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ON $(n-1,n)$-ϕ-PRIME IDEALS IN SEMIRINGS

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Abstract. Let S be a commutative semiring and $T(S)$ be the set of all ideals of S. Let $\phi\colon T(S) \to T(S) \cup \{\emptyset\}$ be a function. A proper ideal I of a semiring S is called an $(n-1,n)$-ϕ-prime ideal of S if $a_1a_2\cdots a_n \in I \setminus \phi(I)$, $a_1, a_2, \ldots, a_n \in S$ implies that $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$ for some $i \in \{1,2,\ldots,n\}$. In this paper, we prove several results concerning $(n-1,n)$-ϕ-prime ideals in a commutative semiring S with non-zero identity connected with those in commutative ring theory.

1. Introduction

Anderson and Bataineh [3] have introduced the concept of ϕ-prime ideals in a commutative ring as a generalization of weakly prime ideals in a commutative ring introduced by Anderson and Smith [4]. After that several authors [2,6–11,16], etc. explored this concept in different ways either in commutative ring or semiring theory. Ebrahimpour and Nekooei [13] generalized the concept of ϕ-prime ideals in terms of $(n-1,n)$-ϕ-prime ideals in commutative rings with non-zero identity and extended several results connected to [3]. In this paper, we introduce the notion of $(n-1,n)$-ϕ-prime ideals in a commutative semiring and prove several results connected with ring theory. Most of the results are inspired by [3,9,13,14].

A commutative semiring is a commutative semigroup $(S,\cdot)$ and a commutative monoid $(S,+,0_S)$ in which the multiplication is distributive with respect to the addition both from the left and from the right and 0_S is the additive identity of S and also $0_S \cdot x = x \cdot 0_S = 0_S$ for all $x \in S$. A non-empty subset I of a semiring S is called an ideal of S if $a, b \in I$ and $s \in S$ imply $a+b \in I$ and $sa, as \in I$. An ideal I of a semiring S is said to be proper if $I \neq S$. An ideal I of a semiring S is called subtractive (also, a k-ideal) if $a, a+b \in I$, $b \in S$ imply $b \in I$. An ideal I of a semiring S is called prime (weakly prime, almost prime, n-almost prime) if $ab \in I$ (respectively, $ab \in I \setminus \{0\}$, $ab \in I \setminus I^2$, $ab \in I \setminus I^n$) implies that either $a \in I$ or $b \in I$. A non-zero element $a \in S$ is said to be a semi-unit in S if there exist $r, s \in S$ such that $1 + ra = sa$. A proper ideal I of a semiring S is called

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2-absorbing (respectively, weakly 2-absorbing) if $abc \in I$ (respectively, $0 \neq abc \in I$) implies $ab \in I$ or $ac \in I$ or $bc \in I$. For the rest of the concepts and terminologies used in semiring theory, we refer to [15]. Throughout this paper, S will always denote a commutative semiring with identity $1 \neq 0$.

2. $(n-1,n)$-ϕ-prime ideals

In this section, we introduce the notion of $(n-1,n)$-ϕ-prime ideals of a semiring S and analyse some properties related to them.

Definition 2.1. Let S be a semiring and $T(S)$ be the set of all ideals of S. Let $\phi\colon T(S) \to T(S) \cup \{\emptyset\}$ be a function. A proper ideal I of a semiring S is called an $(n-1,n)$-ϕ-prime ideal of S, if $a_1a_2\cdots a_n \in I \setminus \phi(I)$, $a_1, a_2, \ldots, a_n \in S$ implies that $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$ for some $i \in \{1,2,\ldots,n\}$.

Note that for $n \geq 2$, $(n-1,n)$-prime ideal denotes $(n-1)$-absorbing ideal I (analogous to [2]) of S, that is, a proper ideal I of S is called an n-absorbing ideal if whenever $a_1a_2\cdots a_{n+1} \in I$ for $a_1, a_2, \ldots, a_{n+1} \in S$, then there are n of the a_i's whose product is in I. Thus, a $(1,2)$-prime ideal is just a prime ideal, a $(2,3)$-prime ideal is a 2-absorbing ideal and an $(n-1,n)$-prime ideal is an $(n-1)$-absorbing ideal of S. Similarly, a proper ideal I of a semiring S is called an $(n-1,n)$-weakly prime ideal of S if $a_1a_2\cdots a_n \in I \setminus \{0\}$, $a_1, a_2, \ldots, a_n \in S$ implies $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$ for some $i \in \{1,2,\ldots,n\}$. It is clear that every $(n-1,n)$-prime ideal is an $(n-1,n)$-weakly prime ideal of a semiring S but the converse need not be true. For example, by definition $\{0\}$ is an $(n-1,n)$-weakly prime ideal but it is not an $(n-1,n)$-prime ideal of S. Some non-trivial examples of $(n-1,n)$-weakly prime ideals that are not $(n-1,n)$-prime are given as follows.

(i). Let $S = Z_{2^n}$ $(n \geq 2)$, where Z is the set of all positive integers. Consider $I = \{0, 2^{n-1}\}$. Clearly, I is an ideal of S. Now, let $S_1 = S \times I$ and $N = \{(0,0),(0,2^{n-1})\}$. Then N is a non-zero $(n-1,n)$-weakly prime ideal of S_1 but it is not an $(n-1,n)$-prime ideal of S_1, since $(2,0)^n \in N$ but $(2,0)^{n-1} \notin N$.

(ii). Let $S = Z_{2^n p}$ $(n \geq 2)$, where Z is the set of all positive integers and p is any prime number. Let $I = \{0, 2^{n-1}p\}$. Let $S_1 = S \times I$ and $N = \{(0,0),(0,2^{n-1}p)\}$. Clearly, N is a non-zero $(n-1,n)$-weakly prime ideal of S_1 but it is not an $(n-1,n)$-prime ideal of S_1, since $(2,0)^n(p,0) \in N$ but neither $(2,0)^n \in N$ nor $(2,0)^{n-1}(p,0) \in N$.

