МАТЕМАТИЧКИ ВЕСНИК
59 (2007), 57–64

UDK 515.124
оригинални научни рад
research paper

ON PSEUDO-SEQUENCE-COVERING π IMAGES OF LOCALLY SEPARABLE METRIC SPACES

Shengxiang Xia

Abstract. In this paper, pseudo-sequence-covering π images and pseudo-sequence-covering, s, π images of locally separable metric spaces are discussed, their internal characterizations are given, which extends and improves the study of images of locally separable metric spaces.

1. Introduction and definitions

To find nice internal characterizations of certain images of metric spaces is one of main tasks on general topology. Recently, many results about the study on various images of locally separable metric spaces have been obtained (see [2,4,6,8,9,10,12,13,16,17]).

The following theorem was proved by Y. Ge, which answered a question in [8].

THEOREM 1.1. [4] *The following are equivalent for a space X:*

(1) *X is a quotient compact image of a locally separable metric space;*

(2) *X is a pseudo-sequence-covering quotient compact image of a locally separable metric space.*

It is known that every compact mapping from a metric space is an s, π mapping, so, it is natural to ask the following question.

QUESTION 1.2. Suppose X is a quotient, s, π image of a locally separable metric space. Is X a pseudo-sequence-covering, quotient, s, π image of a locally separable metric space?

In [5], the definition of the concept of *wcs*-covers was introduced, and by using it, characterizations of pseudo-sequence-covering π images of metric spaces are given. What is a nice characterization of pseudo-sequence-covering, π images of locally separable metric spaces? There is no answer to this question now. In this paper, by using *wcs*-covers, characterizations of pseudo-sequence-covering, π

AMS Subject Classification: 54E20, 54E40, 54D55

Keywords and phrases: Pseudo-sequence-covering mappings; π mappings; *wcs*- covers; point-star networks.

images and pseudo-sequence-covering, s, π images of locally separable metric spaces are given, which affirmatively answer Question 1.2 and extends the study of images of locally separable metric spaces.

In this paper, all spaces are regular T_1, and all mappings are continuous and surjective. N denotes the set of all natural numbers, and $\omega = N \cup \{0\}$. We suppose that every convergent sequence contains its limit point.

DEFINITION 1.3. [15] Let $f\colon X \to Y$ be a mapping, and (X, d) be a metric space. f is called a π mapping, if $d(f^{-1}(y), X \setminus f^{-1}(U)) > 0$ for every $y \in Y$ and every open neighborhood U of y in Y.

DEFINITION 1.4. Let $f\colon X \to Y$ be a mapping.

(1) f is called pseudo-sequence-covering [7,8], if for every convergent sequence S in Y, there exists a compact subset K of X such that $f(K) = S$;

(2) f is called subsequence-covering [12], if for every convergent sequence S in Y, there exists a compact subset K of X such that $f(K)$ is an infinite subsequence of S;

(3) f is called sequentially-quotient [1], if for every convergent sequence S in Y, there exists a convergent sequence L of X such that $f(L)$ is an infinite subsequence of S.

DEFINITION 1.5. [10] Let $\{\mathcal{P}_n\}$ be a sequence of covers of a space X. $\{\mathcal{P}_n\}$ is called a point-star network of X, if $\{st(x, \mathcal{P}_n)\}$ is a network for each $x \in X$.

DEFINITION 1.7. Let $\mathcal{P}$ be a cover of a space X.

(1) $\mathcal{P}$ is called a cs^*-cover [10] for X, if for every convergent sequence S in X, there is $P \in \mathcal{P}$ and a subsequence S' of S such that S' is eventually in P;

(2) $\mathcal{P}$ is called a wcs-cover [5] for X, if for every convergent sequence S converging to $x \in X$, there exists a finite subfamily $\mathcal{P}'$ of $(\mathcal{P})_x$ such that S is eventually in $\bigcup \mathcal{P}'$.

We recall that a space is called a cosmic space [14], if it has a countable network. It is known that a space is a cosmic space if and only if it is an image of a separable metric space.

