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# COPPEL–CONTI SETS OF LINEAR SYSTEMS

Abstract. The paper contains a brief review of results, obtained by the authors, on certain topics in the theory of Coppel-Conti sets of linear systems: the solution of Conti's problem on the inclusion property as the parameter increases; the construction of criteria for the roughness of these sets and their limit sets under uniformly small or integrable perturbations; applications to the investigation of bounded solutions of perturbed nonhomogeneous linear systems.

რეზიუმე. ნაშრომი შეიცავს ავტორების მიერ კოპელ-კონტის სიმრავლეების თეორიის ზოგიერთი საკითხის გარშემო მიღებული შედეგების მოკლე მიმოხილვას. ეს საკითხებია; კონტის პრობლემის გადაწყვეტა პარამეტრის ზრდისას ჩართულობის თვისების შესახებ; ზემოხსენებული სიმრავლეებისა და მათი ზღვრული სიმრავლეების თანაბრად მცირე ან ინტეგრებადი შეშფოთებების მიმართ მდგრადობის კრიტერიუმების აგება; გამოყენებანი შეშფოთებული არაერთგვაროვანი წრფივი სისტემების შემოსაზღვრული ამონახსნების გამოკვლევისათვის.

We consider the Coppel–Conti sets of linear systems

$$\dot{x} = A(t)x \tag{$1_A$}$$

with piecewise continuous real coefficients $A(\cdot) : [0, +\infty) \to \mathrm{Hom}(R^n, R^n)$, generally speaking, unbounded on the semiaxis $t \geq 0$. These sets deal with the problem of boundedness of solutions of the nonhomogeneous linear systems

$$\dot{y} = A(t)y + f(t) \tag{2}$$

raised in 1930 by O. Perron [1].

System $(1_A)$ can be identified with its matrix $A(\cdot)$ and for convenience will be referred to as system A.

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## 1. R. Conti's Problem

We investigate the sets $L^pS$ of linear systems $(1_A)$ with Cauchy matrix $X_A(t,s)$ satisfying

$$C_p(A) \equiv \sup_{t\geq 0} \int\limits_0^{+\infty} \|X_A(t,s)\|^p ds < +\infty. \tag{3}$$

The sets $L^pS$ were introduced by W. Coppel [2] for $p=1$ and by R.Conti [3] for $p>1$. In these cases they are well studied. It was proved in the cited papers that for $p \geq 1$ the inclusion $A \in L^pS$ is equivalent to the boundedness of all solutions of system (2) with any piecewise continuous nonhomogeneous term $f$ that is bounded (if $p=1$) or $p/(p-1)-st$ power intergrable (if $p>1$) on the semiaxis $t \geq 0$. We extend the definition of $L^pS$ from $p \geq 1$ to all $p>0$ by using the condition (3).

R. Conti [4, 5] investigated the sets $L^pS$ as a function of the parameter $p$ and proved [6] that the inclusion $L^pS \subset L^qS$ does not hold for arbitrary $q>p\geq 1$ and posed the following question:

Does the inclusion $L^pS \subset L^qS$ hold for constants $p$ and $q$ such that $p>q\geq 1$?

In our paper [7] we obtained the positive answer to this question.

**Theorem 1 ([7]).** *The inclusion $L^pS \subset L^qS$ is valid for all $p>q>0$.*

Therefore, there exist limit sets $\operatorname*{Lim}\limits_{p\to q\pm 0} L^pS$, $q>0$, and they satisfy the inclusions

$$\operatorname*{Lim}_{p\to q-0} L^pS \supset L^qS \supset \operatorname*{Lim}_{p\to q+0} L^pS.$$

Moreover, there is no left or right continuity with respect to the parameter $p>0$.

Indeed,

$$L^qS \setminus \operatorname*{Lim}_{p\to q+0} L^pS \neq \varnothing, \quad \operatorname*{Lim}_{p\to q-0} L^pS \setminus L^qS \neq \varnothing, \quad q>0$$

(see [6]).

The following criterion for a system $(1_A)$ to belong to the limit set has important applications in investigating the interiors of limit sets.

