm{GeV}$ particle accelerators.

What else? Why quantum gravity?

Ann. Phys. **17**, 549 (1905)

5. *Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen;*

von A. Einstein.

In dieser Arbeit soll gezeigt werden, daß nach der molekularkinetischen Theorie der Wärme in Flüssigkeiten suspendierte Körper von mikroskopisch sichtbarer Größe infolge der Molekularbewegung der Wärme Bewegungen von solcher Größe ausführen müssen, daß diese Bewegungen leicht mit dem Mikroskop nachgewiesen werden können. Es ist möglich, daß die hier zu behandelnden Bewegungen mit der sogenannten „Brownschen Molekularbewegung" identisch sind; die mir erreichbaren Angaben über letztere sind jedoch so ungenau, daß ich mir hierüber kein Urteil bilden konnte.

Wenn sich die hier zu behandelnde Bewegung samt den

Quantum gravity

These are the scales of the *elementary world of quantum space-time.*
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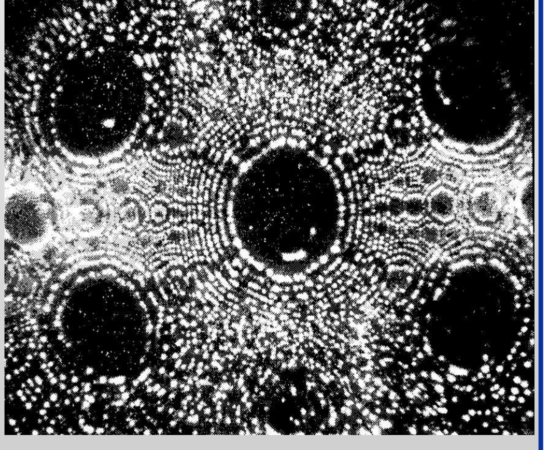
What else? Why quantum gravity?

Indirect effects rather than direct observations.

Example: Brownian motion as convincing indication for atomic structure of matter well before Erwin Müller's field ion microscopy.



$\longrightarrow$ *Potential for observations,* after all: small extension of space and high energies at big bang, combined with long evolution.

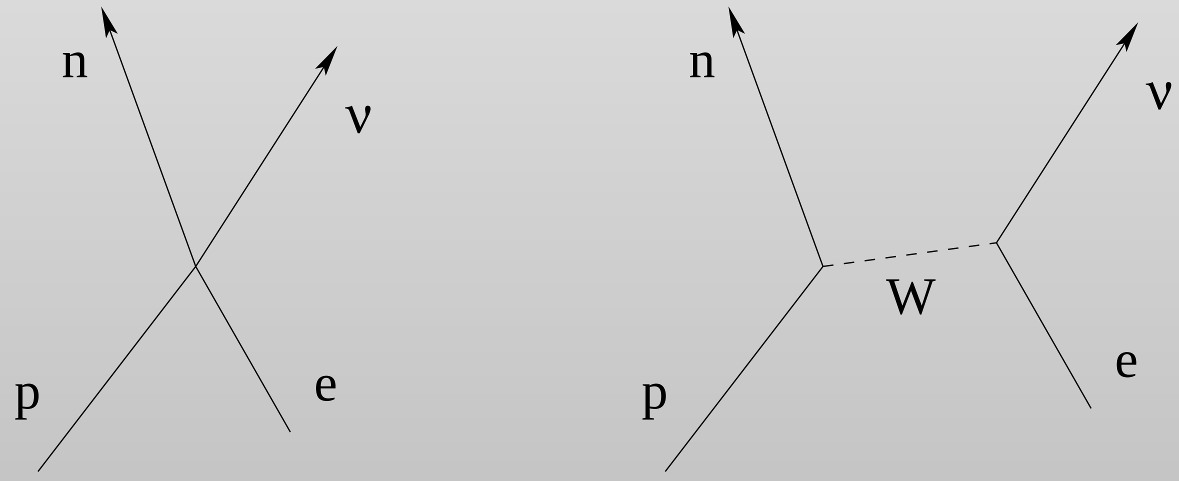
$\longrightarrow$ Quantum effects in the gravitational force may change attraction, *resolve singularities* (as they do for hydrogen).

Microscopic degrees of freedom

At some point, energies will be so high that quantum physics of the gravitational field itself becomes important.

But what are the correct *quantum degrees of freedom* of space-time and their dynamics?

Further input needed. Example from particle physics: *renormalizability* leads from 4-fermion interaction of β-decay to electroweak theory.



Quantized field: W-boson and its interactions.

Principles for quantum gravity

Using *renormalizability* as a guide to find quantum gravity degrees of freedom leads to *string theory.*

Combines gravitational waves with matter, viewed as small fields on given space-time: unified theory.

Strong gravitational fields of big bang and black holes: interaction of matter with space-time metric, must consider *quantum nature of full space-time.*

Alternative principle: *background independence,* quantize full metric $g_{\mu\nu}$. Realized in *loop quantum gravity.*

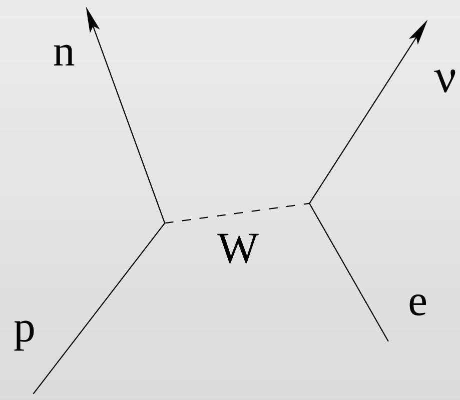
Based on different principles, string theory and loop quantum gravity are quite different from each other, but complementary.

