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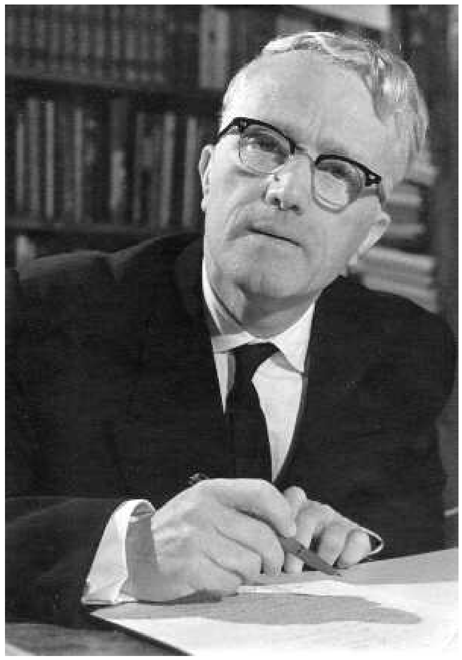

VIKTOR KUPRADZE 110

110 years have passed since the birthday of the outstanding Georgian scientist, a public figure and a statesman, academician Viktor Kupradze. Mathematicians and mechanicians throughout the world are well familiar with his name. Academician Viktor Kupradze made a tremendous contribution to the theory of differential and integral equations, problems of mathematical physics, the theory of elasticity and applied mathematics.

Viktor Kupradze was born on 2 November 1903 in village Kela in Georgia, in a railway worker's family. Little Viktor went to the specialized school in Kutaisi, where a comparatively extended course in mathematics was taught. Viktor's turn for mathematics attracted the attention of his teacher and, following his advice, in 1922 Kupradze became a student of the physico-mathematical faculty of the Tbilisi State University. In 1927 he graduated from the University with honours and as nominee of professors Andria Razmadze and Nikoloz Muskhelishvili, founders of the worldwide known Georgian mathematical school, was left at the University to be prepared for research work. He became an assistant of A. Razmadze in mathematical analysis and an assistant of N. Muskhelishvili in theoretical mechanics.

He also delivered lectures at the Tbilisi Polytechnical Institute. The scientific supervisor of Viktor Kupradze, professor N. Muskhelishvili wrote in the testimonial: “The post-graduate student has mastered quite well the main academic disciplines. He has invariably shown the ability to independent, creative and critical thinking. I can say with confidence that under proper conditions he will become an outstanding specialist in applied mathematics”.

In 1930–1933 he was a post-graduate student at the Academy of Sciences of the USSR in Leningrad (St. Petersburg), where his supervisors were the prominent Russian scientists Alexei Krilov and Vladimir Smirnov.

In the period from 1933 to 1935 Kupradze worked as scientific secretary at the Steklov Mathematical Institute of the Academy of Sciences of USSR. In 1935 he defended his doctor’s thesis (skipping the candidate thesis) on the topic: “Boundary Value Problems of the Electromagnetic Wave Theory”. In the same year Kupradze returned to Tbilisi where he was appointed director of the Tbilisi Mathematical Institute (now Andrea Razmadze Mathematical Institute). During the Great Patriotic War (the World War II) V. Kupradze served in the Soviet Army, participated in the cruel battles for Crimean Peninsula. Due to his fluent German, he was the Executive secretary of Editorial Board of the military newspaper “Zoldatenvaarheit” published in German. In 1943 he was demobilized and appointed pro-rector of the Tbilisi State University, responsible for research work.

From 1944 to 1953 Kupradze was the Minister of Education of Georgia.

In 1946 he was elected Full Member of the Academy of Sciences of Georgia.

In 1954–1958 he held the position of the rector of Tbilisi State University.

In 1962 the Georgian Mathematical Society was re-founded and V. Kupradze was elected its second president. The Society was first founded in 1923 by Andrea Razmadze, who was the president until he passed away in 1929.