2. Main results

THEOREM 2.1. *The following are equivalent for a space X:*

(1) *X is a pseudo-sequence-covering, π image of a locally separable metric space;*

(2) *X has a cover $\{X_\alpha : \alpha \in A\}$, each subspace X_α has a sequence of countable covers $\{\mathcal{P}_{\alpha,n}\}_{n\in\omega}$ (where $\mathcal{P}_{\alpha,0} = \{X_\alpha\}$) satisfying the following:*

(a) $\{\mathcal{P}_n\}_{n\in\omega}$ is a point-star network of X, where $\mathcal{P}_n = \bigcup_{\alpha\in A} \mathcal{P}_{\alpha,n}$ and consisting of cosmic subspaces;

(b) For every sequence S converging to $x \in X$, there exists a finite subset A' of A such that for each $n \in \omega$, $(\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})$ is a wcs-cover for S (i.e. S is eventually in $\bigcup \mathcal{P}'_n$ for some finite subfamily $\mathcal{P}'_n$ of $(\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})_x$).

Proof. (1) $\Rightarrow$ (2). Let $f\colon M \to X$ be a pseudo-sequence-covering, π mapping, M be a locally separable metric space. By 4.4.F in [3], $M = \bigoplus_{\alpha\in A} M_\alpha$, where M_α is a separable metric space. Let $\mathcal{B}_n$ be a locally finite open cover of M and refine $\{B(z, \frac{1}{2n})\}_{z\in M}$, set

$$X_\alpha = f(M_\alpha), \quad \mathcal{P}_{\alpha,n} = \{f(M_\alpha \cap B) : B \in \mathcal{B}_n\}.$$

Then $\{X_\alpha : \alpha \in A\}$ is a cover of X, and $\{\mathcal{P}_{\alpha,n}\}_{n\in N}$ is a sequence of countable covers of X_α. Put

$$\mathcal{P}_{\alpha,0} = \{X_\alpha\}, \quad \alpha \in A, \quad \mathcal{P}_n = \bigcup_{\alpha\in A} \mathcal{P}_{\alpha,n}, \quad n \in \omega;$$

then $\mathcal{P}_n$ consists of cosmic subspaces. For each $x \in X$, and an open neighborhood U of x, then $d(f^{-1}(x), M \setminus f^{-1}(U)) > \frac{1}{n_0}$ for some $n_0 \in N$. Taking $m \geq 2n_0$, we have $st(f^{-1}(x), \mathcal{B}_m) \subset B(f^{-1}(x), \frac{1}{m}) \subset f^{-1}(U)$. Hence we have $st(x, \mathcal{P}_m) \subset f(st(f^{-1}(x), \mathcal{B}_m)) \subset U$. Thus, $\{\mathcal{P}_n\}_{n\in\omega}$ is a point-star network of X.

Let S be a sequence $\{x_n\}$ converging to $x \in X$. Since f is a pseudo-sequence-covering mapping, there exists a compact subset K of M such that $f(K) = S$. As K intersects only finitely many M_α, there is a finite subset A' of A such that $K \subset \bigcup_{\alpha\in A'} M_\alpha$, and we may assume that $M_\alpha \cap K \neq \phi$ for each $\alpha \in A'$. Hence $S \subset \bigcup_{\alpha\in A'} X_\alpha$. Note that $f^{-1}(x) \cap K$ is a compact subset of M, and thus there is a finite subfamily $\mathcal{B}'_n$ of $\mathcal{B}_n$ covering $f^{-1}(x) \cap K$ for each $n \in \omega$, and we may assume that $B \cap f^{-1}(x) \cap K \neq \phi$ for each $B \in \mathcal{B}'_n$. Set

$$\mathcal{P}'_n = \{f(B \cap M_\alpha) : B \in \mathcal{B}'_n, \alpha \in A', f^{-1}(x) \cap B \cap M_\alpha \neq \phi\}.$$

It is clear that $\mathcal{P}'_n$ is finite and $\mathcal{P}'_n \subset (\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})_x$. In the following, we show that S is eventually in $\bigcup \mathcal{P}'_n$. If not, there is a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $x_{n_j} \notin \bigcup \mathcal{P}'_n$ for each $j \in N$. Note that $f(K) = S$, so that there exists $a_j \in K$ such that $f(a_j) = x_{n_j}$ for each $j \in N$. Put $\mathcal{G} = \{G = M_\alpha \cap B : B \in \mathcal{B}'_n, \alpha \in A', f^{-1}(x) \cap B \cap M_\alpha \neq \phi\}$; then $\mathcal{P}'_n = f(\mathcal{G})$. Hence $a_j \notin \bigcup \mathcal{G}$. Since $K \setminus \bigcup \mathcal{G}$ is a compact subset of M, there exists a subsequence $\{a_{j_k}\}$ of $\{a_j\}$ with $\{a_{j_k}\}$ converging to $a \in K \setminus \bigcup \mathcal{G}$. Hence $f(a) \neq x$, which contradicts the continuity of f. Thus, S is eventually in $\bigcup \mathcal{P}'_n$.