Similarly, an $(n-1,n)$-ϕ-prime ideal I of S can be elucidated as follows:

If $\phi_0(I) = \{0\}$, then an $(n-1,n)$-ϕ_0-prime ideal is called an $(n-1,n)$-weakly prime ideal. Similarly, if $\phi_2(I) = I^2$, then an $(n-1,n)$-ϕ_2-prime ideal is called an $(n-1,n)$-almost prime ideal, $(1,2)$-ϕ_0-prime ideal means weakly prime ideal and $(2,3)$-ϕ_0-prime ideal means weakly 2-absorbing ideal of a commutative semiring.

Since $I \setminus \phi(I) = I \setminus (I \cap \phi(I))$, so without loss of generality, we assume, throughout the paper that $\phi(I) \subseteq I$. Let S be a semiring and $T(S)$ be the set of all ideals of S. Define the following functions $\phi_\alpha : T(S) \to T(S) \cup \{\emptyset\}$ and their corresponding ϕ_α-prime ideals as follows: $\phi_\emptyset(I) = \{\emptyset\}$; $\phi_0(I) = \{0\}$; $\phi_1(I) = I$;

$\phi_2(I) = I^2$; $\phi_n(I) = I^n, n \geq 2$; $\phi_\omega(I) = \bigcap_{n=1}^{\infty} I^n$ for all $I \in T(S)$. Clearly, $\phi_\emptyset$, ϕ_0, ϕ_2 and ϕ_n are prime, weakly prime, almost prime and n-almost prime ideals respectively.

Definition 2.2. [12, Definition 2.2] Let S be a semiring and $\phi\colon T(S) \to T(S) \cup \{\emptyset\}$ be a function, where $T(S)$ is the set of all ideals of S and let I be an ideal of S. Then I is said to be a ϕ-subtractive ideal of S if I and $\phi(I)$ are subtractive ideals of S. Similarly, we define the following functions $\phi_\alpha\colon T(S) \to T(S) \cup \{\emptyset\}$ and their corresponding ϕ_α-prime ideals such that $\phi_\emptyset(I) = \{\emptyset\}$; $\phi_0(I) = \{0\}$; $\phi_1(I) = I$; $\phi_2(I) = I^2$; $\phi_n(I) = I^n, n \geq 2$ for all $I \in T(S)$. Then I is said to be a ϕ_i-subtractive ideal of S if I and $\phi_i(I)$ are subtractive ideals of S, where $2 \leq i \leq n$.

Several examples have been studied in [12]. For the sake of completeness, we consider the set $S = Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then S forms a semiring under addition and multiplication modulo 8. If we take the set $I = \{0, 2, 4, 6\}$, then it is easy to check that I and $\phi_i(I)$ (as defined above) are subtractive ideals of S and therefore I is a ϕ_i-subtractive ideal of S, where $2 \leq i \leq n$.

Proposition 2.3. *Let S be a semiring and I be an ideal of S. Let $x \in S$. Then $(I : x)$ and $(0 : x)$ are also subtractive ideals of S, where $(I : x) = \{r \in S : rx \in I\}$ and $(0 : x) = \{r \in S : rx = 0\}$.*

Proof. The proof is straightforward. ■

Result 2.4. [16] *If I and J are two subtractive ideals of S, then $I \cup J$ is a subtractive ideal of S if and only if $I \cup J = I$ or $I \cup J = J$.*

Theorem 2.5. *Let $\phi : T(S) \to T(S) \cup \{\emptyset\}$ be a function and I be a subtractive proper ideal of S. Then the following statements are equivalent:*

(i) *I is $(n-1, n)$-ϕ-prime;*

(ii) *for $a_1 a_2 \cdots a_{n-1} \in S \setminus I$, $(I : a_1 a_2 \cdots a_{n-1}) = \bigcup_{i=1}^{n-1} (I : a_1 a_2 \cdots a_{i-1} a_{i+1} \cdots a_{n-1}) \cup (\phi(I) : a_1 a_2 \cdots a_{n-1})$.*

Proof. (i) $\Rightarrow$ (ii) Let $a_1 a_2 \cdots a_{n-1} \in S \setminus I$ and let $x \in (I : a_1 a_2 \cdots a_{n-1})$. Then $a_1 a_2 \cdots a_{n-1} x \in I$. If $a_1 a_2 \cdots a_{n-1} x \notin \phi(I)$, then $a_1 a_2 \cdots a_{n-1} x \in I \setminus \phi(I)$. Since I is $(n-1, n)$-ϕ-prime, then we have $a_1 a_2 \cdots a_{i-1} a_{i+1} \cdots a_{n-1} x \in I$ for some $i \in \{1, 2, \ldots, n-1\}$. Hence $x \in (I : a_1 a_2 \cdots a_{i-1} a_{i+1} \cdots a_{n-1})$. If $a_1 a_2 \cdots a_{n-1} x \in \phi(I)$, then $x \in (\phi(I) : a_1 a_2 \cdots a_{n-1})$. Thus, $(I : a_1 a_2 \cdots a_{n-1}) \subseteq \bigcup_{i=1}^{n-1} (I : a_1 a_2 \cdots a_{i-1} a_{i+1} \cdots a_{n-1}) \cup (\phi(I) : a_1 a_2 \cdots a_{n-1})$. Clearly, $\bigcup_{i=1}^{n-1} (I : a_1 a_2 \cdots a_{i-1} a_{i+1} \cdots a_{n-1}) \cup (\phi(I) : a_1 a_2 \cdots a_{n-1}) \subseteq (I : a_1 a_2 \cdots a_{n-1})$, since $\phi(I) \subseteq I$. Therefore, $(I : a_1 a_2 \cdots a_{n-1}) = \bigcup_{i=1}^{n-1} (I : a_1 a_2 \cdots a_{i-1} a_{i+1} \cdots a_{n-1}) \cup (\phi(I) : a_1 a_2 \cdots a_{n-1})$.