Background independence

Quantum field theory on a background space-time:
Mathematical operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger}$ to describe the annihilation and creation of particles of momentum $\mathbf{k}$.

Using $a_{\mathbf{k}}^{\dagger}$ introduces a new particle,
and *increases the total energy.*

Products of operators to obtain interactions.



Problem: Particles can only be created on a given space-time, whose metric is used in the definition of $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger}$.

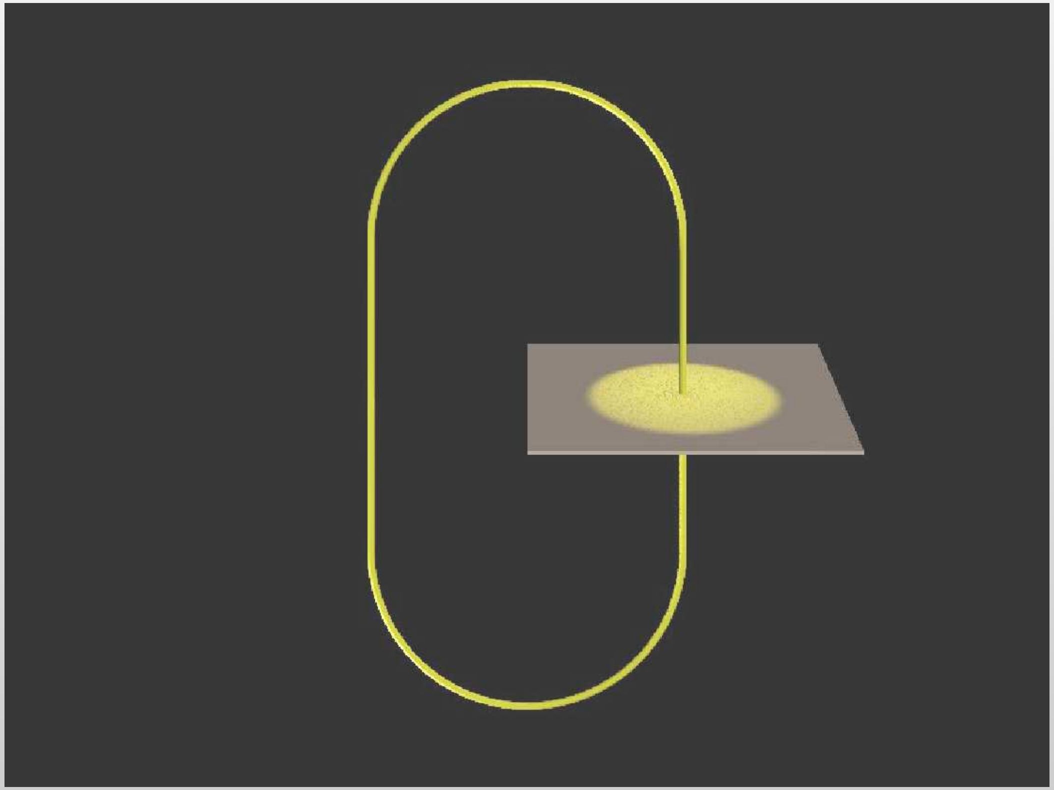
Solution: Define operators for space-time itself.
Increase distances, areas and volumes, not energy.

Realized in *loop quantum gravity:*
"holonomies" h_I create geometry. [Ashtekar, Rovelli, Smolin]

Excitations of geometry

Holonomies exist for all curves in space, create *spatial geometry.*

Single “space-atom”:



