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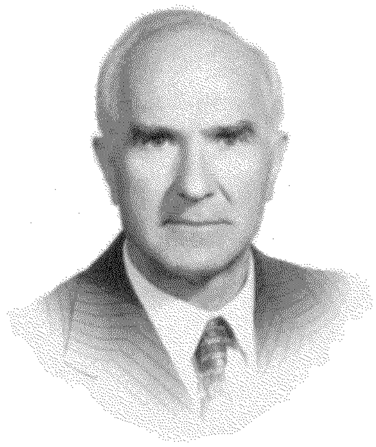
Obituary

PROFESSOR IOTAM KARTSIVADZE



On November 6, 1997 the Georgian public was bereaved of the well-known mathematician Iotam Kartsivadze.

Iotam Kartsivadze was born on September 11, 1918, in Tbilisi, in the family of Nikoloz Kartsivadze who was a man of a progressive outlook, well-educated, a democrat and patriot and one of the leaders of the abortive uprising in 1924 against Red Russia. In the years of repression in the 30ies he was shot without trial, while his wife, Iotam's mother, was exiled for many years to the north of Russia.

After finishing secondary school in 1935, Iotam Kartsivadze entered the mathematical faculty of Tbilisi State University, which he graduated in 1940. The same year he was admitted to a post-graduate course. In 1946 he defended the dissertation for the Candidate of Science Degree. In 1977 he was awarded the title of professor. In different years he worked at University, the Institutes of Mathematics and Computational mathematics of the Georgian Academy of Sciences, and from 1975 to his last day headed the mathematical analysis chair at Tbilisi State University.

Iotam Kartsivadze was a pupil of the famous scientist Academician Niko Muskhelishvili, the founder of the Georgian mathematical school of the theory of elasticity. Naturally, at the beginning of his research career I. Kartsivadze was concerned with problems of this theory and related questions. With time, the range of his interests broadened: for a long time he worked on problems of the potential theory, dealt successfully with some issues of functional analysis, and whatever work he did bore the mark of his outstanding talent.

When a post-graduate student, in his first research paper (*Trudy Tbiliss. Mat. Inst. Razmadze Akad. Nauk Gruzin. SSR* **12**(1943)) I. Kartsivadze generalized Muskhelishvili's method for a half-plane to a circle. As is known, displacements and stresses in deformed two-dimensional bodies can be expressed by means of the Kolosov-Muskhelishvili formulas with two analytic functions. By continuing one of these functions to the exterior of a circle, the problem can be reduced to a simple boundary value problem for analytic functions whose solution can be written in explicit form through Cauchy-type integrals of boundary values. Later, I. Kartsivadze developed this method for arbitrary simply connected domains mapped onto the circle by means of a rational function (*Soobshch. Akad. Nauk Gruzin. SSR* **7**(1946), No. 8). However, in such a general situation, the analytic continuation to the exterior of a circle leads to the appearance of some poles which can be eliminated by an appropriate choice of the parameter values contained in the continuation formula. This choice has a certain mechanical meaning and is eventually reduced to finding solvability conditions of problems of elasticity. The suggested method is highly effective and described in the well-known monograph by Niko Muskhelishvili "Some Problems of the Mathematical Theory of Elasticity" (Moscow, 1954, Ch.VI, §§121–123, and Ch. VII, §§125–127).

I. Kartsivadze was an active participant in N. Muskhelishvili's popular seminar on the theory of singular integral equations and conjugation problems of the theory of analytic functions. During that time he obtained a number of important results on the seminar's topics. He investigated the general properties of one-dimensional singular operators containing integrals in the sense of Cauchy's principal value and studied the possibility of permutating of integrals, when at least one of them is singular, for various classes of functions (*Soobshch. Akad. Nauk Gruzin. SSR* **15**(1955), No. 1). Special mention should be made of one result closely related to the theory of singular integral equations. When the integration contour is an open curve and the admissible functions belong to the Hölder class everywhere on the contour, the latter theory makes essential use of the fact that a Cauchy-type integral operator of the density, satisfying the Hölder condition on the contour, has singularities of the order less than one near the contour ends. I. Kartsivadze studied the behaviour of a Cauchy-type integral near the ends for other density classes and found an exceptional irregularity of the behaviour of a Cauchy-type integral near the ends of the integration path for densities satisfying the Hölder condition only inside the integration contour. This result indicated that it was necessary to use subtler methods when investigating singular operators in the case of non-Hölder classes of functions (*Trudy Tbiliss. Mat. Inst. Razmadze Akad. Nauk Gruzin. SSR* **18**(1957)). From the same standpoint, no less importance is attached to I. Kartsivadze's paper (*Dokl. Akad. Nauk SSSR* **109**(1956), No. 3) where he constructed a refined example of the singular operator

