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ON ϕ-SYMMETRIC LP-SASAKIAN MANIFOLDS ADMITTING SEMI-SYMMETRIC METRIC CONNECTION

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Abstract. The object of the present paper is to study locally ϕ-symmetric LP-Sasakian manifolds admitting a semi-symmetric metric connection and obtain a necessary and sufficient condition for a locally ϕ-symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection to be locally ϕ-symmetric LP-Sasakian manifold with respect to the Levi-Civita connection.

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1. Introduction

Analogously to the Sasakian manifolds, in 1989 Matsumoto [12] introduced the notion of LP-Sasakian manifolds. Again the same notion was studied by Mihai and Rosca [13] and they obtained many results. LP-Sasakian manifolds were also studied by De et. al. [8], Shaikh et. al. ([15], [16], [17], [19]), Taleshian and Asghari [27], Venkatesha and Bagewadi [28] and many others. The notion of a local ϕ-symmetry on a 3-dimensional LP-Sasakian manifold was studied by Shaikh and De [20].

In 1924 Friedmann and Schouten [10] introduced the notion of a semi-symmetric linear connection on a differentiable manifold. Then in 1932 Hayden [11] introduced the idea of metric connection with torsion on a Riemannian manifold. A systematic study of the semi-symmetric metric connection on a Riemannian manifold has been given by Yano [29] in 1970. Also semi-symmetric metric connection on a Riemannian manifold has been studied by Barua and Mukhopadhyay [1], Binh [3], Chaki and Chaki [5], Chaturvedi and Pandey [6], Shaikh and Hui [22], Sharfuddin and Hussain [24] and many others. Recently Shaikh and Jana studied the quarter-symmetric metric connection on a (k, μ)-contact metric manifold [23].

The study of Riemann symmetric manifolds began with the work of Cartan [4]. A Riemannian manifold (M^n, g) is said to be locally symmetric due to Cartan [4] if its curvature tensor R satisfies the relation $\nabla R = 0$, where ∇

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denotes the operator of covariant differentiation with respect to the metric tensor g. As a weaker version of local symmetry, the notion of a locally ϕ-symmetric Sasakian manifold was introduced by Takahashi [26]. Shaikh and Baishya [15] studied locally ϕ-symmetric LP-Sasakian manifolds in the sense of Takahashi,. The notion of locally ϕ-symmetric manifolds in different structures has been studied by several authors (see, [7], [15], [18], [21], [26]). An LP-Sasakian manifold is said to be ϕ-symmetric [7] if it satisfies the condition

$$\phi^2((\nabla_W R)(X,Y)Z) = 0 \tag{1.1}$$

for arbitrary vector fields X, Y, Z and W on M.

In particular, if X, Y, Z, W are horizontal vector fields, i.e., orthogonal to ξ, then it is called a locally ϕ-symmetric LP-Sasakian manifold [26].

It is easy to check that an LP-Sasakian manifold is ϕ-symmetric if and only if it is locally symmetric or locally ϕ-symmetric.

Recently De and Sarkar [9] studied ϕ-Ricci symmetric Sasakian manifolds. In this connection Shukla and Shukla [25] studied ϕ-Ricci symmetric Kenmotsu manifolds. An LP-Sasakian manifold is said to be ϕ-Ricci symmetric [9] if it satisfies

$$\phi^2((\nabla_X Q)(Y)) = 0, \tag{1.2}$$

where Q is the Ricci operator, i.e., $g(QX,Y) = S(X,Y)$ for all vector fields X, Y.

If X, Y are horizontal vector fields then the manifold is said to be locally ϕ-Ricci symmetric.

It is easy to check that an LP-Sasakian manifold is ϕ-Ricci symmetric if and only if it is Ricci symmetric or locally ϕ-Ricci symmetric.