(ii) $\Rightarrow$ (i) Let $a_1 a_2 \cdots a_n \in I \setminus \phi(I)$. If $a_1 a_2 \cdots a_{n-1} \in I$, then we are done. So assume that $a_1 a_2 \cdots a_{n-1} \notin I$. Therefore, we have $(I : a_1 a_2 \cdots a_{n-1}) = \bigcup_{i=1}^{n-1} (I : a_1 a_2 \cdots a_{i-1} a_{i+1} \cdots a_{n-1}) \cup (\phi(I) : a_1 a_2 \cdots a_{n-1})$. Since $a_1 a_2 \cdots a_n \in I$, we have $a_n \in (I : a_1 a_2 \cdots a_{n-1})$. Also $a_n \notin (\phi(I) : a_1 a_2 \cdots a_{n-1})$. Therefore, $(I :$

$a_1a_2\cdots a_{n-1}) \neq (\phi(I) : a_1a_2\cdots a_{n-1})$. Thus $a_n \in (I : a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_{n-1})$ for some $i \in \{1, 2, \ldots, n-1\}$. Hence $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_{n-1}a_n \in I$ gives I is $(n-1,n)$-ϕ-prime. ∎

Corollary 2.6. *Let I be a proper subtractive ideal of S. Then the following statements are equivalent:*

(i) *I is $(n-1,n)$-weakly prime;*

(ii) *for $a_1a_2\cdots a_{n-1} \in S \setminus I$, $(I : a_1a_2\cdots a_{n-1}) = \bigcup_{i=1}^{n-1}(I : a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_{n-1}) \cup (0 : a_1a_2\cdots a_{n-1})$.*

Theorem 2.7. *Let S be a semiring, I be a proper ϕ-subtractive ideal of S and $\phi : T(S) \to T(S) \cup \{\emptyset\}$ be a function. If I is an $(n-1,n)$-ϕ-prime ideal of S with $I^n \nsubseteq \phi(I)$, then I is an $(n-1,n)$-prime ideal of S.*

Proof. Let I be an $(n-1,n)$-ϕ-prime ideal of S with $I^n \nsubseteq \phi(I)$ and let $a_1a_2\cdots a_n \in I$ for some $a_1, a_2, \ldots, a_n \in S$. If $a_1a_2\cdots a_n \notin \phi(I)$, then $a_1a_2\cdots a_n \in I \setminus \phi(I)$, which gives $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$ for some $i \in \{1, 2, \ldots, n\}$, since I is an $(n-1,n)$-ϕ-prime ideal of S. So assume that $a_1a_2\cdots a_n \in \phi(I)$. First, suppose that $a_1a_2\cdots a_{n-m}I^m \nsubseteq \phi(I)$ for all $m \in \{1, 2, \ldots, n-1\}$. Therefore there exists $i_1, i_2, \ldots, i_m \in I$ such that $a_1a_2\cdots a_{n-m}i_1i_2\cdots i_m \notin \phi(I)$. Then $a_1a_2\cdots a_{n-m}(a_{n-m+1} + i_1)(a_{n-m+2} + i_2)\cdots(a_n + i_m) \in I \setminus \phi(I)$, since I is a ϕ-subtractive ideal of S. So $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$, since I is ϕ-subtractive $(n-1,n)$-ϕ-prime ideal. Now, suppose that $a_1a_2\cdots a_{n-m}I^m \subseteq \phi(I)$ for all $m \in \{1, 2, \ldots, n-1\}$. Similarly, we can assume that for all $l_1, l_2, \ldots, l_{n-m}$ from $\{1, 2, \ldots, n\}$, $a_{l_1}a_{l_2}\cdots a_{l_{n-m}}I^m \subseteq \phi(I)$, $1 \leq m \leq n-1$. Since $I^n \nsubseteq \phi(I)$, there exist $m_1, m_2, \ldots, m_n \in I$ such that $m_1m_2\cdots m_n \notin \phi(I)$. Then $(a_1 + m_1)(a_2 + m_2)\cdots(a_n + m_n) \in I \setminus \phi(I)$, since I is ϕ-subtractive. Thus, $(a_1 + m_1)(a_2 + m_2)\cdots(a_{i-1} + m_{i-1})(a_{i+1} + m_{i+1})\cdots(a_n + m_n) \in I$ for some $i \in \{1, 2, \ldots, n\}$. Therefore, $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$, since I is a ϕ-subtractive $(n-1,n)$-ϕ-prime ideal. Hence I is an $(n-1,n)$-prime ideal of S. ∎

Corollary 2.8. *Let S be a semiring and I be a proper subtractive ideal of S. If I is an $(n-1,n)$-weakly prime ideal of S that is not an $(n-1,n)$-prime ideal of S, then $I^n = 0$.*

Corollary 2.9. *Let S be a semiring and I be a proper subtractive ideal of S. If I is an $(n-1,n)$-weakly prime ideal of S that is not an $(n-1,n)$-prime ideal of S, then $\sqrt{I} = \sqrt{0}$.*

Proof. Clearly, $\sqrt{0} \subseteq \sqrt{I}$. Also by Corollary 2.8, we have $I^n = 0$, which gives $I = \sqrt{0}$ and hence $\sqrt{I} \subseteq \sqrt{0}$. Thus we have $\sqrt{I} = \sqrt{0}$. ∎

If I is a proper ideal of a semiring S such that $I^n = \{0\}$, then it need not be an $(n-1,n)$-weakly prime ideal of S $(n \geq 2)$. For example, let $S = Z_{2^{n+1}3}$ be a semiring, where Z is the set of positive integers. If we take an ideal $I = \{0, 2^n3\}$, then $I^n = \{0\}$ but it is not an $(n-1,n)$-weakly prime ideal of S, since $0 \neq 2^n3 \in I$ but neither $2^n \in I$ nor $2^{n-1}3 \in I$.