$$s\varphi \equiv \alpha(x)\varphi(x) + \frac{\beta(x)}{\pi i} \int\limits_{c} \frac{\varphi(t)dt}{t-x},$$

which is not normally solvable even if $\alpha \pm \beta$ is different from zero everywhere on c. Here c is the unit circle, while α and β are smooth everywhere on c

except one point on c at which these functions, remaining bounded, have a discontinuity point of second kind.

I. Katsivadze devoted a series of papers to problems of the Newtonean potential, i. e., to studying the properties of functions of the form $u_\mu(x) = \int \frac{\mu(dy)}{|x-y|}$, where the integral is taken over the support of a generalized measure μ and $|x-y|$ denotes the Euclidean distance in R^3.

I. Kartsivadze studied the conditions which make it possible to represent the function u defined in the domain D as a potential u_μ, where μ is a distribution with a support lying outside D. In particular, he formulated the following theorem (*Soobshch. Akad. Nauk Gruzin. SSR* **32**(1963), No. 1; *Trudy Vichisl. Tsentra Akad. Nauk Gruzin. SSR* **4**(1963)): the function u can be represented as a potential u_μ, where μ is the measure lying entirely outside D, if and only if for any generalized measure ν having a support in D and being such that the potential u_ν is positive on the boundary u_ν of D we have $\int u_\nu d\nu \geq 0$. Another refined result obtained by I. Kartsivadze concerns the uniqueness of solution of the classical boundary value problems of harmonic functions (*Trudy Tbiliss. Univ. Mat. Mekh. Astronom.* **110**(1965)). Let D be a two-dimensional simply connected domain whose boundary L is composed of two smooth open arcs L_1 and L_2 which are interconnected at the points t_1 and t_2 and form the internal angles ν_1 and ν_2 at t_1 and t_2, respectively $(0 < \nu_i \leq 2\pi)$. Let u be a harmonic function in D, having continuous partial derivatives of first order in $D \cup L \backslash \{t_1, t_2\}$. In that case, if L satisfies the boundary conditions

$$u|_{L_1} = 0, \quad \frac{\partial u}{\partial n}|_{L_2} = 0,$$

everywhere on L except perhaps the points t_1 and t_2, then u has the form

$$u(t) = \sum_{k \in Z} c_k u_k(z),$$

where $u_k(z) = \operatorname{Re} w_k(z)$ and $w_k(z)$ realizes a conformal and bijective mapping of the domain D onto the many-sheeted domain B_k of the form

$$B_k = \begin{cases} \{\zeta : -\frac{\pi}{2} < \arg\zeta < k\pi\} & \text{for} \quad k \geq 0, \\ \{\zeta : k\pi < \arg\zeta < -\frac{\pi}{2}\} & \text{for} \quad k < 0. \end{cases}$$

This implies that in the class of harmonic functions with the above-mentioned properties there does not exist a function for which the estimates

$$\alpha|z-t_1|^\lambda \leq u \leq \beta|z-t_i|^\lambda$$

are valid (here α and β are positive numbers and $\frac{m\pi}{2\lambda_i} < \lambda < \frac{(m+1)\pi}{2\nu_i}$, where m is a positive number).

I. Kartsivadze introduced the notion of "the regularity of a point" of the boundary of a domain and used it to prove the following theorem: the distribution μ concentrated on the boundary of D and representing a given potential u is unique if and only if all points of the boundary are regular. Analogous questions of the potential theory were studied in collaboration with J. V. Kapanadze for domains with a noncompact boundary (*Trudy Inst. Prikl. Mat. I. N. Vekua* **2**(1969)).