**Theorem 2 ([8]).** $A \in \operatorname*{Lim}\limits_{p\to q-0} L^pS$, *where* $0<q\leq+\infty$, *if and only if*

$$2 \min_{\tau\in[t-T,t]} \|X_A(t,\tau)\| \leq 1 \quad \forall t \geq T = T_A \geq 1,$$

$$\lim_{\substack{t\to+\infty \\ t-\tau\to+0}} \int\limits_\tau^t \|X_A(t,s)\|^p ds = 0 \quad \forall p \in (0,q).$$

It was also established in [8] that the following two properties of the constants $C_p(A)$ regarded as functions of the parameter $p > 0$ are satisfied for a fixed $A \in \operatorname*{Lim}_{p\to q-o} L^pS$, $q > 0$ : 1) the function $C_{(\cdot)}(A) : (0, +q) \to R^+$ is continuous and 2) there exists a system $A \in \operatorname*{Lim}_{p\to\infty} L^pS$ such that the function $C_{(\cdot)}(A) : (0, +\infty) \to R^+$ has the characteristic exponent $\lambda[C_{(\cdot)}(A)] = +\infty$.

## 2. The Structure of the Interior of the Set $L^pS$

We define the interior $\operatorname{Int} L^pS$ of the set $L^pS$ as the set consisting of all $A \in L^pS$ such that $A + Q \in L^pS$ for any piecewise continuous $n \times n$ matrix $Q(t)$ satisfying $\|Q(t)\| < \varepsilon_A$ for all $t \geq 0$ and some $\varepsilon_A > 0$.

**Theorem 3 ([7]).** $\operatorname{Int} L^pS = L^pS$ *if and only if* $p \geq 1$.

Another Conti problem on the interior of the set $\bigcap_{p>0} L^pS$ to coincide with the set itself is solved (for $q = +\infty$) by the first of the following two theorems about the properties of the interior of the limit sets.

**Theorem 4 ([8]).** $\operatorname{Int} \operatorname*{Lim}_{p\to q-0} L^pS = \operatorname*{Lim}_{p\to q-0} L^pS$ *if and only if* $1 < q \leq +\infty$.

**Theorem 5 ([8]).** $\operatorname{Int} \operatorname*{Lim}_{p\to q+0} L^pS = \operatorname*{Lim}_{p\to q+0} L^pS$ *if and only if* $1 \leq q < +\infty$.

We also considered [9] the similar problem whether systems $(1_A)$ and $(1_B)$ with coefficients close in some integral metric simultaneously belong to either of the sets $L^pS$, $\operatorname*{Lim}_{\gamma\to p-0} L^\gamma S$ and $\operatorname*{Lim}_{\gamma\to p+0} L^\gamma S$. We obtained the following general result for the integral interior $\operatorname{Int}_q L^pS \equiv \{A \in L^pS : B \in L^pS$, for $\|B - A\|_q \equiv \{\int_0^{+\infty} \|B(r) - A(\tau)\|^q d\tau\}^{1/q} < +\infty\}$, $q > 0$, of the set $L^pS$ and for the similar interiors $\operatorname{Int}_q \operatorname*{Lim}_{\gamma\to p-0} L^\gamma S$ and $\operatorname{Int}_q \operatorname*{Lim}_{\gamma\to p+0} L^\gamma S$ of the sets $\operatorname*{Lim}_{\gamma\to p-0} L^\gamma S$ and $\operatorname*{Lim}_{\gamma\to p+0} L^\gamma S$.

**Theorem 6 ([9]).** $\operatorname{Int}_q M = M$ *if and only if*
1) $p > 1$ *and* $q \geq p/(p-1)$ *if* $M = L^pS$;
2) $p > 1$ *and* $q > p/(p-1)$ *if* $M = \operatorname*{Lim}_{\gamma\to p-0} L^\gamma S$;
3) $p > 1$ *and* $q \geq p/(p-1)$ *if* $M = \operatorname*{Lim}_{\gamma\to p+0} L^\gamma S$.

Since inclusions $L^qS \subset L^pS$ are valid for all $q > p > 0$, the similar inclusions $\operatorname{Int} L^qS \subset \operatorname{Int} L^pS$ are valid for their interiors. For the integral interiors $\operatorname{Int}_q L^pS$ with different $q > 0$ but the same $p$ the opposite inclusion is valid, at least for $p \geq 1$. This is given by the following theorem.

**Theorem 7 ([9]).** $\operatorname{Int}_q L^pS \subset \operatorname{Int}_l L^pS$ *for* $p \geq 1$ *and* $q < l$.

The interior $\mathrm{Int}_q\ L^pS$ of $L^pS$, which is clearly a part of the interior $\mathrm{Int}_{q0}\ L^pS \equiv \{A \in L^pS : A + Q \in L^pS$ for any $Q(t) \to 0,\ t \to +\infty$, and $\|Q\|_q < +\infty\}$ of this set for all $p > 0$ and $q > 0$, does not coincide with the latter for some $p > 0$ and $q > 0$. The following assertion is valid in the case of small perturbations vanishing at infinity.