The object of the present paper is to study the locally ϕ-symmetric and locally ϕ-Ricci symmetric LP-Sasakian manifolds admitting semi-symmetric metric connection. The paper is organized as follows. Section 2 is concerned with some preliminaries about LP-Sasakian manifolds and semi-symmetric metric connections. Section 3 is devoted to the study of locally ϕ-symmetric LP-Sasakian manifolds admitting a semi-symmetric metric connection and obtained a necessary and sufficient condition for a locally ϕ-symmetric LP-Sasakian manifold with respect to semi-symmetric metric connection to be locally ϕ-symmetric LP-Sasakian manifold with respect to the Levi-Civita connection. Section 4 deals with the study of locally ϕ-Ricci symmetric LP-Sasakian manifolds admitting semi-symmetric metric connection.

2. Preliminaries

An n-dimensional smooth manifold M is said to be an LP-Sasakian manifold ([13], [16]) if it admits a (1, 1) tensor field ϕ, a unit timelike vector field ξ, an 1-form η and a Lorentzian metric g, which satisfy

$$\eta(\xi) = -1, \quad g(X,\xi) = \eta(X), \quad \phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

(2.2) $$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad \nabla_X \xi = \phi X,$$

(2.3) $$(\nabla_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g. It can be easily seen that in an LP-Sasakian manifold, the following relations hold:

(2.4) $$\phi\xi = 0, \quad \eta \circ \phi = 0, \quad \text{rank } \phi = n - 1.$$

Again, if we take
$$\Omega(X, Y) = g(X, \phi Y)$$
for any vector fields X, Y, then the tensor field $\Omega(X, Y)$ is a symmetric (0,2) tensor field [12]. Also, since the vector field η is closed in an LP-Sasakian manifold, we have ([8], [12])

(2.5) $$(\nabla_X \eta)(Y) = \Omega(X, Y), \quad \Omega(X, \xi) = 0$$

for any vector fields X and Y.

Let M be an n-dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g). Then the following relations hold ([15], [16]):

(2.6) $$R(X, Y)\xi = \eta(Y)X - \eta(X)Y,$$

(2.7) $$\eta(R(X, Y)Z) = \eta(X)g(Y, Z) - \eta(Y)g(X, Z),$$

(2.8) $$S(X, \xi) = (n - 1)\eta(X),$$

(2.9) $$S(\phi X, \phi Y) = S(X, Y) + (n - 1)\eta(X)\eta(Y),$$

(2.10) $$(\nabla_W R)(X, Y)\xi = \Omega(Y, W)X - \Omega(X, W)Y - R(X, Y)\phi W,$$

(2.11) $$(\nabla_W R)(X, \xi)Y = \Omega(W, Z)X - g(X, Z)\phi W - R(X, \phi W)Z$$

for any vector fields X, Y, Z, where R is the curvature tensor of g.

Let M be an n-dimensional LP-Sasakian manifold and ∇ be the Levi-Civita connection on M. A linear connection $\widetilde{\nabla}$ on M is said to be semi-symmetric if the torsion tensor τ of the connection $\widetilde{\nabla}$
$$\tau(X, Y) = \widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X - [X, Y]$$
satisfies

(2.12) $$\tau(X, Y) = \eta(Y)X - \eta(X)Y$$

for all $X, Y \in \chi(M)$; $\chi(M)$ being the Lie algebra of all smooth vector fields on M. A semi-symmetric connection $\widetilde{\nabla}$ is called semi-symmetric metric connection if it further satisfies

$$\widetilde{\nabla} g = 0. \tag{2.13}$$

A semi-symmetric metric connection $\widetilde{\nabla}$ in an LP-Sasakian manifold is defined by ([24],[29]):

$$\widetilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi. \tag{2.14}$$

If R and $\widetilde{R}$ are respectively the curvature tensor of the Levi-Civita connection ∇ and the semi-symmetric metric connection $\widetilde{\nabla}$ in an LP-Sasakian manifold, then we have [14]

$$\begin{aligned} \widetilde{R}(X,Y)Z &= R(X,Y)Z - \alpha(Y,Z)X + \alpha(X,Z)Y \\ &- g(Y,Z)LX + g(X,Z)LY, \end{aligned} \tag{2.15}$$