Of the problems considered in I. Kartsivadze's papers in other fields of mathematics we should mention the following results which are of special interest. In the paper written jointly with N. N. Vakhania (*Mat. Zametki* **3**(1968), No. 2), the properties of intersections of sequences of nested closed sets in metric spaces are studied. Various "abnormalities" of the behaviour of such sequences are indicated. In particular, when the enveloping space E is the Banach one, the intersection of a sequence of decremental closed balls is always nonempty (despite the noncompactness of balls). However this is not true, generally speaking, for such a sequence of closed convex bodies in E. Let $\varkappa(F)$ be a numerical characterictic F determined by the formula

$$\varkappa(F) = \sup_{x \in F} \frac{\Gamma_x(F)}{R_x(F)},$$

where $R_x(F) = \sup\limits_{y \in F} \|x - y\|$, $\Gamma_x(F) = \inf_{y \in F} \|x - y\|$.

It appears that if a sequence of closed convex bodies $(F_n)_{n \geq 1}$ $(F_n \supset F_{n+1})$ possesses the property $\limsup \varkappa(F_n) > \frac{1}{2}$, then the intersection of all F_n's is nonempty. Moreover, for any $\varkappa < \frac{1}{2}$ one can find a Banach space and a sequence of closed convex sets $F_1 \supset F_2 \supset \cdots$, in this space such that $\limsup \varkappa(F_n) = \varkappa$ and $\cap F_n = \varnothing$.

Another result we wish to mention is contained in the paper written by I. Kartsivadze in collaboration with I. Kandelaki and T. Chantladze (*Trudy Tbiliss. Univ. Mat. Mekh. Astronom.* **179**(1979)). A method for constructing an orthogonal multiplication is exposed and its properties are studied in the Hilbert space H. The term "orthogonal multiplication" is applied to any bilinear mapping $f : H \times H \to H$ which possesses the property $\|f(x, y)\| = \|x\|\|y\|$, i. e., the norm of the product is equal to the product of the norms of cofactors. As is known, in the finite-dimensional case this is possible only when the dimensions of a vector space are $n = 1, 2, 4$ or 8, giving, respectively, real, complex, quaternion and Kelly numbers. Like in the finite-dimensional case, for an infinite Hilbert space H the consideration of an orthogonal multiplication can be reduced to the construction of an infinite sequence of orthogonal operators $\{u_i\}_{i \in N}$ such that

(1) $u_i^2 = -1$,

(2) $u_i u_j + u_j u_i = 0$ for $i \neq j$.

This is possible by introducing the multiplication by the formula $f(x, y) = \sum (x|l_k) u_k(y)$, where $(\cdot|\cdot)$ is the scalar product and $\{l_k\}_{k \in N}$ is an orthogonal basis in H. Such a multiplication was constructed in the paper, but it turned out that, unlike the finite-dimensional case, the orthogonal sequence of vectors from H: $x, u_1(x), u_2(x), \ldots, u_n(x), \ldots$ is not a basis for any $x \in H$. Moreover, the following theorem is valid: for any sequence of operators

$\{u_i\}_{i\in N}$ with the above properties, the codimension of the subspace of H generated by the vectors $\{u_i(x)\}_{i\in N}$ is infinite for any $x \in H$. In the next paper (*Trudy Tbiliss. Univ. Mat. Mekh. Astronom.* **197**(1978)) I. Kartsivadze clarified the reasons of this fact.

I. Kartsivadze began the teaching career in his young years at the mathematical analysis chair first as assistant lecturer, then as senior lecturer, assistant professor and, finally, as professor and head of the chair. His bright personality, charm, perfect knowledge of the subject and audience, ability to express profound ideas in a simple form – all these features made him highly popular among students. His lectures invariably attracted an overcrowded audience. The teaching world highly appreciated I. Kartsivadze's work as a lecturer. He was awarded several honorary diplomas. I. Kartsivadze put his teaching experience in the text-books on mathematical analysis which he wrote in collaboration with a group of colleagues. He was also the author of an excellent two-volume text-book written at a modern mathematical level.

His broad outlook, friendliness, modesty, adherence to high moral standards, acute sense of responsibility and impartiality won him love and respect of everybody who came in contact with him. The light memory of Iotam Kartsivadze will remain for ever in our hearts.

T. Chantladze, N. Kandelaki, A. Lomtatidze, D. Ugulava,