REVAZ GAMKRELIDZE IS 70

Not long ago the mathematical community warmly congratulated prominent mathematician Revaz Gamkrelidze, Corresponding Member of the Russian Academy of Sciences and Member of the Georgian Academy of Sciences, on his 70th birthday.

These 70 years are marked by remarkable deeds and interesting events.

Revaz Gamkrelidze was born on February 4, 1927, in the Georgian city of Kutaisi. His father Valerian Gamkrelidze was a public figure who contributed much to the publishing of scientific literature in Georgia.

When Revaz Gamkrelidze studied in a secondary school, he read Courant's excellent book "What is Mathematics" and acquired a permanent passion for this science. After school, Revaz Gamkrelidze continued his education at Tbilisi State University. As a gifted sophomore, he was sent to Moscow to study at the mechanics and mathematics faculty of Moscow State University. Here he met Lev Pontryagin, an outstanding contemporary mathematician, who lost his eyesight at the age of 12 but whom Fate rewarded with an insight into the world of science. Pontryagin's maximum principle (1956) is a fine example of this insight.

The meeting of Lev Pontryagin and Revaz Gamkrelidze was that of a teacher and a pupil, one of those rare lucky chances which leave an indelible mark in one's life.

Revaz Gamkrelidze's first studies were dedicated to algebraic geometry and algebraic topology. He determined characteristic classes of complex algebraic manifolds (1953). In the mathematical literature this result is referred to as Gamkrelidze's formula.

In 1954, Revaz Gamkrelidze took up research into nonclassical variational problems related to the theory of automatic control systems for the rocket industry. He achieved remarkable results in this area. Along with L. Pontryagin, V. Boltyanskiĭ and E. Mishchenko, he laid the foundation of the modern mathematical theory of optimal processes. The first paper by the four authors was published in 1956 and evoked a great interest among specialists. In the paper, a general optimal control problem was formulated for the first time and the maximum principle was proposed as a method for its solution. The maximum principle was at that time the most beautiful and sophisticated theorem formulated as Pontryagin's hypothesis. Its proof required four years of continuous strenuous research. The findings were presented in the monograph "Mathematical Theory of Optimal Processes" which in 1961 won the authors the State Lenin Prize. The monograph was translated into several foreign languages , and in many countries it stimulated research in this specific mathematical area.

The achievements of Pontryagin's school were reported at the international conference on mathematical control theory held in 1967 at the University of Southern California (USA). The university situated in a picturesque

suburb of Los Angeles was regarded by mathematicians as a temple of dynamic programming with Richard Bellman as head priest. Prior to Pontryagin's maximum principle, Bellman's method of dynamic programming was the only one used in investigating optimal control systems. R. Bellman's school carried out research within the framework of the program specially developed for the US Air Forces and financed by the gigantic RAND Corporation.

Bellman's method of dynamic programming has certainly played a very important role, but its main disadvantage is that it can not be substantiated in general terms and does not hold for some cases. In this respect, Pontryagin's maximum principle has no alternative, the fact recognized by the participants of the above-mentioned conference chaired by R. Bellman himself.

However, it took long years for these events to take place. In the meantime the maximum principle remained unproven, the Lenin prize was never thought of, the gold star of honour did not decorate the lapel of Pontryagin's suit.

In 1957 Revaz Gamkrelidze made the initial important step; he proved the maximum principle for time-optimal problem in the case of linear systems. Pontryagin's maximum principle was completely proved only three years later. Even in a very concise form, the proof occupied 45 typewritten pages.

To the same period belong Revaz Gamkrelidze's fundamental investigations into the theory of optimal processes, as well as into problems with bounded phase coordinates. The results he obtained formed a part of his doctoral thesis (1959).

In the subsequent years, Revaz Gamkrelidze discovered and worked on optimal sliding states. He developed the notion of a quasiconvex set in a linear topological space, which underlies the general theory of extremal problems. These results were reported at the International Congress of Mathematicians in Nice in 1970.

Revaz Gamkrelidze studied quasilinear differential games and worked out an evasion strategy for them. Further, he constructed an exponential representation of streams and invented a technique of chronological calculus.

In recent years Revaz Gamkrelidze has been carrying out research and teaching activities in the USA, Germany, France and Great Britain.

Revaz Gamkrelidze pioneered and contributed much to the development of the optimal control theory in Georgia. For a few decades he has been heading the control theory chair at Tbilisi State University. A course of lectures he read at the university was included in his monograph "Fundamentals of Optimal Control"(1975) which was translated into English and published in the United States. In Georgia the monograph was awarded the Razmadze Prize.

Apart from his research and teaching work, Revaz Gamkrelidze is also concerned with publishing matters. He is the editor-in-chief of the "Referativnyĭ Zhurnal Matematika" (Moscow), the series "Modern Problems of

Mathematics" (Russian, Moscow), Modern Mathematical Encyclopaedia (Russian). He is the scientific editor of the American-British series "Modern Soviet Mathematics", German series "Soviet Mathematics", and the series "Russian Classics of Mathematics".

On his 70th birthday Revaz Gamkrelidze is, as always, full of intentions, ideas and energy. We wish him a long successful way in life strewn with further laurels.

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March 8, 1997, Tbilisi

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