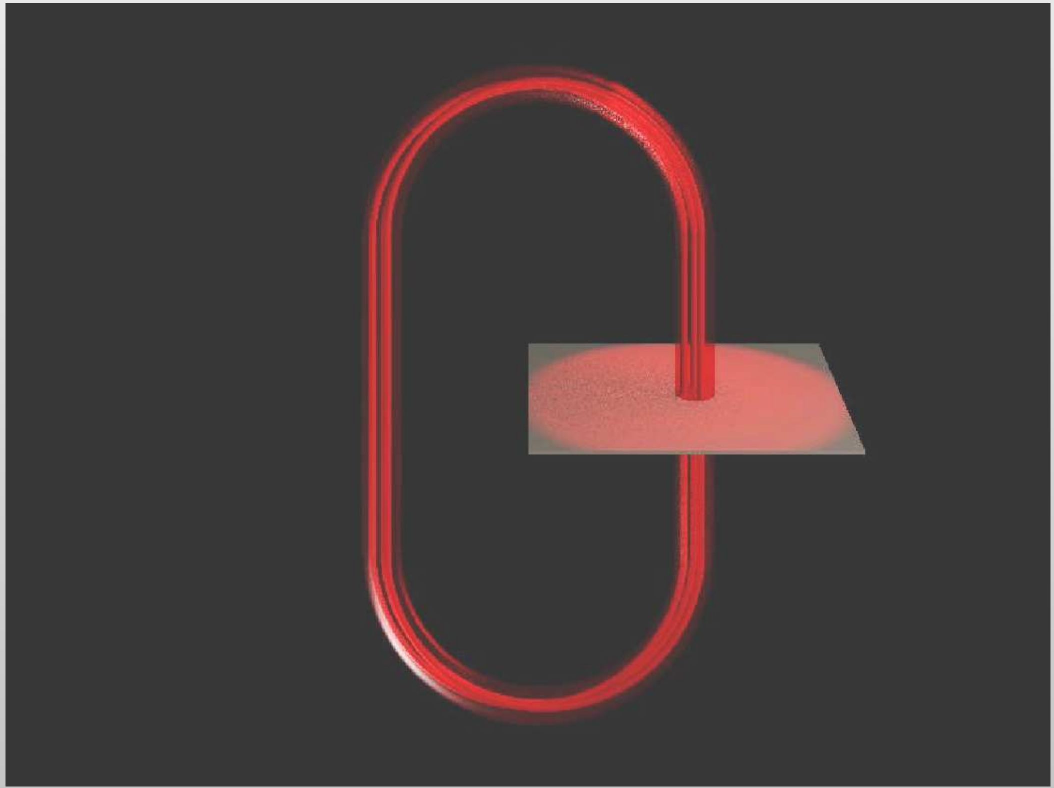
Loop as visualization of state. Physical meaning through measurement of, e.g., area illustrated by intersecting surface.

Excitations of geometry

Holonomies exist for all curves in space, create *spatial geometry.*

Higher excitations in two ways:

(i) use operators for the same loop



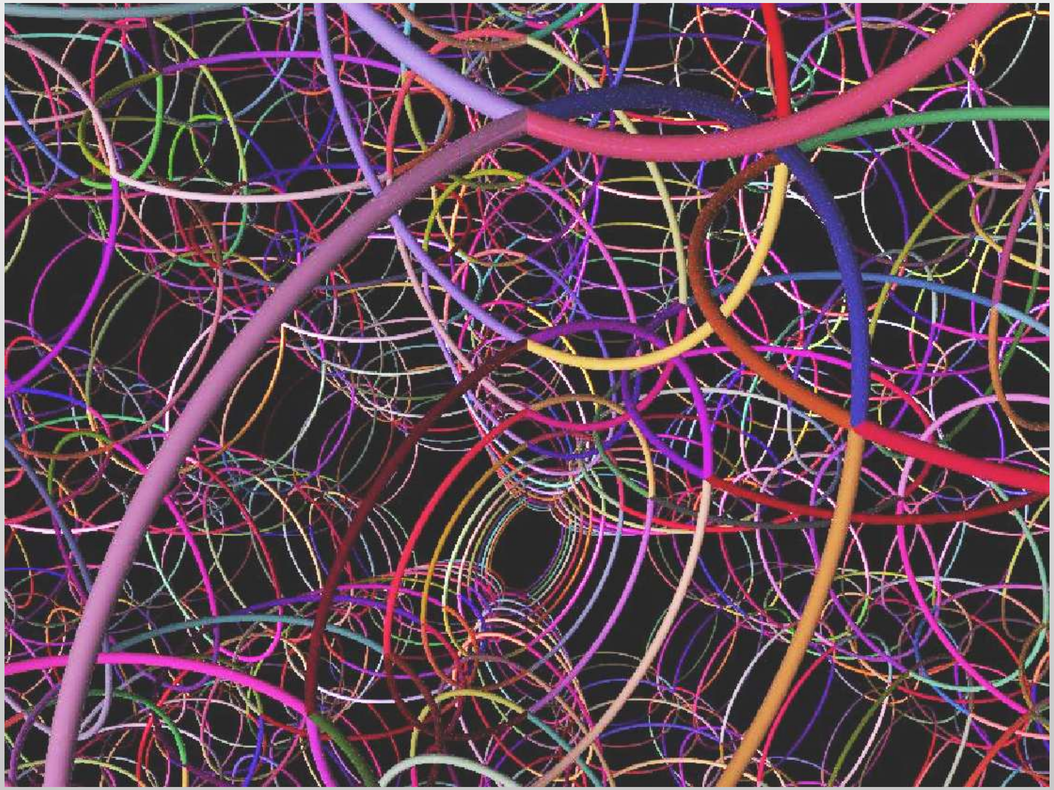
Excitations of geometry

Holonomies exist for all curves in space, create *spatial geometry.*

Higher excitations in two ways:

(i) use operators for the same loop

or (ii) use different loops.



Strong excitation necessary for macroscopic geometry: "many particles."