Corollary 2.10. *If I is a proper ϕ-subtractive $(n-1,n)$-ϕ-prime ideal of S with $\phi \leq \phi_{n+1}$, then I is $(n-1,n)$-ω-prime, where $n \geq 2$.*

Proof. Let I be $(n-1,n)$-prime. Then I is $(n-1,n)$-ϕ-prime for each ϕ and hence I is $(n-1,n)-\omega$-prime. So, assume that I is not $(n-1,n)$-prime. Therefore, by Theorem 2.7, $I^n \subseteq \phi(I) \subseteq I^{n+1}$. Thus, $\phi(I) = I^m$ for each $m \geq n$. Hence I is $(n-1,n)-\omega$-prime. ∎

Definition 2.11. [5, Definition 1(i)] A proper ideal I of a semiring S is said to be a strong ideal if for each $a \in I$ there exists $b \in I$ such that $a + b = 0$.

Theorem 2.12. *Let S and S' be semirings, $f\colon S \to S'$ be an epimorphism such that $f(0) = 0$ and I be a ϕ-subtractive strong ideal of S. If I is an $(n-1,n)$-ϕ-prime ideal of S with $I^n \nsubseteq \phi(I)$ and ker $f \subseteq I$, then $f(I)$ is an $(n-1,n)$-ϕ-prime ideal of S'.*

Proof. Let I be an $(n-1,n)$-ϕ-prime ideal of S with $I^n \nsubseteq \phi(I)$ and $a_1a_2\cdots a_n \in f(I) \setminus \phi(f(I))$ for some $a_1, a_2, \ldots, a_n \in S'$. Since $a_1a_2\cdots a_n \in f(I)$, therefore there exists an element $m \in I$ such that $a_1a_2\cdots a_n = f(m)$. Since f is an epimorphism and $a_1, a_2, \ldots, a_n \in S'$, then there exist $p_1, p_2, \ldots, p_n \in S$ such that $a_1 = f(p_1)$, $a_2 = f(p_2)$, $\ldots$, $a_n = f(p_n)$. As $m \in I$ and I is a strong ideal of S, there exists $l \in I$ such that $m + l = 0$, which implies $f(m + l) = 0$. This gives that $f(p_1p_2\cdots p_n + l) = 0$ implies $p_1p_2\cdots p_n + l \in ker\, f \subseteq I$. This implies $p_1p_2\cdots p_n \in I$, since I is subtractive. Since I is an $(n-1,n)$-ϕ-prime ideal with $I^n \nsubseteq \phi(I)$, we have that I is an $(n-1,n)$-prime ideal by Theorem 2.7. Therefore, we have $p_1p_2\cdots p_{i-1}p_{i+1}\cdots p_n \in I$ for some $i \in \{1, 2, \ldots, n\}$. Thus $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in f(I)$, for some $i \in \{1, 2, \ldots, n\}$. Hence $f(I)$ is an $(n-1,n)$-ϕ-prime ideal of S'. ∎

Definition 2.13. A semiring S is said to be cancellative, if whenever $xc = yc$ and $cx = cy$ in S, then $x = y$.

Theorem 2.14. *Let S be a cancellative semiring and $x \in S$. Let Sx be a ϕ_2-subtractive ideal of S. Then Sx is $(n-1,n)$-ϕ_2-prime if and only if Sx is an $(n-1,n)$-prime ideal of S.*

Proof. First, suppose that Sx is $(n-1,n)$-ϕ_2-prime and $a_1, a_2, \ldots, a_n \in S$ are such that $a_1a_2\cdots a_n \in Sx$. If $a_1a_2\cdots a_n \notin \phi_2(Sx)$, then $a_1a_2\cdots a_n \in Sx \setminus \phi_2(Sx)$, which gives $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in Sx$ for some $i \in \{1, 2, \ldots, n\}$, since Sx is $(n-1,n)$-ϕ_2-prime. Let $a_1a_2\cdots a_n \in \phi_2(Sx)$. Also, $a_1\cdots a_{n-1}(a_n + x) \in Sx$. If $a_1\cdots a_{n-1}(a_n + x) \notin \phi_2(Sx)$, then $a_1\cdots a_{n-1}(a_n + x) \in Sx \setminus \phi_2(Sx)$. This gives $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in Sx$ for some $i \in \{1, 2, \ldots, n\}$, since Sx is ϕ_2-subtractive $(n-1,n)$-ϕ_2-prime. If $a_1\cdots a_{n-1}(a_n + x) \in \phi_2(Sx)$, then we have $a_1\cdots a_{n-1}x \in \phi_2(Sx) = (Sx)^2$. Therefore, $a_1\cdots a_{n-1}x = (s_1x)(s_2x) = (s_1s_2)x^2$ for some $s_1, s_2 \in S$. This gives $a_1\cdots a_{n-1} = s_1s_2x$, as S is cancellative. Thus, $a_1\cdots a_{n-1} \in Sx$ and hence Sx is an $(n-1,n)$-prime ideal of S.

The converse is obvious. ∎

Theorem 2.15. *Let S be a cancellative semiring and $a \in S$ be a non-unit. Let $\langle a\rangle$ be a ϕ_2-subtractive ideal of S. Then $\langle a\rangle$ is an $(n-1,n)$-ϕ_2-prime ideal if and only if $\langle a\rangle$ is an $(n-1,n)$-prime ideal ($n \geq 2$).*

Proof. Let $\langle a\rangle$ be an $(n-1,n)$-ϕ_2-prime ideal of S and $a_1a_2\cdots a_n \in \langle a\rangle$ for some $a_1, a_2, \ldots, a_n \in S$. If $a_1a_2\cdots a_n \notin \langle a\rangle^2$, then $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in \langle a\rangle$ for some $i \in \{1, 2, \ldots, n\}$. So assume that $a_1a_2\cdots a_n \in \langle a\rangle^2$. Also $(a_1 + a)a_2\cdots a_n \in \langle a\rangle$. If $(a_1 + a)a_2\cdots a_n \notin \langle a\rangle^2$, then $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in \langle a\rangle$, for some $i \in \{1, 2, \ldots, n\}$, since $\langle a\rangle$ is a ϕ_2-subtractive $(n-1,n)$-ϕ_2-prime ideal of S. So assume that $(a_1 + a)a_2\cdots a_n \in \langle a\rangle^2$. Then $aa_2\cdots a_n \in \langle a\rangle^2$, as $a_1a_2\cdots a_n \in \langle a\rangle^2$ and $\langle a\rangle$ is ϕ_2-subtractive. Hence $a_2\cdots a_n \in \langle a\rangle$, since S is cancellative. Thus, $\langle a\rangle$ is an $(n-1,n)$-prime ideal.