where α is a symmetric (0,2) tensor field given by

$$\alpha(X,Y) = (\widetilde{\nabla}_X \eta)(Y) + \frac{1}{2}g(X,Y), \tag{2.16}$$

$$LX = \widetilde{\nabla}_X \xi + \frac{1}{2}X = \phi X - \frac{1}{2}X - \eta(X)\xi \tag{2.17}$$

and

$$g(LX,Y) = \alpha(X,Y). \tag{2.18}$$

Lemma 2.1. [14] *In an LP-Sasakian manifold with semi-symmetric metric connection $\widetilde{\nabla}$, we have*

$$\widetilde{R}(X,Y)Z + \widetilde{R}(Y,Z)X + \widetilde{R}(Z,X)Y = 0, \tag{2.19}$$

$$g(\widetilde{R}(X,Y)Z,U) = -g(\widetilde{R}(Y,X)Z,U), \tag{2.20}$$

$$g(\widetilde{R}(X,Y)Z,U) = -g(\widetilde{R}(X,Y)U,Z), \tag{2.21}$$

$$g(\widetilde{R}(X,Y)Z,U) = g(\widetilde{R}(Z,U)X,Y). \tag{2.22}$$

Lemma 2.2. [14] *In an n-dimensional LP-Sasakian manifold the Ricci tensor $\widetilde{S}$ and scalar curvature $\widetilde{r}$ with respect to the semi-symmetric metric connection $\widetilde{\nabla}$ are given by*

$$\widetilde{S}(X,Y) = S(X,Y) - (n-2)\alpha(X,Y) - ag(X,Y) \tag{2.23}$$

and

$$\widetilde{r} = r - 2(n-1)a, \tag{2.24}$$

where $a =$ tr. α, S and r denote the Ricci tensor and scalar curvature of the Levi-Civita connection ∇ respectively.

Lemma 2.3. [14] *Let M be an n-dimensional LP-Sasakian manifold with the semi-symmetric metric connection $\widetilde{\nabla}$. Then we have*

$$g(\widetilde{R}(X,Y)Z,\xi) = \eta(\widetilde{R}(X,Y)Z) = (\widetilde{\nabla}_X\eta)(Z)\eta(Y) - (\widetilde{\nabla}_Y\eta)(Z)\eta(X), \tag{2.25}$$

$$\widetilde{R}(\xi,X)\xi = -\widetilde{\nabla}_X\xi = X + \eta(X)\xi - \phi X, \tag{2.26}$$

$$\widetilde{R}(X,Y)\xi = \eta(X)\widetilde{\nabla}_Y\xi - \eta(Y)\widetilde{\nabla}_X\xi, \tag{2.27}$$

$$\widetilde{R}(\xi,X)Y = \eta(Y)\widetilde{\nabla}_X\xi - g(Y,\widetilde{\nabla}_X\xi)\xi, \tag{2.28}$$

$$\widetilde{S}(X,\xi) = \Big(\frac{n}{2} - a\Big)\eta(X), \tag{2.29}$$

$$\begin{aligned} \widetilde{S}(\phi X,\phi Y) &= S(X,Y) + \Big(\frac{n}{2} - a\Big)\eta(X)\eta(Y) \\ &- (n-2)\alpha(X,Y) - ag(X,Y) \end{aligned} \tag{2.30}$$

for arbitrary vector fields X, Y and Z.

From (2.2), (2.3), (2.5), (2.14) and (2.17), we get

$$\begin{aligned} (\widetilde{\nabla}_W\widetilde{R})(X,Y)\xi &= R(X,Y)W - R(X,Y)\phi W + \alpha(X,W)Y \\ &- \alpha(Y,W)X + g(X,W)LY - g(Y,W)LX \\ &+ \alpha(Y,\phi W)X - \alpha(X,\phi W)Y + \Omega(Y,W)LX \\ &- \Omega(X,W)LY + g(X,W)Y - g(Y,W)X \\ &+ g(Y,W)\phi X - g(X,W)\phi Y + \Omega(Y,W)X \\ &- \Omega(X,W)Y + \Omega(X,W)\phi Y - \Omega(Y,W)\phi X \\ &+ \eta(X)[g(Y,W) - \Omega(Y,W)]\xi \\ &- \eta(Y)[g(X,W) - \Omega(X,W)]\xi \end{aligned} \tag{2.31}$$