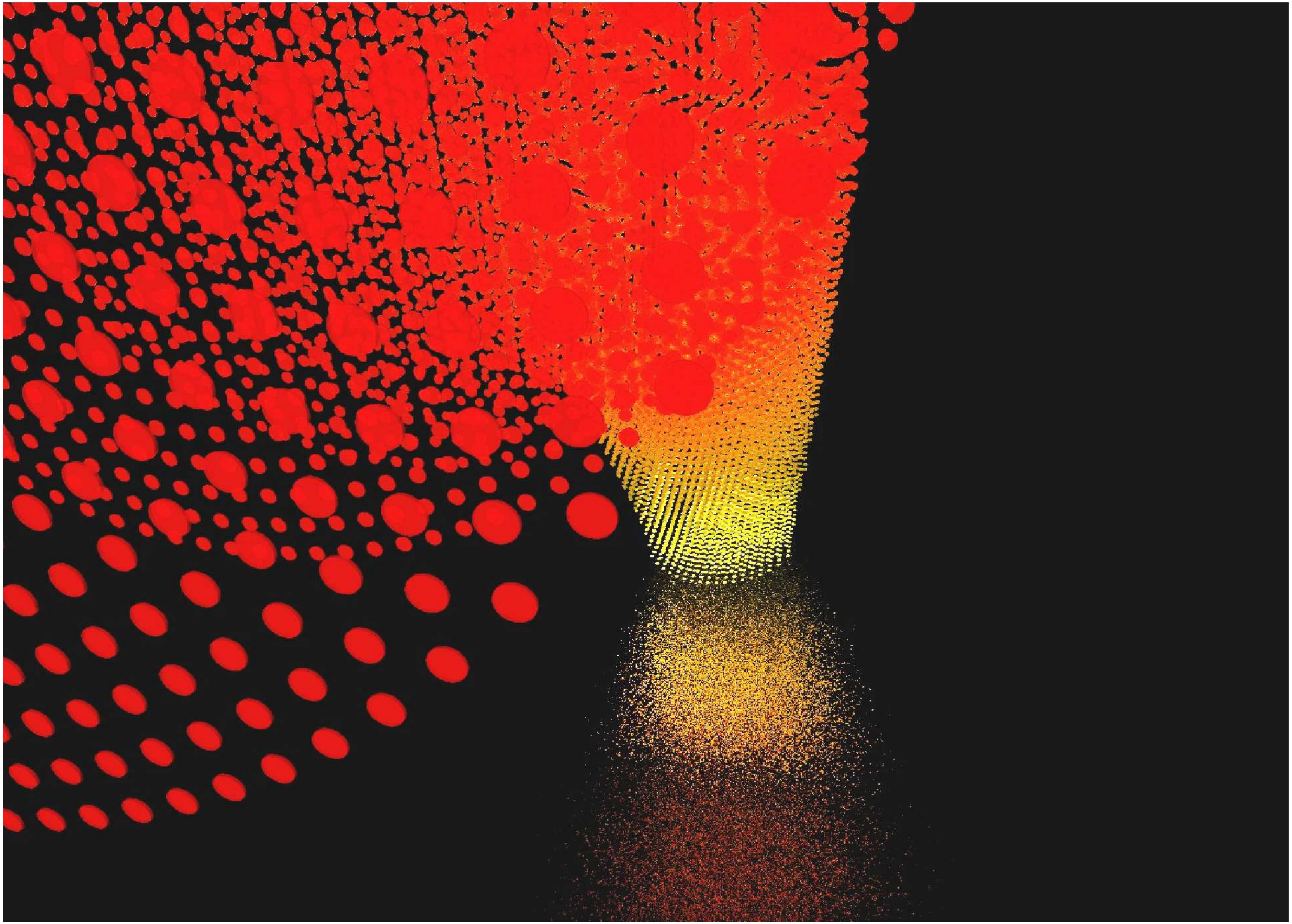
Dynamics

[Thiemann]

$$\hat{H} = \sum_{v,IJK} \epsilon^{IJK} \mathrm{tr}(h_{v,I} h_{v+I,J} h^{-1}_{v+J,I} h^{-1}_{v,J} h_{v,K} [h^{-1}_{v,K}, \hat{V}])$$

as (simplified) Hamiltonian: *excitations of geometry take place dynamically.* Depends on geometry through volume operator $\hat{V}$.

Universe as growing crystal of discrete space: atoms of space created and excited as universe expands.



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as (simplified) Hamiltonian: *excitations of geometry take place dynamically.* Depends on geometry through volume operator $\hat{V}$.

Universe as growing crystal of discrete space: atoms of space created and excited as universe expands.

Significant at *high densities* (big bang), or if many small corrections add up in a *large universe* (dark energy, perhaps).