(2) $\Rightarrow$ (1). Put

$$\mathcal{P}_{\alpha,n} = \{P_\beta : \beta \in B_{\alpha,n}\}, \quad \alpha \in A, \quad B_n = \bigcup_{\alpha\in A} B_{\alpha,n}, \quad n \in \omega.$$

Each B_n is endowed with the discrete topology, and put $M = \{a = (\gamma_n) \in \prod_{n\in\omega} B_n$: there is a finite subset A' of A such that $P_{\gamma_n} \in \bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$, and $\{P_{\gamma_n} : n \in \omega\}$ is a network for some point $x_a \in X\}$. Then M is a metric space. Define $f\colon M \to X$ by $f(a) = x_a$; it is easy to check that f is a mapping.

(i) M is locally separable.

If $a = (\gamma_n) \in M$, then there exists a finite subset A' of A such that $P_{\gamma_n} \in \bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$ for each $n \in \omega$. Set $M_a = \{b = (\beta_n) \in M : \beta_0 = \gamma_0, P_{\beta_n} \in \bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$ for each $n \in N\}$; then M_a is an open neighborhood of a in M and $M_a \subset \prod_{n\in\omega}(\bigcup_{\alpha\in A'} B_{\alpha,n})$. As $\bigcup_{\alpha\in A'} B_{\alpha,n}$ is countable for each $n \in \omega$, M_a is separable, hence M is locally separable.

(ii) f is a π mapping.

For $a = (\alpha_n) \in M$, $\quad b = (\beta_n) \in M$, define

$$d(a,b) = \begin{cases} 0, & a = b, \\ max\{\frac{1}{k+1},\ \alpha_k \neq \beta_k\}, & a \neq b. \end{cases}$$

Then d is a distance function on M. Since the topology of M is introduced as the subspace topology of the product of discrete topologies on B_n's, d is a metric on M. Let $x \in X$, U be an open neighborhood of x. As $\{\mathcal{P}_n\}$ is a point-star network of X, $st(x, \mathcal{P}_m) \subset U$ for some $m \in \omega$. Then $d(f^{-1}(x), M \setminus f^{-1}(U)) \geq \frac{1}{2m+1} > 0$. In fact, let $a = (\alpha_n) \in M$ with $d(f^{-1}(x), a) < \frac{1}{2m+1}$; then $d(a,b) < \frac{1}{m+1}$ for some $b = (\beta_n) \in f^{-1}(x)$. Hence we have $\alpha_k = \beta_k$, when $k \leq m$. Noting that $x \in P_{\beta_m} \in \mathcal{P}_m$, $P_{\alpha_m} = P_{\beta_m}$. So $f(a) \in P_{\alpha_m} = P_{\beta_m} \subset st(x, \mathcal{P}_m) \subset U$. Therefore $a \in f^{-1}(U)$. We have proved that if $a \in M \setminus f^{-1}(U)$, then $d(f^{-1}(x), a) \geq \frac{1}{2m+1}$. Hence $d(f^{-1}(x), M \setminus f^{-1}(U)) \geq \frac{1}{2m+1} > 0$, and f is a π mapping.

(iii) f is a pseudo-sequence-covering mapping.