In 1963 Kupradze was elected academician-secretary of the department of mathematics and physics of the Academy of Sciences of Georgia, where he worked fruitfully till 1981. At the same time he headed the chair of differential and integral equations of the Tbilisi State University. From 1947 to 1985 Kupradze was a member of Presidium of the Georgian Academy of Sciences.

V. Kupradze widely participated also in the public life of Georgia and the former USSR. In 1947 he took part in the Congress of Asiatic and African Peoples held in Delhi. From 1954 to 1963 he was Chairman of the Supreme Soviet (Parliament) of Georgia. In 1955 he was sent to the USA (New York) as a member of Soviet delegation to the 10-th Session of the UN General Assembly. V. Kupradze was actively involved in the international scientific cooperation. Being member of various reputable organizations such as the National Committee of Soviet Mathematicians, National Committee on Theoretical and Applied Mathematics, Bureau of the Scientific Council on Plasticity and Strength of the Academy of Sciences of the USSR. V. Kupradze played a significant role in strengthening scientific contacts between the scientists of different countries. He was a member of the editorial boards of domestic and international scientific journals, including “Uspekhi Matematicheskikh Nauk”, “Differentsial’nye Uravneniya”, “Journal of Thermal Stresses” etc.

Special tribute must be given to V. Kupradze as an excellent teacher, thesis adviser, and lecturer with a considerable personal charisma. For over 40 years he had been the head of the chair of differential and integral equations at Tbilisi State University and brought up several generations of Georgian mathematicians. He had many disciples and followers throughout the countries he visited. Attracted by Kupradze's charisma, many of his pupils became famous scientists and fruitfully continue mathematical scientific and academic activities both in Georgia and abroad.

V. Kupradze passed away on 25 April 1985, about 28 years ago, but all those people who knew him will cherish the memory of his warm, unforgettable personality and his profound intelligence.

The mathematical heritage of V. Kupradze is very rich. He began his scientific activities in the late twenties of the 20th century. His fruitful and tireless work actually has lasted about 55 years. V. Kupradze's contributions to mathematics and mechanics can be divided into six large groups:

- Problems related to the justification of Sommerfeld's Radiation Conditions and boundary value problems (BVP) for the Helmholtz equation;
- Diffraction and scattering of electro-magnetic waves;
- Mathematical problems of the theory of elasticity (BVPs of statics and steady state oscillations, and initial boundary value problems of general dynamics);
- Theory of one- and multi-dimensional singular integral equations and their applications;
- Investigation of refined models of the theory of elasticity (Thermoelasticity, Cosserat model etc.);
- Problems of numerical simulation and approximate solutions of BVPs of mathematical physics, Method of Fundamental Solutions.

A short account of V. Kupradze's contribution to the listed issues reads as follows.

1. Sommerfeld's radiation principle originally formulated in 1912 by the outstanding German physicist and mathematician A. Sommerfeld, concerns the existence and uniqueness of a solution to boundary value problems for the Helmholtz equation,

$$\Delta\, u(x) + k^2 u(x) = 0, \quad x \in \Omega, \tag{1}$$

where Δ is the Laplace operator, k^2 is a real valued constant, called the wavenumber, and Ω is an unbounded domain in the n-dimensional Euclidean space $\mathbb{R}^n$, $n = 2, 3, \ldots$. The basic Dirichlet and Neumann boundary value problems, when either the traces of the solution itself or of its normal derivative are prescribed on the boundary $\Gamma := \partial\Omega$, have unique solutions only under special constraints on the growth of $u(x)$ at infinity

$$u(x) = \mathscr{O}\big(|x|^{\frac{1-n}{2}}\big) \ \text{ as } \ |x| \to \infty, \tag{2}$$

$$\frac{\partial u(x)}{\partial r} \pm ikr = \mathscr{o}\big(|x|^{\frac{1-n}{2}}\big) \ \text{ as } \ r := |x| \to \infty. \tag{3}$$

In 1934, V. Kupradze managed to substantiate this principle mathematically. He reduced these problems to Fredholm type boundary integral equations and showed the existence of a solution under sufficiently general conditions. Ten years later, the same result was obtained by H. Weyl. Moreover, Kupradze predicted and later I. Vekua and F. Rellich proved that the condition (2) is not independent and follows from the radiation condition (3).