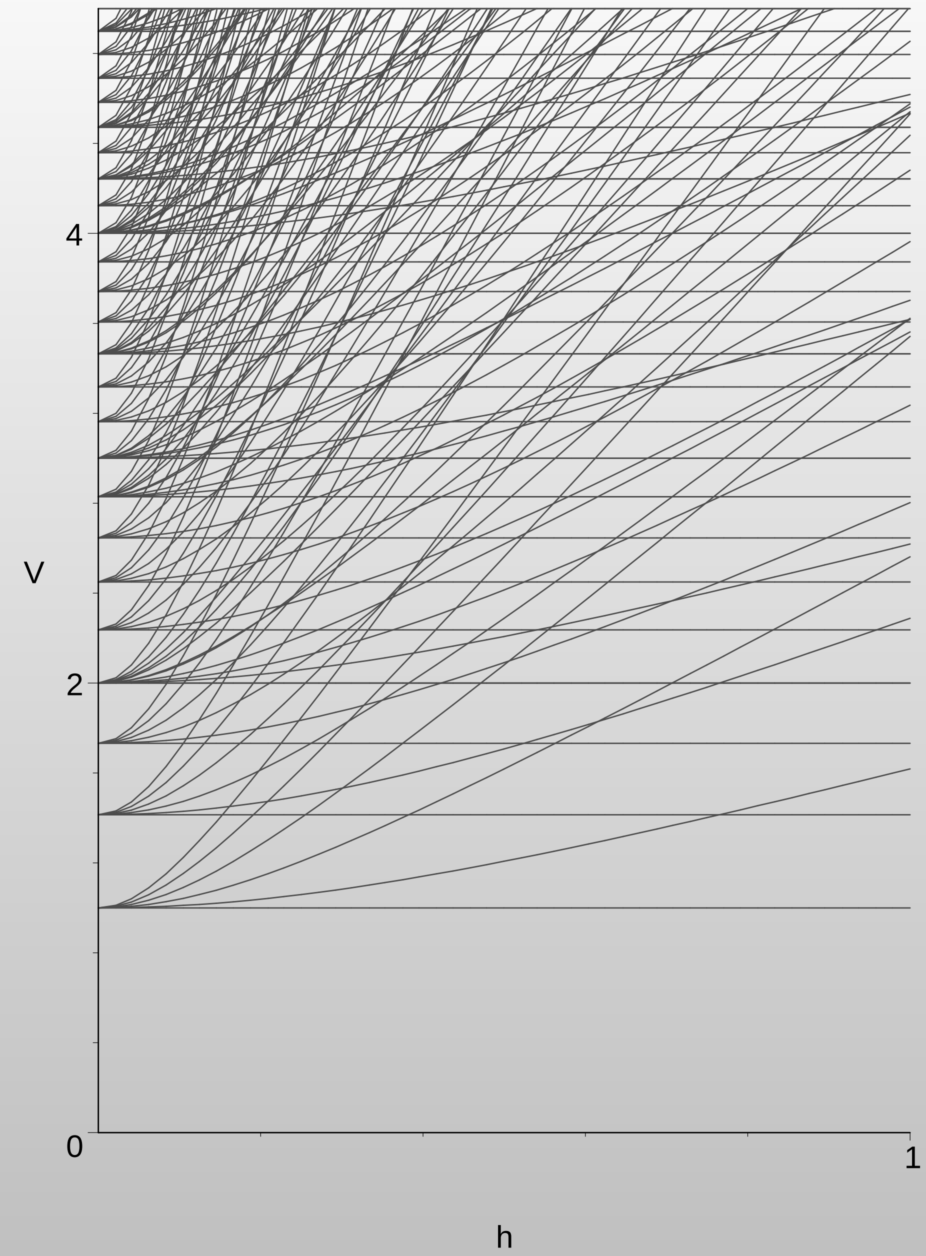
Important: *Spectra of geometry* (volume operator) and their dynamical transitions.

Being analyzed numerically. [Brunnemann, Rideout]

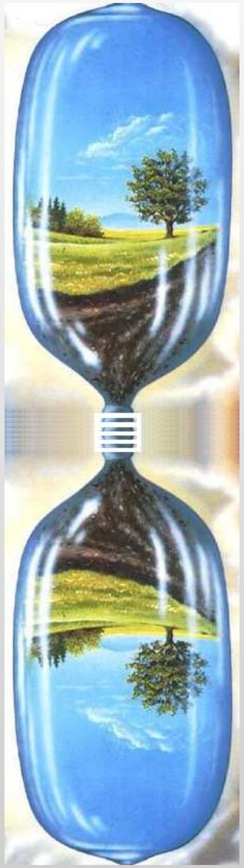
Spectroscopy of geometry

Example: *Volume spectrum* splits when symmetry is relaxed from homogeneity to spherical symmetry.

Most symmetric systems easiest to analyze, also concerning dynamics: *quantum cosmology.*



Loop quantum cosmology



Difference equation for wave function of the universe

$$C_{+}\psi_{\mu+1}(\phi) - C_{0}\psi_{\mu}(\phi) + C_{-}\psi_{\mu-1}(\phi) = -\hat{H}\psi_{\mu}(\phi)$$

depending on matter energy $\hat{H}$.