Let S be a sequence converging to $x \in X$. By (b) of (2), there exists a finite subset A' of A satisfying: there is a finite subfamily $\mathcal{P}'_n$ of $(\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})_x$ such that S is eventually in $\bigcup \mathcal{P}'_n$ for each $n \in \omega$. We may assume that each $\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$ is a cover of S. Hence there exists a finite subset C_n of $\bigcup_{\alpha\in A'} B_{\alpha,n}$ such that $S \subset \bigcup_{\gamma_n\in C_n} P_{\gamma_n}$ and $P_{\gamma_n} \cap S$ closed in X for each $n \in \omega$. Put $K = \{(\gamma_n) \in \prod_{n\in\omega} C_n : \bigcap_{n\in\omega}(P_{\gamma_n} \cap S) \neq \phi\}$. Then $K \subset M$ and $f(K) \subset S$. In fact, let $b = (\gamma_n) \in K$; then $\bigcap_{n\in\omega}(P_{\gamma_n} \cap S) \neq \phi$, and $\gamma_n \in C_n$. Take $z \in \bigcap_{n\in\omega}(P_{\gamma_n} \cap S)$; as $\{\mathcal{P}_n\}$ is a point-star network of X, $\{P_{\gamma_n} : n \in \omega\}$ is a network of z in X. By the definition of C_n, we have $b \in M$ and $f(b) = z$, so $f(K) \subset S$. On the other hand, let $z \in S$; taking $\gamma_n \in C_n$ with $z \in P_{\gamma_n}$ for each $n \in \omega$, we have $\bigcap_{n\in\omega}(P_{\gamma_n} \cap S) \neq \phi$, and thus $\{P_{\gamma_n} : n \in \omega\}$ is a network of z in X. Put $b = (\gamma_n)$; then $b \in K$ and $z = f(b)$. Thus, $S \subset f(K)$. Therefore $f(K) = S$. In the following, we show that K is a compact subset of M. It is clear that $K \subset \prod_{n\in\omega} C_n$, and $\prod_{n\in\omega} C_n$ is a compact subset of $\prod_{n\in\omega} B_n$. If $a = (\gamma_n) \in \prod_{n\in\omega} C_n \setminus K$, then $\bigcap_{n\in\omega}(P_{\gamma_n} \cap S) = \phi$. Noting that each $P_{\gamma_n} \cap S$ is a compact subset, then $\bigcap_{j\leq n_0}(P_{\gamma_j} \cap S) = \phi$ for some $n_0 \in \omega$. Set $U = \{(\beta_n) \in \prod_{n\in\omega} C_n : \beta_n = \gamma_n$, when $\quad n \leq n_0\}$; then U is an open neighborhood of a in $\prod_{n\in\omega} C_n$ and $U \cap K = \phi$. Hence K is closed in $\prod_{n\in\omega} C_n$, K is a compact subset of M. Therefore, f is a pseudo-sequence-covering mapping. ∎

LEMMA 2.2. [4] *Let $f\colon X \to Y$ be a subsequence-covering mapping. If points in X are G_δ, then f is sequentially-quotient.*

LEMMA 2.3. [5] *Let $\mathcal{P}$ be a cover of a space X. If $\mathcal{P}$ is a point countable cs*-cover, then $\mathcal{P}$ is a wcs-cover.*

THEOREM 2.4. *The following are equivalent for a space X:*

(1) *X is a pseudo-sequence-covering, s, π image of a locally separable metric space;*

(2) *X is a subsequence-covering, s, π image of a locally separable metric space;*

(3) *X is a sequentially-quotient, s, π image of a locally separable metric space;*

(4) *X has a point countable cover $\{X_\alpha : \alpha \in A\}$, each subspace X_α has a sequence of countable covers $\{\mathcal{P}_{\alpha,n}\}_{n\in\omega}$ (where $\mathcal{P}_{\alpha,0} = \{X_\alpha\}$) satisfying the following:*

(a) $\{\mathcal{P}_n\}_{n\in\omega}$ is a point-star network of X, where $\mathcal{P}_n = \bigcup_{\alpha\in A} \mathcal{P}_{\alpha,n}$ and consisting of cosmic subspaces;

(b) For every convergent sequence S , there exists a countable subset A' of A such that $(\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})$ is a cs-cover of S for each $n \in \omega$;*

(5) *X has a point countable cover $\{X_\alpha : \alpha \in A\}$, each subspace X_α has a sequence of countable covers $\{\mathcal{P}_{\alpha,n}\}_{n\in\omega}$ (where $\mathcal{P}_{\alpha,0} = \{X_\alpha\}$) satisfying the following:*

(a) $\{\mathcal{P}_n\}_{n\in\omega}$ is a point-star network of X, where $\mathcal{P}_n = \bigcup_{\alpha\in A} \mathcal{P}_{\alpha,n}$ and consisting of cosmic subspaces;

(b) For every convergent sequence S in X, there exists a countable subset A' of A such that $(\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})$ is a wcs-cover of S for each $n \in \omega$.

Proof. (1) $\Rightarrow$ (2) is obvious.