The converse is obvious. ■

Corollary 2.16. *Let S be a cancellative semiring and $a \in S$ be non-unit. Let $\langle a\rangle$ be a subtractive ideal of S. Then $\langle a\rangle$ is an $(n-1,n)$-weakly prime ideal if and only if $\langle a\rangle$ is an $(n-1,n)$-prime ideal ($n \geq 2$).*

Let S_1 and S_2 be commutative semirings. We know that the prime ideals of $S_1 \times S_2$ have the form $I_1 \times S_2$ or $S_1 \times I_2$ where I_1 is a prime ideal of S_1 and I_2 is a prime ideal of S_2. Define multiplication on $S_1 \times S_2$ as $(a_1, a_2)(b_1, b_2) = (a_1b_1, a_2b_2)$ for all $a_1, b_1 \in S_1$ and $a_2, b_2 \in S_2$. Now, we prove the following theorem.

Theorem 2.17. *Let S_1 and S_2 be commutative semirings and I_1 be an $(n-1,n)$-weakly prime ideal of S_1. Then $I = I_1 \times S_2$ is an $(n-1,n)$-ϕ-prime ideal of $S = S_1 \times S_2$ for each ϕ with $\phi_\omega \leq \phi \leq \phi_1$.*

Proof. Let I_1 be an $(n-1,n)$-weakly prime ideal of S_1. First, suppose that I_1 be an $(n-1,n)$-prime ideal of S. Then I is also an $(n-1,n)$-prime ideal and hence is an $(n-1,n)$-ϕ-prime ideal of S for all ϕ. So, suppose that I_1 is not $(n-1,n)$-prime. Then $I_1^n = 0$. Therefore, we have $I^n = 0^n \times S_2$ and hence $\phi_\omega(I) = \{0\} \times S_2$. Now, $I \setminus \phi_\omega(I) = (I_1 \times S_2) \setminus (\{0\} \times S_2)$. Thus, $(a_1, b_1)(a_2, b_2)\cdots(a_n, b_n) \in I \setminus \phi_\omega(I)$. This gives $a_1a_2\cdots a_n \in I_1 \setminus \{0\}$. Thus, $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I_1$, for some $i \in \{1, 2, \ldots, n\}$ which implies $(a_1, b_1)(a_2, b_2)\cdots(a_{i-1}, b_{i-1})(a_{i+1}, b_{i+1})\cdots(a_n, b_n) \in I$. Hence I is an $(n-1,n)$-ϕ_ω-prime and hence $(n-1,n)$-ϕ-prime. ■

Theorem 2.18. *Let I be a proper ϕ-subtractive ideal of a semiring S. Suppose that I is $(n-1,n)$-ϕ-prime with $\phi \leq \phi_{n+1}$. Then, either I is $(n-1,n)$-weakly prime or I^n is an idempotent.*

Proof. If I is $(n-1,n)$-prime, then I is $(n-1,n)$-weakly prime. So, there is nothing to prove. Now, assume that I is not $(n-1,n)$-prime. Therefore, by Theorem 2.7, $I^n \subseteq \phi(I)$. Since, $\phi \leq \phi_{n+1}$, therefore $I^n \subseteq \phi_{n+1}(I) = I^{n+1}$, which gives $I^n = I^{n+1}$. Hence $I^n = I^{2n}$. Thus, I^n is idempotent. ■

3. $(n-1,n)$-weakly prime ideals

In this section, we study the concept of $(n-1,n)$-weakly prime ideals of a commutative semiring S which is a particular case of $(n-1,n)$-ϕ-prime ideals and prove some results related to them.

Let I be an $(n-1,n)$-weakly prime ideal of a semiring S and $a_1,a_2,\ldots,a_n \in S$. Then $(a_1,a_2,\ldots,a_n)$ is an n-zero of I, if $a_1a_2\cdots a_n = 0$ and $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \notin I$ for all $i \in \{1,2,\ldots,n\}$ $(n \geq 2)$.

THEOREM 3.1. *Let I be a subtractive $(n-1,n)$-weakly prime ideal of a semiring S and $(a_1,a_2,\ldots,a_n)$ is an n-zero of I for some $a_1,a_2,\ldots,a_n \in S$. Then $a_{i_1}\cdots a_{i_{n-k}}I^k = 0$ for all $k \in \{1,2,\ldots,n\}$ and $i_1,i_2,\ldots,i_{n-k} \in \{1,2,\ldots,n\}$. In particular, $I^n = 0$.*