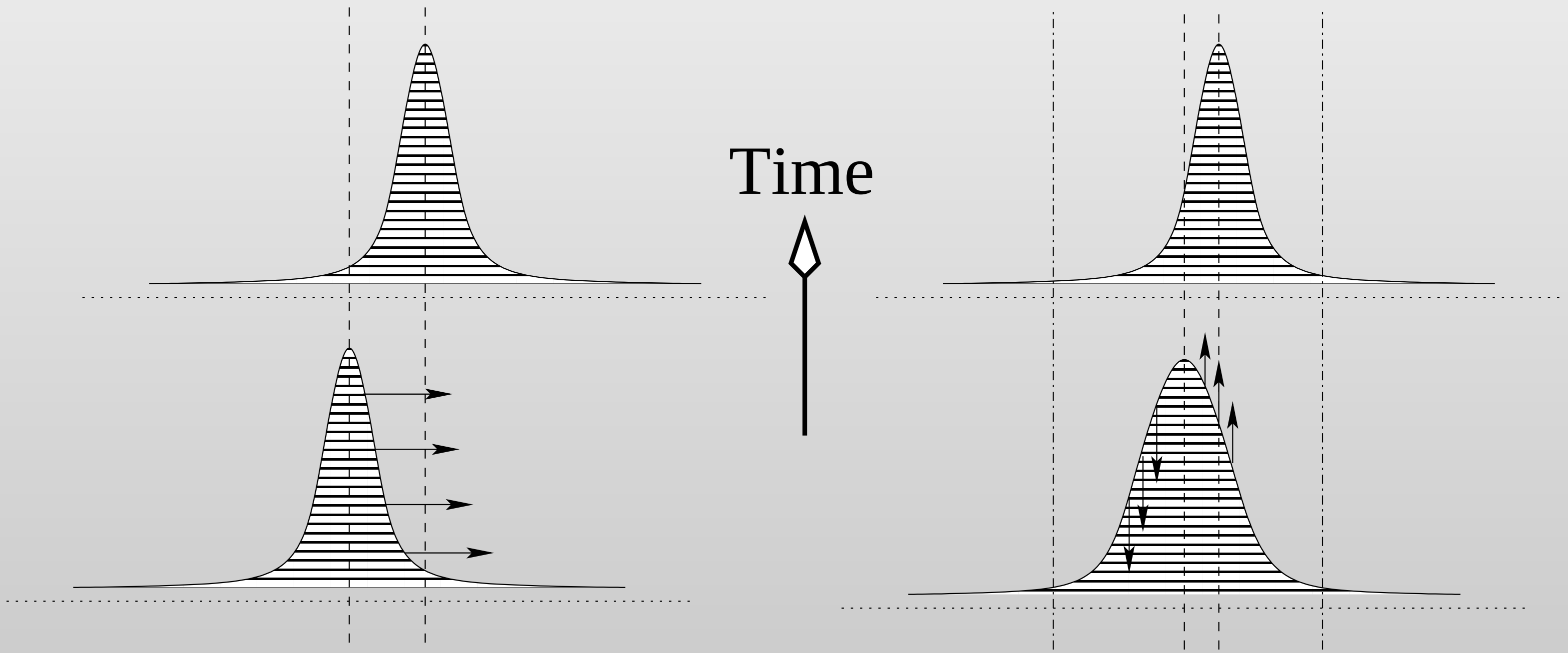
Non-singular: wave function evolves uniquely across classical singularity ($\mu = 0$).

Physical explanation: limited storage for energy in discrete space-time $\longrightarrow$ *repulsive force.*

Geometrical picture in general not available in strong quantum regime.

Quantum back-reaction

General behavior of wave functions quite involved:



Quantum forces possible, which are not expected classically.

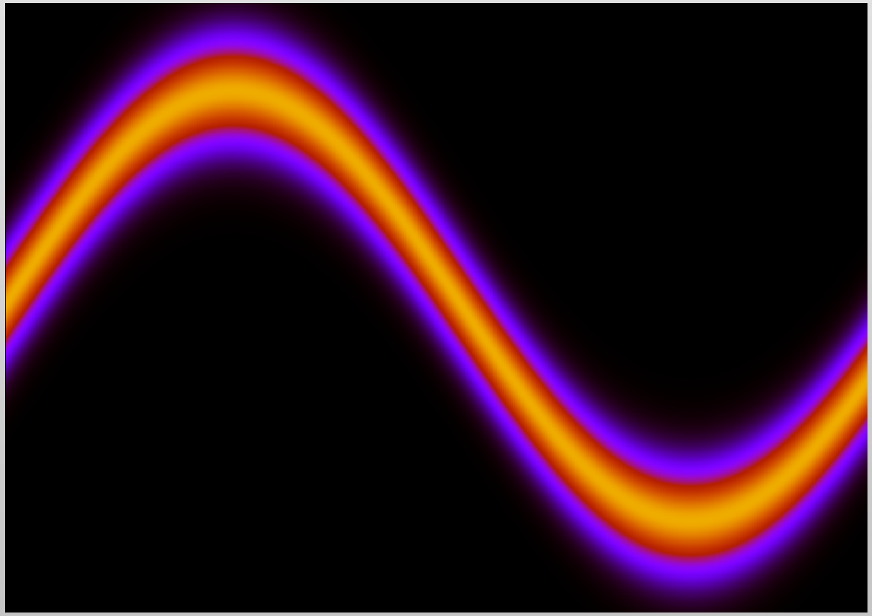
Harmonic oscillator

Simple system in quantum mechanics: harmonic oscillator,

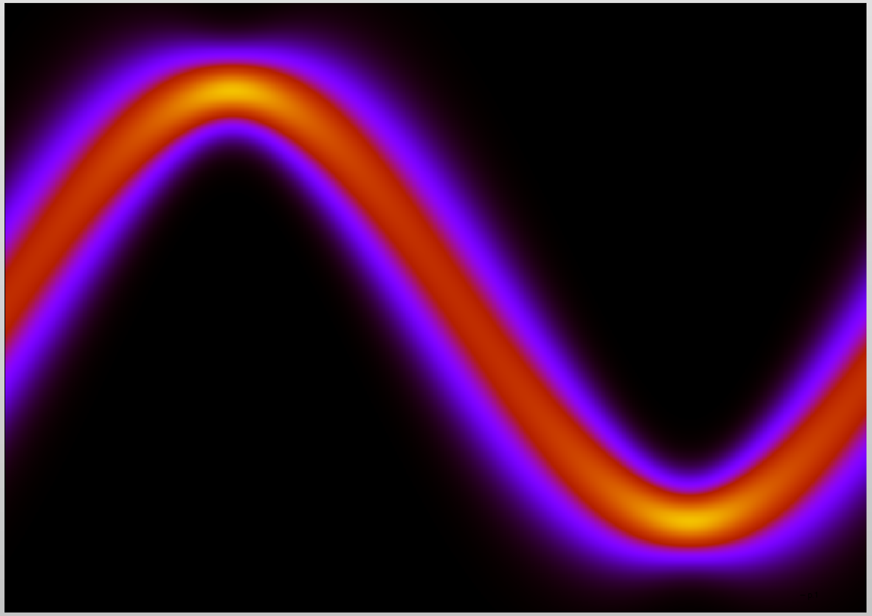
$$[\hat{q}, \hat{p}] = i\hbar \quad , \quad [\hat{q}, \hat{H}] = i\hbar \frac{\hat{p}}{m} \quad , \quad [\hat{p}, \hat{H}] = -i\hbar m\omega^2 \hat{q}$$