2. Electromagnetic wave diffraction problems. A series of V. Kupradze's investigations are devoted to the diffraction of electromagnetic sinusoidal waves around an arbitrary plane contour, described by the Maxwell's equations

$$\begin{cases} \mathbf{curl}\, H + i\omega\varepsilon E = 0 \\ \mathbf{curl}\, E - i\omega\mu H = 0 \end{cases} \quad \text{in } \Omega \subset \mathbb{R}^2 \tag{4}$$

with corresponding boundary and transmission conditions.

These problems were previously solved by A. Sommerfeld, V. Sternberg, H. Freudental and other researchers for special domains with particular geometry. V. Kupradze made essential use of the method of integral equations.

He reduced the diffraction problems to equivalent boundary integral equations and proved their unique solvability.

For these results, in 1938 Viktor Kupradze was awarded the prize at the All-Union Competition of Young Scientists. It was included into the well-known V. Smirnov's university course on higher mathematics and translated into nearly all languages of the world.

3. Basic boundary value problems of statics and stationary oscillations of the elasticity theory. The approach developed for the Helmholtz equation, Viktor Kupradze generalized to investigate the system of stationary oscillation equations of elasticity

$$\mu\Delta\, U(x) + (\lambda+\mu)\,\mathbf{grad}\,\mathbf{div}\, U(x) + \varrho\omega^2 U(x) = 0, \quad x \in \Omega, \tag{5}$$

where $U(x) := (U_1(x), U_2(x)U_3(x))^\top$ is the displacement vector, λ and μ are Lamé constants, ϱ is the density of the elastic material, while ω is the oscillation frequency. On the boundary of the domain Ω (occupied by an elastic body) there is prescribed either the displacement vector

$$U^+(x) = F(x), \quad x \in \partial\Omega, \tag{6}$$

or the stress vector

$$(TU)^+(x) := 2\mu\,\frac{\partial U(x)}{\partial n} + \\ + \lambda n(x)\,\mathbf{div}\, U + \mu n(x) \times \mathbf{curl}\, U(x) = G(x), \quad x \in \partial\Omega. \tag{7}$$

For the system (5) endowed with the boundary condition either (6) or (7), V. Kupradze proved the uniqueness of a classical solution. Then he constructed solutions of three types, which he called a simple-, a double- and an antenna-layer potential. He investigated the fundamental properties of these potentials and derived jump formulas; Theorems analogous to the Lyapunov–Tauber theorem were proved, stating that the normal derivative of a regular harmonic double-layer potential is continuous up to the domain boundary (in contrast to the double layer potential and the normal derivative of a single-layer potential, which are discontinuous at the boundary). Furthermore, he proved an important fact that the above-mentioned boundary value problems are solvable under quite general conditions.

One of the first significant results obtained by V. Kupradze jointly with S. Sobolev concerns the wave propagation on the elastic body-fluid interface. The existence of a wave of a new type was established by mathematical means.

The basic boundary value problems of statics and steady-state oscillations of the elasticity theory with the first and second type boundary conditions, V. Kupradze reduced to equivalent systems of singular integral equations. In particular, he investigated the mentioned BVPs for homogeneous and piecewise-homogeneous elastic bodies showed that the corresponding boundary singular integral operators are of normal type.

