Mem. Differential Equations Math. Phys. 5(1995), 122–124

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ON OSCILLATORY PROPERTIES OF SECOND ORDER SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

(Reported on May 8, 1995)

Consider the system

$$\begin{aligned} u' &= p(t)v, \\ v' &= -q(t)u, \end{aligned} \tag{1}$$

where $p, q : [0, +\infty[\to [0, +\infty[$ are locally summable functions.

Definition. The system (1) is said to be oscillatory if it has at least one oscillatory solution and nonoscillatory otherwise.

It is known (see [1]) that if

$$\int^{+\infty} p(s)\,ds = +\infty \quad \text{and} \quad \int^{+\infty} q(s)\,ds = +\infty,$$

then the system (1) is oscillatory, but if

$$\int^{+\infty} p(s)\,ds < +\infty \quad \text{and} \quad \int^{+\infty} q(s)\,ds < +\infty,$$

then it is nonoscillatory.

Therefore we will assume that

$$\int^{+\infty} p(s)\,ds = +\infty \quad \text{and} \quad \int^{+\infty} q(s)\,ds < +\infty. \tag{2}$$

(The case where $\int^{+\infty} p(s)\,ds < +\infty$ and $\int^{+\infty} q(s)\,ds = +\infty$ can be easily reduced to the case (2)).

As shown in [2], if the condition (2) is fulfilled and for some $\lambda < 1$

$$\int^{+\infty} f^{\lambda}(s)q(s)\,ds = +\infty,$$

where

$$f(t) = \int_0^t p(s)\,ds, \tag{3}$$

1991 *Mathematics Subject Classification.* 34C10.

Key words and phrases. Second order system of ordinary differential equations, oscillatory solution.

then the system (1) is oscillatory.

Therefore we will consider the case where

$$\int^{+\infty} p(s)\,ds = +\infty \quad \text{and} \quad \int^{+\infty} f^{\lambda}(s)q(s)\,ds < +\infty \ \text{ for all } \ \lambda < 1,$$

where f is the function defined by (3).

Introduce the notation

$$g_*(\lambda) = \liminf_{t\to+\infty} f^{1-\lambda}(t) \int_t^{+\infty} f^{\lambda}(s)q(s)\,ds,$$

$$g^*(\lambda) = \limsup_{t\to+\infty} f^{1-\lambda}(t) \int_t^{+\infty} f^{\lambda}(s)q(s)\,ds$$

for $\lambda < 1$ and

$$g_*(\lambda) = \liminf_{t\to+\infty} f^{1-\lambda}(t) \int_0^{t} f^{\lambda}(s)q(s)\,ds,$$

$$g^*(\lambda) = \limsup_{t\to+\infty} f^{1-\lambda}(t) \int_0^{t} f^{\lambda}(s)q(s)\,ds$$

for $\lambda > 1$.

Theorem 1. *Let $g_*(0) \le \frac{1}{4}$, $g_*(2) \le \frac{1}{4}$ and*

$$g^*(0) > g_*(0) + \frac{1}{2}\Big(\sqrt{1-4g_*(0)} + \sqrt{1-4g_*(2)}\,\Big).$$

Then the system (1) *is oscillatory.*

Theorem 2. *Let $g_*(0) \le \frac{1}{4}$ and $g_*(2) \le \frac{1}{4}$. Moreover, let either*

$$g^*(\lambda) > \frac{\lambda^2}{4(1-\lambda)} + \frac{1}{2}\Big(1 + \sqrt{1-4g_*(2)}\,\Big)$$

for some $\lambda < 1$ or

$$g^*(\lambda) > \frac{\lambda^2}{4(\lambda-1)} - \frac{1}{2}\Big(1 - \sqrt{1-4g_*(0)}\,\Big)$$

for some $\lambda > 1$. Then the system (1) *is oscillatory.*

Corollary 1. *Let*

$$\lim_{\lambda\to1} |1-\lambda| g^*(\lambda) > \frac{1}{4}.$$

Then the system (1) *is oscillatory.*

Corollary 2. *Let for some $\lambda \ne 1$*

$$|1-\lambda| g_*(\lambda) > \frac{1}{4}.$$

Then the system (1) *is oscillatory.*

Theorem 3. *If for some* $\lambda \neq 1$

$$|1-\lambda| g_*(\lambda) > \frac{(2\lambda-1)(3-2\lambda)}{4}, \quad |1-\lambda| g^*(\lambda) < \frac{1}{4},$$

then the system (1) *is nonoscillatory.*

References

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