(2) $\Rightarrow$ (3) follows from Lemma 2.2.

(3) $\Rightarrow$ (4). Let $f\colon M \to X$ be a sequentially-quotient, s, π mapping, M be a locally separable metric space. By 4.4.F in [3], $M = \bigoplus_{\alpha\in A} M_\alpha$, where M_α is a separable metric space. Let $\mathcal{B}_n$ be a locally finite open cover of M and refine $\{B(z, \frac{1}{2n})\}_{z\in M}$, set

$$X_\alpha = f(M_\alpha), \quad \mathcal{P}_{\alpha,n} = \{f(M_\alpha \cap B) : B \in \mathcal{B}_n\}.$$

Then $\{X_\alpha : \alpha \in A\}$ is a point countable cover of X, and $\{\mathcal{P}_{\alpha,n}\}_{n\in N}$ is a sequence of countable covers of X_α. Put $\mathcal{P}_{\alpha,0} = \{X_\alpha\}$, $\alpha \in A$, $\mathcal{P}_n = \bigcup_{\alpha\in A} \mathcal{P}_{\alpha,n}$, $n \in \omega$; then $\mathcal{P}_n$ consists of cosmic subspaces. For each $x \in X$, and an open neighborhood U of x, then $d(f^{-1}(x), M \setminus f^{-1}(U)) > \frac{1}{n_0}$ for some $n_0 \in N$. Taking $m \geq 2n_0$, then $st(f^{-1}(x), \mathcal{B}_m) \subset B(f^{-1}(x), \frac{1}{m}) \subset f^{-1}(U)$. Hence we have $st(x, \mathcal{P}_m) \subset f(st(f^{-1}(x), \mathcal{B}_m)) \subset U$. Thus, $\{\mathcal{P}_n\}_{n\in\omega}$ is a point-star network of X.

Let S be a sequence $\{x_n\}$ converging to $x \in X$, and $A' = \{\alpha \in A : x \in X_\alpha\}$. Then A' is countable. Since f is a sequentially-quotient mapping, there exists a sequence L converging to $a \in M$ such that $f(L)$ is a subsequence of S. Suppose $a \in M_{\alpha'}$, hence L is eventually in $M_{\alpha'}$. Since $f(a) = x$, $\alpha' \in A'$. As $\mathcal{B}_n$ is an open cover of M, there exists a $B_n \in \mathcal{B}_n$ such that $a \in B_n$, hence L is eventually in $B_n \cap M_{\alpha'}$. Thus $f(L)$ is eventually in $P_n = f(B_n \cap M\alpha') \in \mathcal{P}_{\alpha',n}$ for each $n \in \omega$, and $\alpha' \in A'$. i.e. S has a subsequence eventually in P_n. Noting that $\mathcal{P}_{\alpha,0} = \{X_\alpha\}$, thus we have shown that $\{X_\alpha : \alpha \in A\}$ is a point countable cs^*-cover of X. By Lemma

2.3, $\{X_\alpha : \alpha \in A\}$ is a wcs-cover for X. So S is eventually in $\bigcup\{X_\alpha : \alpha \in A'\}$. We may assume that $\{X_\alpha : \alpha \in A'\}$ covers S. Thus $S \subset \bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$ for each $n \in \omega$.

In the following we show that $(\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})$ is a cs^*-cover of S for each $n \in \omega$. Let S_1 be a subsequence $\{x_{n_j}\} \cup \{x\}$ of S. Since f is a sequentially-quotient mapping, there exists a sequence L_1 converging to $a_1 \in M$ such that $f(L_1)$ is a subsequence of S_1. Suppose $a_1 \in M_{\alpha''}$, hence L_1 is eventually in $M_{\alpha''}$. Since $f(a_1) = x$, $\alpha'' \in A'$. As $\mathcal{B}_n$ is a open cover of M, there exists a $B'_n \in \mathcal{B}_n$ such that $a_1 \in B'_n$, hence L_1 is eventually in $B'_n \cap M_{\alpha''}$. Thus, $f(L_1)$ is eventually in $P'_n = f(B'_n \cap M\alpha'') \in \mathcal{P}_{\alpha'',n}$ for each $n \in \omega$, and $\alpha'' \in A'$. i.e. S has a subsequence eventually in P'_n. Hence $(\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n})$ is a cs^*-cover of S for each $n \in \omega$.