Proof. We prove it by induction on k. For $k=1$, suppose that $a_{i_1}a_{i_2}\cdots a_{i_{n-1}}I \neq 0$. Then, there exists an element $x \in I$ such that $a_{i_1}a_{i_2}\cdots a_{i_{n-1}}x \neq 0$. So, $0 \neq a_{i_1}a_{i_2}\cdots a_{i_{n-1}}(a_{i_n}+x)$. This gives $a_{i_1}a_{i_2}\cdots a_{i_{n-1}}(a_{i_n}+x) \in I \setminus \{0\}$. Since I is subtractive $(n-1,n)$-weakly prime, we have $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$ for some $i \in \{1,2,\ldots,n\}$, a contradiction. Thus $a_{i_1}a_{i_2}\cdots a_{i_{n-k}}I = 0$. Now, let $a_{i_1}\cdots a_{i_{n-(k-1)}}I^{k-1} = 0$ for some $k \geq 2$ and for all possible $i_1,i_2,\ldots,i_{n-(k-1)} \subseteq \{1,2,\ldots,n\}$. Assume that $a_{i_1}\cdots a_{i_{n-k}}I^k \neq 0$. Then $a_{i_1}a_{i_2}\cdots a_{i_{n-k}}p_1p_2\cdots p_k \neq 0$ for some $p_1,p_2,\ldots,p_k \in I$. Thus, $a_{i_1}a_{i_2}\cdots a_{i_{n-k}}(a_{i_{n-k+1}}+p_1)(a_{i_{n-k+2}}+p_2)\cdots(a_{i_n}+p_k) \in I \setminus \{0\}$. Consequently, $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$ for some $i \in \{1,2,\ldots,n\}$ (since I is subtractive $(n-1,n)$-weakly prime), a contradiction. Hence $a_{i_1}\cdots a_{i_{n-k}}I^k = 0$. In particular, $I^n = 0$. ■

THEOREM 3.2. *Let I be a subtractive $(n-1,n)$-weakly prime ideal of a semiring S that is not an $(n-1,n)$-prime ideal. If $a \in Nil(S)$, then either $a^{n-1} \in I$ or $a^{n-k}I^k = 0$ for all $k \in \{1,2,\ldots,n-1\}$.*

Proof. We prove it by induction on k. Suppose $k=1$. Let $a \in Nil(S)$ and $a^{n-1}I \neq 0$. Then there exists $y \in I$ such that $a^{n-1}y \neq 0$ and let m be the least positive integer such that $a^m = 0$. Then $m \geq n$ and $0 \neq a^{n-1}y = a^{n-1}(y+a^{m-n+1}) \in I$. Therefore, either $a^{n-1} \in I$ or $a^{m-1} \in I$. If $a^{n-1} \in I$, then there is nothing to prove. So, assume that $0 \neq a^{m-1} \in I$. Thus, $a^{n-1} \in I$ (since $m \geq n$ and I is subtractive $(n-1,n)$-weakly prime). Hence for each $a \in Nil(S)$, we have $a^{n-1} \in I$ or $a^{n-1}I = 0$. Next, assume that $v^{n-1} \notin I$, where $v \in Nil(S)$. Therefore $v^{n-1}I = 0$. Suppose that it is true for $n = k$ that $v^{n-k}I^k = 0$ for all $k \in \{1,2,\ldots,n-1\}$. Suppose that $v^{n-k}I^k \neq 0$. Then there exist $i_1,i_2,\ldots,i_k \in I$ such that $v^{n-k}i_1i_2\cdots i_k \neq 0$. Since $v \in Nil(S)$, we have $v^m = 0$ where m is the least positive integer. If $m < n$, then $v^{n-1} = 0 \in I$, a contradiction. So, $m \geq n$. Now, $0 \neq v^{n-k}i_1i_2\cdots i_k = v^{n-k}(v+i_1)(v+i_2)\cdots(v+i_{k-1})(v^{m-n+1}+i_k) \in I$. This gives $v^{n-1} \in I$ or $v^{m-1} \in I$ and hence $v^{n-1} \in I$, (since $m \geq n$, $v^{m-1} \neq 0$ and I is an $(n-1,n)$-weakly prime). Hence $v^{n-1} \in I$, a contradiction. Thus, $v^{n-k}I^k = 0$. ■

THEOREM 3.3. *Let S be a semiring and $\{I_i\}_{i\in\Delta}$ be a family of subtractive*

$(n-1,n)$-weakly prime ideals of S that are not $(n-1,n)$-prime ideals of S. Then $I=\bigcap_{i\in\Delta} I_i$ is an $(n-1,n)$-weakly prime ideal of S.

Proof. Let $\{I_i\}_{i\in\Delta}$ be a family of $(n-1,n)$-weakly prime ideals of S that are not $(n-1,n)$-prime ideals of S and $I=\bigcap_{i\in\Delta} I_i$. Therefore by Corollary 2.9, we have $\sqrt{I_i}=\sqrt{0}$ for all $i\in\Delta$. This gives $\bigcap_{i\in\Delta}\sqrt{I_i}=\sqrt{0}$. Thus, we have $\sqrt{I}=\sqrt{0}$, since $\bigcap_{i\in\Delta}\sqrt{I_i}=\sqrt{I}$. Next, let $a_1a_2\cdots a_n\in I\setminus\{0\}$ for some $a_1,a_2,\ldots,a_n\in S$. If $a_1a_2\cdots a_{n-1}\in I$, then there is nothing to prove. So, let $a_1a_2\cdots a_{n-1}\notin I$. Then there exists $i\in\Delta$ such that $a_1a_2\cdots a_{n-1}\notin I_i$ and we also have $a_1a_2\cdots a_n\in I_i\setminus\{0\}$. This gives $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n\in I_i$ for some $i\in\{1,2,\ldots,n-1\}$, since I_i is an $(n-1,n)$-weakly prime ideal of S and $a_1a_2\cdots a_{n-1}\notin I_i$. Thus, $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n\in I_i\subseteq\sqrt{I_i}=\sqrt{0}=\sqrt{I}$ for some $i\in\{1,2,\ldots,n-1\}$. Hence I is an $(n-1,n)$-weakly prime ideal of S. ∎

THEOREM 3.4. *Let $S=S_1\times S_2\times\cdots\times S_n$, where S_i is a commutative semiring for all $i\in\{1,2,\ldots,n\}$. If I is an $(n-1,n)$-weakly prime ideal of S, then either $I=0$ or $I=I_1\times I_2\times\ldots\times I_{i-1}\times S_i\times I_{i+1}\times\ldots\times I_n$ for some $i\in\{1,2,\ldots,n\}$ and if $I_j\neq S_j$ for $j\neq i$, then I_j is an $(n-1,n)$-prime ideal in S_j.*

