Mem. Differential Equations Math. Phys. 10(1997), 138–139

R. Koplatadze and G. Kvinikadze

ON OSCILLATION OF SECOND ORDER LINEAR DIFFERENCE EQUATIONS WITH DEVIATED ARGUMENTS

(Reported on September 23–30, 1997)

Consider the equation

$$\Delta^2 u(k) + \sum_{j=1}^{m} p_j(k) u\big(\tau_j(k)\big) = 0 \quad k = 1, 2, \dots, \tag{1}$$

where $\Delta u(k) = u(k+1) - u(k)$, $p_j : N \to R_+$, $\tau_j : N \to N$, and $\lim\limits_{k\to+\infty} \tau_j(k) = +\infty$ $(j = 1, \dots, m)$. A sequence $\{u(k)\}_{k=1}^{+\infty}$ is said to be a proper solution of the equation (1) if it satisfies (1) for any $k = 1, 2, \dots$ and

$$\sup\Big\{|u(i)| : i \geq k\Big\} > 0 \quad \text{for} \quad k \in N.$$

A solution $u(k)$ of the equation (1) is said to be nonoscillatory if there exists $k_0 \in N$ such that either $u(k) > 0$ or $u(k) < 0$ for $k \geq k_0$. Otherwise the solution is called oscillatory.

Below sufficient conditions are given for all proper solutions of (1) to be oscillatory as well as for a nonoscillatory solution to exist.

Theorem 1. *Suppose that*

$$\varliminf_{k\to+\infty} \frac{\tau_j(k)}{k} > 0 \quad (j = 1, \dots, m) \tag{2}$$

and for any $\lambda \in [0, 1[$ *there exists* $\varepsilon > 0$ *such that*

$$\varliminf_{k\to+\infty} k^{1-\lambda} \sum_{i=k}^{+\infty} \sum_{j=1}^{m} p_j(i) \big(\tau_j(i)\big)^{\lambda} > \lambda + \varepsilon.$$

Then every proper solution of (1) *is oscillatory.*

Theorem 2. *Suppose that* (2) *is fulfilled and for any* $\lambda \in [0, 1[$ *there exists* $\varepsilon > 0$ *such that*

$$\varliminf_{k\to+\infty} k \sum_{i=k}^{+\infty} \sum_{j=1}^{m} p_j(i) \Big(\frac{\tau_j(i)}{i}\Big)^{\lambda} > \lambda(1-\lambda) + \varepsilon.$$

Then every proper solution of (1) *is oscillatory.*

1991 *Mathematics Subject Classification.* 34B05.

Key words and phrases. Linear difference equations, proper solution, oscillatory solution.

Theorem 3. *Suppose that*

$$\varliminf_{k\to+\infty} k\sum_{i=k}^{+\infty} p(i) > \max\left\{\lambda(1-\lambda)\Big(\sum_{j=1}^{m} c_j\alpha_j\Big)^{-\lambda} : \lambda\in[0,1]\right\},$$

where $p : N \to R_+$, $0 < \alpha_j = \varliminf_{k\to+\infty} \dfrac{\tau_j(k)}{k}$, $c_j \in]0,+\infty[$ $(j = 1,\dots,m)$. *Then every proper solution of the equation*

$$\Delta^2 u(k) + p(k)\sum_{j=1}^{m} c_j u\big(\tau_j(k)\big) = 0$$

is oscillatory.

Corollary. *Suppose that*

$$\varliminf_{k\to+\infty} k\sum_{i=k}^{+\infty} p(i) > \frac{1}{4},$$

where $p : N \to R_+$. *Then every solution of the equation*

$$\Delta^2 u(k) + p(k)u(k) = 0$$

is oscillatory.

Theorem 4. *Suppose that for some* $\lambda \in]0,1[$ *there exists* $k_0 \in N$ *such that*

$$k^{1-\lambda}\sum_{i=k}^{+\infty} p(i)\big(\tau(i)\big)^{\lambda} \le \lambda \quad \textit{for} \quad k \ge k_0.$$

Then (1) *has a nonoscillatory solution.*

Authors' addresses:

R. Koplatadze
I. Vekua Institute of Applied Mathematics
I. Javakhishvili Tbilisi State University
2, University St., Tbilisi 380043
Georgia

G. Kvinikadze
A. Razmadze Mathematical Institute
Georgian Academy of Sciences
1, M. Aleksidze St., Tbilisi 380093
Georgia