(4) $\Rightarrow$ (5) holds by Lemma 2.3 and the fact each $\bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$ is countable.

(5) $\Rightarrow$ (1). Put

$$\mathcal{P}_{\alpha,n} = \{P_\beta : \beta \in B_{\alpha,n}\}, \quad \alpha \in A, \quad B_n = \bigcup_{\alpha\in A} B_{\alpha,n}, \quad n \in \omega.$$

Each B_n is endowed with the discrete topology, and put $M = \{a = (\gamma_n) \in \prod_{n\in\omega} B_n$: there is a countable subset A' of A such that $P_{\gamma_n} \in \bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$, and $\{P_{\gamma_n}\}_{n\in\omega}$ is a network for some point $x_a \in X\}$. Then M is a metric space. Define $f\colon M \to X$ by $f(a) = x_a$; it is easy to check that f is a mapping.

(i) M is locally separable.

Let $a = (\gamma_n) \in M$; then there exists a countable subset A' of A such that $P_{\gamma_n} \in \bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$ for each $n \in \omega$. Set $M_a = \{b = (\beta_n) \in M : \beta_0 = \gamma_0, P_{\beta_n} \in \bigcup_{\alpha\in A'} \mathcal{P}_{\alpha,n}$ for each $n \in N\}$. Then M_a is an open neighborhood of a in M and $M_a \subset \prod_{n\in\omega}(\bigcup_{\alpha\in A'} B_{\alpha,n})$. As $\bigcup_{\alpha\in A'} B_{\alpha,n}$ is countable for each $n \in \omega$, M_a is separable, M is locally separable.

(ii) As in the proof (2) $\Rightarrow$ (1) in Theorem 2.1, f is a π mapping.

(iii) f is an s mapping.

For $x \in X$, set $A'' = \{\alpha \in A : x \in X_\alpha\}$; then A'' is countable. Put $B'_n = \{\gamma \in B_{\alpha,n} : x \in P_\gamma, \alpha \in A''\}$; then each B'_n is countable. If $L = \prod_{n\in\omega} B'_n$, then L is a separable subset of $\prod_{n\in\omega} B_n$. If $b = (\gamma_n) \in L$, then $x \in P_{\gamma_n} \in \bigcup_{\alpha\in A''} \mathcal{P}_{\alpha,n}$ for each $n \in \omega$. As $\{\mathcal{P}_n\}$ is a point-star network of X, $\{P_{\gamma_n} : n \in \omega\}$ is a network of x in X. Hence $b \in f^{-1}(x)$, $L \subset f^{-1}(x)$. On the other hand, suppose $b = (\gamma_n) \in f^{-1}(x)$, then $f(b) = x$. So $\{P_{\gamma_n} : n \in \omega\}$ is a network of x in X, and $x \in P_{\gamma_n}$. Thus $\gamma_n \in B'_n$, $b = (\gamma_n) \in \prod_{n\in\omega} B'_n$, $f^{-1}(x) \subset L$. Therefore $f^{-1}(x) = L$, i.e. f is an s mapping.

(iv) f is pseudo-sequence-covering in view of (2) $\Rightarrow$ (1) in Theorem 2.1. ∎

Corollary 2.5. *The following are equivalent for a space X:*

(1) *X is a pseudo-sequence-covering, quotient, s, π image of a locally separable metric space;*

(2) *X is a subsequence-covering, quotient, s, π image of a locally separable metric space;*

(3) *X is a quotient, s, π image of a locally separable metric space;*

(4) *X is a sequentially-quotient, quotient, s, π image of a locally separable metric space;*

(5) *X is sequential and has a point countable cover $\{X_\alpha : \alpha \in A\}$, each subspace X_α has a sequence of countable covers $\{\mathcal{P}_{\alpha,n}\}_{n\in\omega}$ (where $\mathcal{P}_{\alpha,0} = \{X_\alpha\}$) satisfying the following:*

(a) $\{\mathcal{P}_n\}_{n\in\omega}$ is a point-star network of X, where $\mathcal{P}_n = \bigcup_{\alpha\in A}\mathcal{P}_{\alpha,n}$ and consisting of cosmic subspaces;

(b) For every convergent sequence S , there exists a countable subset A' of A such that $(\bigcup_{\alpha\in A'}\mathcal{P}_{\alpha,n})$ is a cs^*-cover of S for each $n \in \omega$;