Proof. Let $I=I_1\times I_2\times\cdots\times I_n$ be an $(n-1,n)$-weakly prime ideal of S and let $I\neq 0$. Then $(0,0,\ldots,0)\neq(a_1,a_2,\ldots,a_n)\in I$. Therefore, $(a_1,a_2,\ldots,a_n)=(a_1,1,\ldots,1)(1,a_2,\ldots,1)\cdots(1,1,\ldots,a_n)\in I$. Since I is $(n-1,n)$-weakly prime, we have $(a_1,a_2,\ldots a_{i-1},1,a_{i+1},\ldots,a_n)\in I$ for some $i\in\{1,2,\ldots,n\}$. Thus $(0,\ldots,1,0,\ldots,0)\in I$. Hence $I=I_1\times I_2\times\cdots\times I_{i-1}\times S_i\times I_{i+1}\times\cdots\times I_n$. Next, suppose $I_j\neq S_j$ for $i\neq j$. Let $i<j$ and $x_1x_2\cdots x_n\in I_j$. Then

$$\begin{aligned}0&\neq(0,0,\ldots,0,1,\ldots,x_1x_2\cdots x_n,0\ldots,0)\\&=(0,0,\ldots,1,0,\ldots,0,x_1,0,0\ldots,0)(0,0,\ldots,1,0,\ldots,0,x_2,0,\ldots,0)\\&\qquad\cdots(0,0,\ldots,1,0,\ldots,0,0,x_n,0,\ldots,0)\in I.\end{aligned}$$

Thus, we have $(0,0,\ldots,0,1,0,\ldots,x_1x_2\cdots x_{l-1}x_{l+1}\cdots x_n,0,\ldots,0)\in I$ for some $l\in\{1,2,\ldots,n\}$. Hence $x_1x_2\cdots x_{l-1}x_{l+1}\cdots x_n\in I_j$. Consequently, I_j is an $(n-1,n)$-prime ideal of S_j. The other cases for $j<i$ are similar. ∎

A semiring S is said to be a local semiring denoted by (S,M) if and only if S has a unique maximal subtractive ideal, say M. Darani [9] proved that the semiring S is a local semiring if and only if the set of non-semi-unit elements of S forms a subtractive ideal. It is also proved that if S is a local semiring then the unique maximal subtractive ideal of S is precisely the set of non-semi-units of S.

THEOREM 3.5. *Let (S,M) be a local semiring with $M^n=0$. Then every proper subtractive ideal of S is $(n-1,n)$-weakly prime.*

Proof. Suppose that $M^n=0$, and let I be a proper subtractive ideal of S and $a_1,a_2,\ldots,a_n\in S$. Suppose that $0\neq a_1a_2\cdots a_n\in I$. Since (S,M) is a local semiring then we have $a_i\in M$ for some $i\in\{1,2,\ldots,n\}$. Since each a_i, $i\in\{1,2,\ldots,n\}$ does not belong to M, at the same time because in this case $a_1a_2\cdots a_n \in M^n = 0$, this gives a contradiction. So a_i for some $i \in \{1,2,\dots,n\}$ must be a semi-unit. Assume that a_i is a semi-unit. Then there exist $r,s \in S$ such that $1+ra_i = sa_i$. So $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n + ra_1a_2\cdots a_n = (1+ra_i)a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n = sa_1a_2\cdots a_n \in I$ and $ra_1a_2\cdots a_n \in I$ imply that $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I$ for some $i \in \{1,2,\dots,n\}$. Thus I is an $(n-1,n)$-weakly prime. ∎

Definition 3.6. [1, Definition (4)] An ideal I of a semiring S is called a $Q-$ideal (partitioning ideal) if there exists a subset Q of S such that

(i) $S = \cup\{q+I : q \in Q\}$

(ii) If $q_1,q_2 \in Q$, then $(q_1+I)\cap(q_2+I) \neq \emptyset \Leftrightarrow q_1 = q_2$.

Let I be a Q-ideal of a semiring S. Then $S/I_{(Q)} = \{q+I : q \in Q\}$ forms a semiring under the following addition '$\oplus$' and multiplication '$\odot$', $(q_1+I)\oplus(q_2+I) = q_3+I$ where $q_3 \in Q$ is unique such that $q_1+q_2+I \subseteq q_3+I$, and $(q_1+I)\odot(q_2+I) = q_4+I$ where $q_4 \in Q$ is unique such that $q_1q_2+I \subseteq q_4+I$. This semiring $S/I_{(Q)}$ is called the quotient semiring of S by I and denoted by $(S/I_{(Q)},\oplus,\odot)$ or just $S/I_{(Q)}$. By definition of a Q-ideal, there exists a unique $q_0 \in Q$ such that $0+I \subseteq q_0+I$. Then q_0+I is a zero element of $S/I_{(Q)}$. Clearly, if S is commutative then so is $S/I_{(Q)}$.

Theorem 3.7. *Let S be a semiring, I be a $Q-$ideal of S and P a subtractive ideal of S such that $I \subseteq P$. Then P is an $(n-1,n)$-prime ideal of S if and only if $P/I_{(Q\cap P)}$ is an $(n-1,n)$-prime ideal of $S/I_{(Q)}$.*

Proof. Let P be an $(n-1,n)$-prime ideal of S. Suppose that $q_1+I, q_2+I,\dots,q_n+I \in S/I_{(Q)}$ are such that $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) = q_r+I \in P/I_{(Q\cap P)}$ where $q_r \in Q\cap P$ is a unique element such that $q_1q_2\cdots q_n+I \subseteq q_r+I \in P/I_{(Q\cap P)}$. So $q_1q_2\cdots q_n = q_r+i$, for some $i \in I$. Since P is a subtractive $(n-1,n)$-prime ideal of S and $I \subseteq P$, therefore $q_1q_2\cdots q_{i-1}q_{i+1}\cdots q_n \in P$ for some $i \in \{1,2,\dots,n\}$. Now $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_{i-1}+I)\odot(q_{i+1}+I)\odot\cdots\odot(q_n+I) = i_1+I$ where $i_1 \in Q$ is a unique element such that $q_1q_2\cdots q_{i-1}q_{i+1}\cdots q_n+I \subseteq i_1+I$. So $i_1+f = q_1q_2\cdots q_{i-1}q_{i+1}\cdots q_n+e$ for some $e,f \in I$. Since P is a subtractive ideal of S and $I \subseteq P$, we have $i_1 \in P$, therefore $i_1 \in Q\cap P$. Hence $P/I_{(Q\cap P)}$ is an $(n-1,n)$-prime ideal of $S/I_{(Q)}$.