**Theorem 8 ([9]).** *The interior* $\mathrm{Int}_0\ L^pS$ *of* $L^pS$ *with respect to perturbations* $Q(t)$ *vanishing at infinity* ($\to 0$ *as* $t \to +\infty$,) *i.e., the set* $\mathrm{Int}_0\ L^pS \equiv \{A \in L^pS : A + Q \in L^pS$ *for any* $Q(t) \to 0$ *as* $t \to +\infty\}$, *coincides for all* $p > 0$ *with the usual interior* $\mathrm{Int}\, L^pS$.

## 3. Some Generalizations

In this section we consider, instead of a constant $p > 0$, a function $p(t) > 0$ piecewise continuous for $t \geq 0$ and equal at the points of discontinuity to one of its limit values $p(t \pm 0) > 0$. We consider two generalizations of the set $L^pS$ and obtain results for them analogous to Theorem 1 and 2.

First we introduce the set

$$L_1^{p(t)} = \left\{ A : \int_0^t \|X_A(t,\tau)\|^{p(t)} d\tau \leq c_p(A) \equiv const < +\infty, \quad t \geq 0 \right\}.$$

We have the following properties of $L_1^{p(t)}S$:

1°. $\bigcup\limits_{p>0} L^pS \subset \bigcup\limits_{p(t)>0} L_1^{p(t)}S$ and $\bigcup\limits_{p(t)>0} L_1^{p(t)}S \setminus \bigcup\limits_{p>0} L^pS \neq \varnothing$;

2°. $\bigcap\limits_{p(t)} L_1^{p(t)}S \neq \varnothing$ and $\bigcap\limits_{p(t)\geq q(t)} L_1^{p(t)} \neq \varnothing$ for each fixed $q(t) > 0$.

The analog of Theorem 1 for the set $L_1^{p(t)}$ is

**Theorem 9 ([7]).** *If* $p(t) > 0$ *is piecewise continuous for* $t \geq 0$ *and such that for some* $c > 0$, $d \geq 0$ *and a measurable set* $M \subset [0, +\infty)$ *with*

$$\varliminf_{t-\tau\to+\infty} \mathrm{mes}\{[\tau, t] \cap M\}/(t - \tau) > 0,$$

*the inequality*

$$\sum_{i=1}^{k} \inf_{\tau\in[0,\Theta]\cap M} \frac{p(t)}{p(t - i\Theta + \tau)} \geq c \ln k - d$$

*holds for the positive integers* $k = 1, \ldots, [t/\Theta]$ *and sufficiently large constants* $\Theta > 1$, *then* $L_1^{p(t)}S \subset L_1^{q(t)}S$ *for each piecewise continuous* $q(t)$ *such that* $1 \geq q(t)/p(t) \geq const > 0$, $t \geq 0$.

The conclusion of Theorem 9 holds for:

1) a function $p(t) \geq const > 0$ bounded on the half-line $t \geq 0$;

2) a function $p(t) > 0$ such that there are constants $a, b \in (0, 1)$ for which $p(t)/p(\tau) \geq a$ when $\tau \in [bt, t]$ and $t \geq 1$;

3) a function $p(t) > 0$ nondecreasing for $t \geq 0$;

4) each power function $p(t) = t^m$ and a piecewise continuous $q(t)$ such that $1 \geq q(t)/p(t) \geq const > 0$, $t \geq 1$.

The structure of the interior of $L_1^{p(t)}S$ for $p(t) \geq 1$ is established by the following analog of Theorem 2.

**Theorem 10 ([7]).** *The equality* $\operatorname{Int} L_1^{p(t)}S = L_1^{p(t)}$ *holds if and only if there is an interval* $[t_0, +\infty)$ *on which* $p(t)$ *is nonicreasing and not smaller than* 1.

Finally we inverstigate linear-system sets

$$L_0^{p(t)}S = \left\{ A : \int\limits_0^{\xi} \|X_A(\xi,\tau)\|^{p(t)} d\tau \leq c_p(A) < +\infty, 0 \leq \xi \leq t < +\infty \right\},$$

corresponding to functions $p(t)$; these sets are clearly empty if $\varliminf\limits_{t\to+\infty} p(t) = 0$. We have the following inclusions

$$\bigcup_{p>0} L^p S \subset \bigcup_{p(t)>0} L_0^{p(t)} S \subset \bigcup_{p(t)>0} L_1^{p(t)} S$$

and each of them is strict.

The properties of these sets ensure that they are nearer to the sets $L^pS$ than to the $L_1^{p(t)}S$. The following result corresponding to Theorem 1 holds for $L_0^{p(t)}S$.