for arbitrary vector fields X, Y and W. Also from (2.14), (2.15) and (2.21), we have

$$g((\widetilde{\nabla}_W\widetilde{R})(X,Y)Z,U) = -g((\widetilde{\nabla}_W\widetilde{R})(X,Y)U,Z). \tag{2.32}$$

From (2.17) we have

$$\alpha(X,\xi) = \frac{1}{2}\eta(X), \tag{2.33}$$

$$(\nabla_W\alpha)(X,\xi) = \frac{1}{2}\Omega(W,X) - \alpha(X,\phi W), \tag{2.34}$$

$$(\nabla_W L)(X) = [g(W,X) - \Omega(W,X)]\xi + \eta(X)[W - \phi W] + 2\eta(X)\eta(W)\xi. \tag{2.35}$$

Again by the virtue of (2.33) - (2.35) we have from (2.14) and (2.15) that

$$\begin{aligned}
&(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z \\
&= (\nabla_W R)(X,Y)Z - g(R(X,Y)Z,W)\xi + [g(W,Y) - \Omega(W,Y)]\eta(Z)X \\
&+ [g(W,Z) - \Omega(W,Z)]\eta(Y)X + 2\eta(Z)\eta(W)[\eta(Y)X - \eta(X)Y] \\
&+ \alpha(Y,Z)[g(X,W)\xi - \eta(X)W] + [\Omega(W,X) - g(W,X)]\eta(Z)Y \\
&+ [\Omega(W,Z) - g(W,Z)]\eta(X)Y + \alpha(X,Z)[\eta(Y)W - g(Y,W)\xi] \\
&- g(Y,Z)[\{g(W,X) - \Omega(W,X) - \alpha(X,W)\}\xi + \eta(X)\{\frac{1}{2}W - \phi W + 2\eta(W)\xi\}] \\
&+ g(X,Z)[\{g(W,Y) - \Omega(W,Y) - \alpha(Y,W)\}\xi + \eta(Y)\{\frac{1}{2}W - \phi W + 2\eta(W)\xi\}].
\end{aligned} \tag{2.36}$$

By the virtue of (2.33) and (2.35) it follows from (2.14) that

$$\begin{aligned}
(\widetilde{\nabla}_X \widetilde{S})(Y,Z) &= (\nabla_X S)(Y,Z) - [S(X,Y) + \alpha(X,Y)]\eta(Z) \\
&+ [\frac{3}{2}g(X,Z) + (n-2)\Omega(X,Z)]\eta(Y) \\
&- (n-2)[g(X,Y) - \Omega(X,Y)]\eta(Z) - da(X)g(Y,Z).
\end{aligned} \tag{2.37}$$

Also from (2.8) we have

$$(\nabla_X S)(Y,\xi) = (n-1)\Omega(X,Y) - S(Y,\phi X). \tag{2.38}$$

3. Locally ϕ-symmetric LP-Sasakian manifolds admitting semi-symmetric metric connection

Definition 3.1. An LP-Sasakian manifold M is said to be locally ϕ-symmetric with respect to a semi-symmetric metric connection if its curvature tensor $\widetilde{R}$ satisfies the condition

$$\phi^2((\widetilde{\nabla}_W \widetilde{R})(X,Y)Z) = 0 \tag{3.1}$$

for all horizontal vector fields X, Y, Z and W.