(6) *X is sequential and has a point countable cover $\{X_\alpha : \alpha \in A\}$, each subspace X_α has a sequence of countable covers $\{\mathcal{P}_{\alpha,n}\}_{n\in\omega}$ (where $\mathcal{P}_{\alpha,0} = \{X_\alpha\}$) satisfying the following:*

(a) $\{\mathcal{P}_n\}_{n\in\omega}$ is a point-star network of X, where $\mathcal{P}_n = \bigcup_{\alpha\in A}\mathcal{P}_{\alpha,n}$ and consisting of cosmic subspaces;

(b) For every convergent sequence S in X, there exists a countable subset A' of A such that $(\bigcup_{\alpha\in A'}\mathcal{P}_{\alpha,n})$ is a wcs-cover of S for each $n \in \omega$.

Proof. Note that quotient mappings preserve sequential spaces. We need only to prove (3) $\Rightarrow$ (4), and it follows from Lemma 1.4.2 in [10]. ■

Recently, Lin [11] proved that there exists a quotient, π mapping f on a metric space such that f is not pseudo-sequence-covering. And the following question is still open. Is every quotient π image of a metric space also a pseudo-sequence-covering, quotient π image of a metric space? [11]. The question and Corollary 2.5 naturally suggest the following question.

QUESTION 2.6. Suppose X is a quotient, π image of a locally separable metric space. Is X a pseudo-sequence-covering, quotient, π image of a locally separable metric space?

ACKNOWLEDGEMENT. The author is very grateful to the referee for valuable comments and corrections.

REFERENCES

[**1**] J. R. Boone, F. Siwiec, *Sequential quotient mappings,* Czech. Math. J., **26** (1976), 174–182.

[**2**] H. Chen, *Weak neighborhoods and Michael-Nagami's question,* Houston J. Math., **25** (1999), 297–309.

[**3**] R. Engelking, *General Topology,* Warzawa: Polish Scientific Publishers, 1977.

[**4**] Y. Ge, *On compact images of locally separable metric spaces,* Topology Proc., **27** (2003), 351–360.

[**5**] Y. Ge, *On pseudo-sequence-covering π-images of metric spaces*, Mat. Vesnik, **57** (2005), 113–120.

[**6**] Y. Ge, S. Lin, *Uniform covers and compact images of metric spaces,* Acta Math. Sinica, **47** (2004), 1149–1154 (in Chinese).

[**7**] G. Gruenhage, E. Michael, Y. Tanaka, *Spaces determined by point-countable covers,* Pacific J. Math., **113** (1984), 303–332.

[**8**] Y. Ikeda, C. Liu, Y. Tanaka, *Quotient compact images of metric spaces, and related matters,* Topology Appl., **122** (2002), 237–252.

[**9**] J.J. Li, W. Y. Cai, *Notes on sequence-covering s-mappings*, Acta Math. Sinica, **43** (2000), 758–762 (in Chinese).

[**10**] S. Lin, *Point-countable covers and sequence-covering mappings,* Beijing: Chinese Science Press, 2002.

[**11**] S. Lin, *A note on sequence-covering mappings,* Acta Math. Hungar., **107** (2005), 187–191.

[**12**] S. Lin, C. Liu, M. M. Dai, *Images on locally separable metric spaces*, Acta Math. Sinica, New Series, **13** (1997), 1–8.

[**13**] C. Liu, Y. Tanaka, *Star-countable k-networks, and quotient images of locally metric spaces,* Topology Appl., **82** (1998), 317–325.

[**14**] E. Michael, $\aleph_0$*-spaces*, J. Math. Mech., **15** (1966), 983–1002.

[**15**] V. I. Ponomarev, *Axioms of countability and continuous mappings,* Bull. Pol. Acta. Math., **8** (1960), 127–133.

[**16**] Y. Tanaka, S. X. Xia, *Certain s-images of locally separable metric spaces*, Questions Answers in General Topology, **14** (1996), 217–231.

[**17**] L. Z. Zhou, *Some sequence-covering s-images of locally separable metric spaces,* Acta Math. Sinica, **42** (1999), 476–582. (in Chinese).

(received 27.10.2006, in revised form 30.03.2007)

College of Science, Shandong Jianzhu University, Jinan 250101 P.R. China

E-mail: xias@sdjzu.edu.cn