spreading wave packets do not disturb mean position.

unsqueezed state:



squeezed state:



Colors: probability for position (vertical) at any time (horizontal)

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Harmonic cosmology

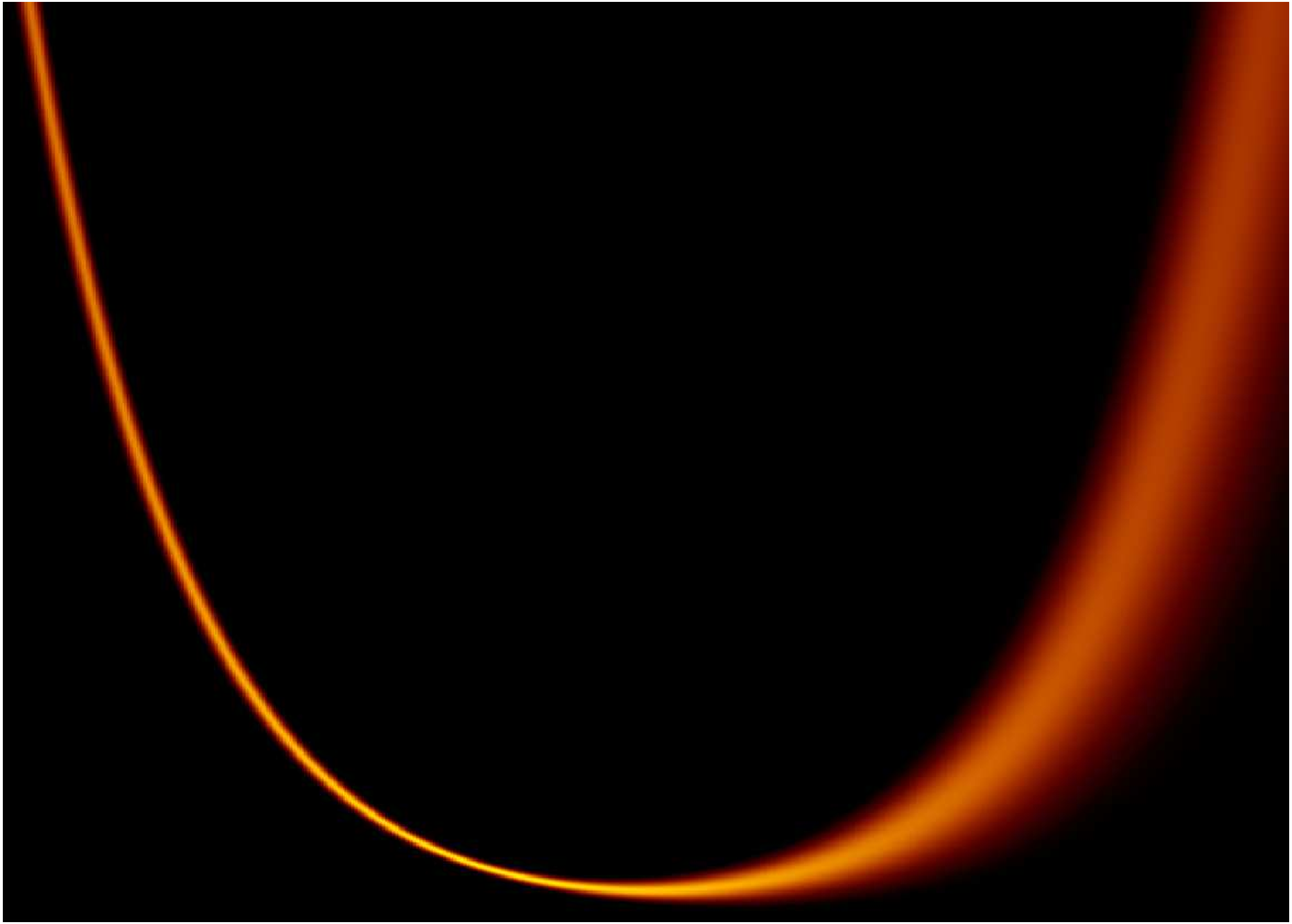
Similar system exists in loop quantum cosmology (Conditions: isotropic, flat space; free, massless scalar).
sl$(2, \mathbb{R})$ algebra of operators:

$$[\hat{p}, \hat{J}] = i\hbar\hat{H} \quad , \quad [\hat{p}, \hat{H}] = -i\hbar\hat{J} \quad , \quad [\hat{J}, \hat{H}] = i\hbar\hat{p}$$

Squeezing: Oscillating fluctuations between different universe phases.