From the 40s investigation of two- and three-dimensional problems of the elasticity theory held an ever growing place in the scientific activities of V. Kupradze and his followers. Building up a strong research team, he was extending, together with his disciples, the potential method to the basic boundary value and nonstandard transmission problems of the mathematical theory of elasticity. He constructed the matrix of fundamental solutions of the system of steady state elastic oscillations explicitly (now called "Kupradze's matrix") and formulated the radiation conditions in the elasticity theory (known as the "Sommerfeld–Kupradze principle") which in the three-dimensional case read as follows,

$$\begin{cases} U = U^{(p)} + U^{(s)}, \\ \Delta U^{(p)} + k_1^2 U^{(p)} = 0, \quad \mathbf{curl}\, U^{(p)} = 0, \quad k_1^2 = \dfrac{\varrho\,\omega^2}{\lambda + 2\mu}\,, \\ \Delta U^{(s)} + k_2^2 U^{(s)} = 0, \quad \mathbf{div}\, U^{(s)} = 0, \quad k_2^2 = \dfrac{\varrho\,\omega^2}{\mu}\,, \\ \dfrac{\partial U^{(p)}(x)}{\partial r} - ik_1 U^{(p)}(x) = \mathscr{o}\big(|x|^{-1}\big) \ \text{ as } \ r = |x| \to \infty, \\ \dfrac{\partial U^{(s)}(x)}{\partial r} - ik_2 U^{(s)}(x) = \mathscr{o}\big(|x|^{-1}\big) \ \text{ as } \ r = |x| \to \infty, \end{cases} \tag{8}$$

where $U^{(p)}$ and $U^{(s)}$ are the so-called longitudinal (potential) and transverse (solenoidal) parts of the displacement vector U. These conditions have a crucial role in the proof of uniqueness theorems for exterior BVPs.

4. Multidimensional singular integral equations and their applications. In 1935, in his doctoral thesis V. Kupradze developed the method of potentials for three-dimensional problems of diffraction. During the subsequent 40 years V. Kupradze, and his collaborators developed and worked out the theory of singular integral equations on manifolds, generalizing results of S. Mikhlin and G. Giraud for multidimensional singular integral equations. They successfully applied the theory of singular potentials and newly created theory of singular integral equations to the analysis of boundary value problems of statics and steady state oscillations, as well as initial boundary value problems of general dynamics of the theory of elasticity. By the same approach, basic problems of some refined models of the theory of elasticity (anisotropic elasticity, thermoelasticity, couple-stress elasticity etc.) have been thoroughly studied. These results are exposed in the fundamental monograph "Three Dimensional Problems of the Mathematical Theory of Elasticity and Thermoelasticity" (V. Kupradze, T. G. Gegelia, M. O. Basheleishvili, T. V. Burchuladze; *North-Holland Publ. Comp., Amsterdam*, 1979). This monograph became a companion desk book for scientists working in the field.

5. Approximate solutions of boundary value problems of mathematical physics. In the early 1960s, by modifying and generalizing Picone's method V. Kupradze found new effective effective of constructing approximate solutions for a wide class of boundary value problems of mathematical physics. The method can be used for plane and spatial, basic and mixed boundary value problems of statics and oscillation theory in the case of homogeneous and piecewise-homogeneous, isotropic and anisotropic bodies. In the scientific literature this method is referred to as "Method of Fundamental Solutions" (MFS).

The main idea of the MFS is to distribute the singularity poles $\{y_k\}_{k=1}^{\infty}$ of the fundamental solution $\Gamma(x-y)$ of the differential operator outside the domain under consideration, construct the set of functions $\{\Gamma(x-y_k)\}_{k=1}^{\infty}$, prove its density properties in appropriate function spaces, and then approximate the solution by a linear combination of the fundamental solutions, $\sum\limits_{k=1}^{N} C_k\Gamma(x-y_k)$ with unknown coefficients C_k which are to be determined by satisfying the corresponding boundary conditions. Starting from 1970s, the MFS gradually became a useful technique for solving a large variety of physical and engineering problems.

The level of present-day computing facilities makes V. Kupradze's methods of constructing effective solutions even more important and enjoys ever growing popularity among mathematicians and engineers.

The theory and the methods developed by V. Kupradze are widely and successfully applied to many theoretical and practical spheres of mathematical physics and engineering even nowadays. That means that Viktor Kupradze as a celebrated scientist is still alive – as an intellectual and spiritual bridge from the 20th century to the 21st one.

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