Conversely, let $P/I_{(Q\cap P)}$ is an $(n-1,n)$-prime ideal of $S/I_{(Q)}$. Let $a_1a_2\cdots a_n \in P$ for some $a_1,a_2,\dots,a_n \in S$. Since I is a $Q-$ideal of S, therefore there exist $q_1,q_2,\dots,q_n \in Q$ such that $a_1 \in q_1+I$, $a_2 \in q_2+I$, $\dots$, $a_n \in q_n+I$ and $a_1a_2\cdots a_n \in (q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) = q_k+I$, for some $q_k \in Q$. So, $a_1a_2\cdots a_n = q_k+i_2 \in P$ for some $i_2 \in I$. Since P is a subtractive ideal of S and $I \subseteq P$, we have $q_k \in P$. So, $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) = q_k+I \in P/I_{(Q\cap P)}$ which gives $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_{i-1}+I)\odot(q_{i+1}+I)\odot\cdots\odot(q_n+I) \in P/I_{(Q\cap P)}$ for some $i \in \{1,2,\dots,n\}$, since $P/I_{(Q\cap P)}$ is an $(n-1,n)$-prime ideal of $S/I_{(Q)}$. Now for some $i \in \{1,2,\dots,n\}$, we have $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_{i-1}+I)\odot(q_{i+1}+I)\odot\cdots\odot(q_n+I) \in P/I_{(Q\cap P)}$. Then there exists $q_h \in Q\cap P$ such that

$a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in (q_1+I)\odot(q_2+I)\odot\cdots\odot(q_{i-1}+I)\odot(q_{i+1}+I)\odot\cdots\odot(q_n+I) = q_h+I$. This gives $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n = q_h+i_3$ for some $i_3 \in I$. This implies $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in P$ for some $i \in \{1,2,\ldots,n\}$. Thus P is an $(n-1,n)$-prime ideal of S. ■

THEOREM 3.8. *Let S be a semiring, I a $Q-$ideal of S and P a subtractive ideal of S such that $I \subseteq P$. Then*

(i) *if P is an $(n-1,n)$-weakly prime ideal of S, then $P/I_{(Q\cap P)}$ is an $(n-1,n)$-weakly prime ideal of $S/I_{(Q)}$;*

(ii) *if I and $P/I_{(Q\cap P)}$ are $(n-1,n)$-weakly prime ideals of S and $S/I_{(Q)}$ respectively, then P is an $(n-1,n)$-weakly prime ideal of S.*

Proof. (i) If $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) \neq 0$ in S/I_Q, then $q_1q_2\cdots q_n \neq 0$ in S and hence the proof follows from the above theorem.

(ii) Let $a_1,a_2,\ldots,a_n \in S$ be such that $0 \neq a_1a_2\cdots a_n \in P$. If $a_1a_2\cdots a_n \in I$ then $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in I \subseteq P$ for some $i \in \{1,2,\ldots,n\}$, since I is an $(n-1,n)$-weakly prime ideal of S. So, assume that $a_1a_2\cdots a_n \notin I$. Then there are elements $q_1,q_2,\cdots q_n \in Q$ such that $a_1 \in q_1+I$, $a_2 \in q_2+I$, $\ldots$, $a_n \in q_n+I$. Therefore for some $i_1,i_2,\ldots,i_n \in I$, $a_1 = q_1+i_1$, $a_2 = q_2+i_2,\ldots,a_n = q_n+i_n$. As $a_1a_2\cdots a_n = q_1q_2\cdots q_n + q_1q_2\cdots q_{n-1}i_n + \cdots + q_ni_1i_2\cdots i_{n-1} + i_1i_2\cdots i_n \in P$ and since P is subtractive, we have $q_1q_2\cdots q_n \in P$. Consider, $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) = q_k+I$ where $q_k \in Q$ is the unique element such that $q_1q_2\cdots q_n+I \subseteq q_k+I$. Since P is subtractive, we have $q_k \in P\cap Q$. Hence $q_1q_2\cdots q_n+I \subseteq q_k+I \in P/I_{Q\cap P}$, that is, $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) \in P/I_{Q\cap P}$. Let $q \in Q$ be the unique element such that $q+I$ is the zero element in S/I_Q. If $(q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) = 0_{S/I_Q} = q+I$, then there exist $r,s \in I$ such that $q_1q_2\cdots q_n+r = q+s \in I$. Therefore, $q_1q_2\cdots q_n \in I$, since I is a Q-ideal of S therefore it is subtractive by [15, Corollary 8.23]. This gives $a_1a_2\cdots a_n \in I$, a contradiction. Hence, $0_{S/I_Q} \neq (q_1+I)\odot(q_2+I)\odot\cdots\odot(q_n+I) \in P/I_{Q\cap P}$. This gives $(q_1+I)\odot(q_2+I)\cdots\odot(q_{i-1}+I)\odot(q_{i+1}+I)\odot\cdots\odot(q_n+I) \in P/I_{Q\cap P}$ for some $i \in \{1,2,\ldots,n\}$, since $P/I_{Q\cap P}$ is an $(n-1,n)$-weakly prime ideal of S/I_Q. Thus, we have $a_1a_2\cdots a_{i-1}a_{i+1}\cdots a_n \in P$ for some $i \in \{1,2,\ldots,n\}$. Hence, P is an $(n-1,n)$-weakly prime ideal of S. ■

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