**Theorem 11 ([7]).** *The inclusion* $L_0^{p(t)}S \subset L_0^{q(t)}S$ *holds for each* $q(t)$ *for which* $1 \geq q(t)/p(t) \geq const > 0$, $t \geq t_0$.

The following assertion distinguishes a difference between properties of $L_0^{p(t)}S$ and $L^pS$.

**Theorem 12 ([7]).** *The inclusion* $L_0^{p(t)} \subset L_0^{q(t)}$ *holds for each function* $q(t)$ *such that* $p(t) \geq q(t) \geq \lambda_q \min\{1, p(t)\}$, *where* $\lambda_q = const \in (0,1)$ *and* $t \geq t_0$, *if and only if* $\lim\limits_{t\to+\infty} p(t) < +\infty$.

We have the following necessary and sufficient condition for the coincidence of the set $L_0^{p(t)}S$ with its interior Int $L_0^{p(t)}S$.

**Theorem 13 ([7]).** $\operatorname{Int} L_0^{p(t)}S = L_0^{p(t)}S \neq \varnothing$ *if and only if* $p(t) > 0$ *is bounded on the half-line* $t \geq 0$ *and is larger than or equal to* 1 *on some interval* $[t_0, +\infty)$.

## 4. The Coppel–Conti sets $M^pS$ of Unstable Linear Systems

We also considered the Coppel–Conti sets $M^pS$ of unstable linear systems $(1_A)$ whose Cauchy matrix $X_A(t,\tau)$ satisfies the inequality

$$\int\limits_t^{+\infty} \|X_A(t,\tau)\|^p d\tau \le c_p(A) < +\infty, \quad t \ge 0.$$

These sets (if $p \ge 1$) connect with the existence of a unique bounded solution of system (2) for any vector-valued function $f \in L_q[0,+\infty)$ with $q = p/(p-1)$ conjugate to $p$.

The Conti problem for these sets is also solved positively.

**Theorem 14 ([10]).** *The inclusion $M^qS \subset M^pS$ is valid for all $q>p>0$.*

For the interior $\operatorname{Int} M^pS$ of the set $M^pS$, we have the assertion analogous to Theorem 2.

**Theorem 15 ([10]).** $\operatorname{Int} M^pS = M^pS$ *if and only if $p \ge 1$.*

## 5. Linear Systems with $L^p$-dichotomy

Finally we consider the general case of linear systems with an $L^p$-dichotomy. This notion is the extension of the concept of exponential dichotomy [11, 12]. It has been inverstigated by W.A.Coppel [2, 12], R.Conti [3–6], P. Talpalaru [13], V.Staikos [14] and other authors. It is known [2, 3], that the system (2) has at least one solution bounded on $R^+$ for any $f \in L_q[0,+\infty)$, $q \ge 1$, if and only if the system $(1_A)$ is $L^p$-dichotomous with $1/p + 1/q = 1$.

We extend the definition of $L^p$-dichotomy from $p \ge 1$ to all $p > 0$.

Denote by $X_A(t)$ the fundamental matrix of $(1_A)$, $X_A(0) = E$.

**Definition.** We say that the system $(1_A)$ is $L^p$-dichotomous on $R^+$, $0 < p < +\infty$, and write $A \in L^pD$ if there exist complementary projectors $P_1$ and $P_2$ such that

$$\int\limits_0^t \|X_A(t)P_1X_A^{-1}(\tau)\|^p d\tau + \int\limits_t^{+\infty} \|X_A(t)P_2X_A^{-1}(\tau)\|^p d\tau \le C_p(A) < +\infty, t \ge 0.$$

The asymptotic behavior of solutions of an $L^p$-dichotomous system is described by the following lemma (see [2, 15] for $p \ge 1$.)

**Lemma 1.** *If the system $(1_A)$ is $L^p$-dichotomous with some $p > 0$, then a) $\lim\limits_{t\to+\infty} x(t) = 0$ for any solution $x(t)$ with $x(0) \in B_1 = P_1\mathbb{R}^n$, b) any solution $x(t)$ with $x(0) \in \mathbb{R}^n \setminus B_1$ satisfies $\overline{\lim\limits_{t\to+\infty}} \|x(t)\| = +\infty$.*

The property of exponential dichotomy is known to be self-dual [11] in the following sense: if a linear system $(1_A)$ is exponentially dichotomous with projectors $P_1$ and $P_2$, then the adjoint linear system $\dot{y} = -A^T(t)y$ is also exponentially dichotomous with projectors $P_2^T$ and $P_1^T$. The property of $L^p$-dichotomy, however, is not self-dual in this sense.