Harmonic systems are among the simplest and can be solved exactly.
But this does not guarantee that all properties are tightly constrained.

Quantum aspects (squeezing, quantum correlations) play large roles in big bang transition, restrict knowledge of state before the big bang even in solvable model: *Cosmic forgetfulness.*



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Follow the bouncing universe

1930's Tolman postulates bounces, discusses *cyclic models;*

1940's–1960's Alpher/Herman to Penzias/Wilson: big bang model;

1979 Novello/Salim, Melnikov/Orlov propose *mechanisms* for bounce;

1996–present Durrer/Laukenmann, Peter/Pinto-Neto, and others: *cosmological implications* of bounces;

2000–present MB, . . . : *loop quantum cosmology;*
Steinhardt/Turok, . . . : *cyclic models motivated from string theory.*

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Follow the bouncing universe

1930's Tolman postulates bounces, discusses *cyclic models;*

Great Depression

1940's–1960's Alpher/Herman to Penzias/Wilson: big bang model;

Keynesianism

1979 Novello/Salim, Melnikov/Orlov propose *mechanisms* for bounce;

Oil Crisis

1996–present Durrer/Laukenmann, Peter/Pinto-Neto, and others: *cosmological implications* of bounces;

Bursting Bubbles

2000–present MB, . . . : *loop quantum cosmology;*
Steinhardt/Turok, . . . : *cyclic models motivated from string theory.*

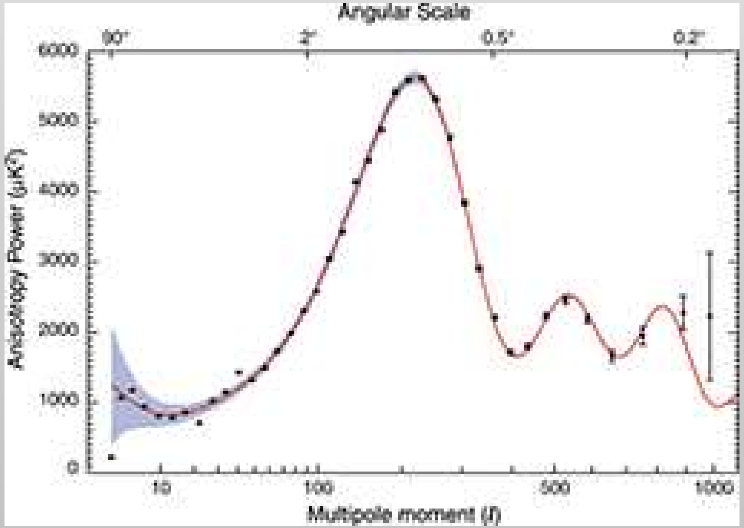
Cosmology

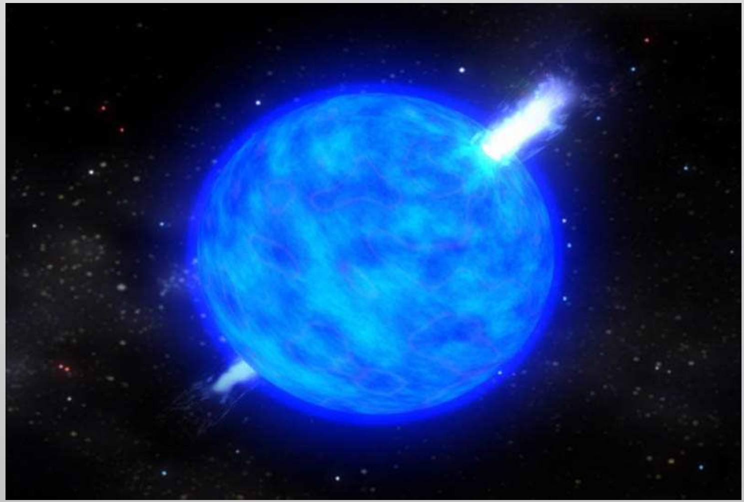
With matter interactions and inhomogeneities, back-reaction results: systematic perturbation theory around solvable model.

Indirect effects of atomic space-time: small individual corrections even at high energies, must add up coherently.

$\longrightarrow$ *cosmology,* high energy density, long evolution

$\longrightarrow$ *high energy particles* from distant sources.





Outlook

Discrete space-time as dispersive medium changes propagation of fields. Does this preserve *covariance?*
(Recent result: Yes, but consistent quantum gravity is delicate.)

Early universe cosmology: Observations of cosmic microwave background, maybe even earlier stages with gravitational waves.

With some luck, *indirect tests of atomic nature of space-time* may become possible. How long will it take?

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With some luck, *indirect tests of atomic nature of space-time* may become possible. How long will it take?

Probably less than it took for first indirect proof of atomic matter.

About 25 centuries passed between ancient Greek atomists and Einstein's analysis of Brownian motion.

Direct picture of space-time atoms?
Eventually information about the universe before the big bang?

This will take much longer.

Further Reading

Living Reviews in Relativity 11 (2008) 4

Scientific American Oct 2008, 44–51

Next Spring ⟶