**Lemma 2 ([16]).** *For any $p > 0$ there exists an $L^p$-dichotomous system such that for any $q > 0$ the adjoint system is not $L^q$-dichotomous.*

We obtained [16] that the sets $L^pD$ satisfy the same narrowing property as its two extreme subsets $L^pS$ and $M^pS$ corresponding to the cases $P_1 = E$ and $P_1 = 0$, respectively.

**Theorem 16 ([16]).** *Any linear system $L^p$-dichotomous with $p > 0$ is also $L^q$-dichotomous with any $q$, $0 < q < p$, and the same projectors.*

This theorem follows from the following criterion for a linear system to be $L^p$-dichotomous.

Introduce the sets

$$T_\alpha^1(t) = \{\tau \in [0,t] : \|X_A(t)P_1X_A^{-1}(\tau)\| \geq \alpha\},$$
$$T_\alpha^2(t) = \{\tau \in [t,+\infty) : \|X_A(t)P_2X_A^{-1}(\tau)\| \geq \alpha\}$$

for any $\alpha > 0$.

**Theorem 17 ([16]).** *A linear system $(1_A)$ is $L^p$-dichotomous with some $p > 0$ and projectors $P_1$ and $P_2$ if and only if the following conditions are satisfied for some $\alpha$, $0 < \alpha < 1$ :*

$$\operatorname{mes}\{T_\alpha^1(t) \bigcup T_\alpha^2(t)\} \leq c(\alpha) < \infty, \quad t \geq 0;$$
$$\int\limits_{T_1^1(t)} \|X_A(t)P_1X_A^{-1}(\tau)\|^p d\tau + \int\limits_{T_1^2(t)} \|X_A(t)P_2X_A^{-1}(\tau)\|^p d\tau \leq C < \infty, \quad t \geq 0.$$

As to the structure of the integral interior of $L^pD$, we have

**Theorem 18 ([13, 16]).** *If $p$ and $q$ are conjugate numbers, then* $\operatorname{Int}_q L^pD = L^pD$.

From here we have the important property of roughness with respect to uniformly small perturbations for the set $L^1D$.

## 6. A Linear Boundary Value Problem on $\mathbb{R}^+$

We consider the perturbed nonhomogeneous linear system

$$\dot{y} = F(t)y + g(t) \tag{4}$$

for which we study the following boundary value problem on $R^+$: the existence and asymptotic behavior of bounded solutions.

Using the foregoing properties of the usual and integral interiors of the Coppel–Conti sets, the inclusion property and the Coppel–Conti theorem [2, 3] we obtain some applications to the above-mentioned boundary problem.

**Theorem 19 ([17]).** *Let $F(t) = A(t) + B(t) + D(t)$, $g(t) = f(t) + \varphi(t)$. If $A \in L^pS$ (respectively, $A \in M^pS$) for some $p > 1$, then there exists an $\varepsilon_A > 0$ such that all solutions of the system (5) are bounded (respectively, there exists a unique bounded solution) for any piecewise continuous matrix $B(\cdot)$ with $\|B(t)\| < \varepsilon_A$ for any $t \geq t_B \geq 0$, for any matrix $D(t)$ with $\|D(t)\| \in L_q[0, +\infty)$, $q \geq p/(p-1)$, for any vector function $f(\cdot)$ bounded on the positive semiaxis, and for any $\varphi(\cdot) \in L_q[0, +\infty)$, $q \geq p/(p-1)$.*

*If $p = 1$, then the matrix $D(\cdot)$ and the function $\varphi(\cdot)$ are to be omitted in this assertion.*

If $p > 1$, then a finite sum of the vector-valued functions $\varphi_i(\cdot) \in \in L_{q(i)}[0, +\infty)$ with arbitrary $q(i) \geq p/(p-1)$ can be taken for $\varphi(\cdot)$.

In the general case where the system $(1_A)$ is $L^p$-dichotomous, $p \geq 1$, the dimension of the subspace of all bounded solutions of $(1_A)$ coincides with the dimension of the corresponding subspace of the system $(1_{A+B})$ if $\|B(t)\| \in \in L_q[0, +\infty)$ with $q = p/(p-1)$.

It follows

**Theorem 20 ([16]).** *Let $F(t) = A(t) + B(t)$. If $A \in L^pD$, $p > 1$, then for any matrix $B(\cdot)$, $\|B(t)\| \in L_q[0, +\infty)$ with $q$ conjugate to $p$, and for any vector-function $g(\cdot) \in L_r[0, +\infty)$ with $r \geq p/(p-1)$, the system (4) has a $k$-parameter family of solutions $y(t)$ such that $\lim\limits_{t \to +\infty} y(t) = 0$, where $k = \operatorname{rank} P_1$.*

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