We now consider a locally ϕ-symmetric LP-Sasakian manifold with respect to a semi-symmetric metric connection. Then by the virtue of (2.1) it follows from (3.1) that

$$(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z + \eta((\widetilde{\nabla}_W \widetilde{R})(X,Y)Z)\xi = 0. \tag{3.2}$$

Using (2.32) in (3.2), we get

$$(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z = g((\widetilde{\nabla}_W \widetilde{R})(X,Y)\xi, Z)\xi. \tag{3.3}$$

In view of (2.31) it follows from (3.3) that

$$
\begin{aligned}
(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z &= \Big[g(R(X,Y)W,Z) - g(R(X,Y)\phi W,Z) + \alpha(X,W)g(Y,Z) \\
&- \alpha(Y,W)g(X,Z) + g(X,W)\alpha(Y,Z) - g(Y,W)\alpha(X,Z) \\
&+ \alpha(Y,\phi W)g(X,Z) - \alpha(X,\phi W)g(Y,Z) + \Omega(Y,W)\alpha(X,Z) \\
&- \Omega(X,W)\alpha(Y,Z) + g(X,W)g(Y,Z) - g(Y,W)g(X,Z) \\
&+ g(Y,W)\Omega(X,Z) - g(X,W)\Omega(Y,Z) + \Omega(Y,W)g(X,Z) \\
&- \Omega(X,W)g(Y,Z) + \Omega(X,W)\Omega(Y,Z) - \Omega(Y,W)\Omega(X,Z)\Big]\xi
\end{aligned} \tag{3.4}
$$

for all horizontal vector fields X, Y, Z and W. Next, let us assume that in an LP-Sasakian manifold, the relation (3.4) holds for all horizontal vector fields X, Y, Z and W. Then it follows from (2.36) that (3.4) holds and consequently the manifold is locally ϕ-symmetric with respect to a semi-symmetric metric connection. This leads to the following:

Theorem 3.1. *An LP-Sasakian manifold is locally ϕ-symmetric with respect to semi-symmetric metric connection if and only if the relation* (3.4) *holds for all horizontal vector fields X, Y, Z and W.*

In view of (2.32), it follows from (3.2) that

$$
(\widetilde{\nabla}_W \widetilde{R})(X,Y)\xi = 0. \tag{3.5}
$$

From (2.31) and (3.5) it follows that

$$
\begin{aligned}
& R(X,Y)W - R(X,Y)\phi W \\
= \; & g(Y,W)X - g(X,W)Y + g(X,W)\phi Y - g(Y,W)\phi X \\
+ \; & \Omega(X,W)Y - \Omega(Y,W)X + \Omega(Y,W)\phi X - \Omega(X,W)\phi Y \\
+ \; & \alpha(Y,W)X - \alpha(X,W)Y + g(Y,W)LX - g(X,W)LY \\
+ \; & \alpha(X,\phi W)Y - \alpha(Y,\phi W)X + \Omega(X,W)LY - \Omega(Y,W)LX
\end{aligned} \tag{3.6}
$$

for horizontal vector fields X, Y and W. Contracting (3.6), we get

$$
\begin{aligned}
S(Y,W) - S(Y,\phi W) &= (n-1+a-\psi)[g(Y,W) - \Omega(Y,W)] \\
&+ (n-2)[\alpha(Y,W) - \alpha(Y,\phi W)],
\end{aligned} \tag{3.7}
$$

where $\psi =$ tr. Ω and $a =$ tr. α. Hence we can state the following:

Theorem 3.2. *In a locally ϕ-symmetric LP-Sasakian manifold with a semi-symmetric metric connection, the curvature tensor and the Ricci tensor are respectively given by* (3.6) *and* (3.7).

We now consider a locally ϕ-symmetric LP-Sasakian manifold with the Levi-Civita connection. Then in [15], Shaikh and Baishya proved that

Theorem 3.3. *An LP-Sasakian manifold (M^n, g) is locally ϕ-symmetric with respect to the Levi-Civita connection if and only if the following relation*

$$
\begin{aligned}
(3.8) \qquad & (\nabla_W R)(X,Y)Z \\
= \;& \big[2\{\Omega(Y,W)g(X,Z) - \Omega(X,W)g(Y,Z)\} \\
+ \;& \Omega(Y,Z)g(X,W) - \Omega(X,Z)g(Y,W) \\
+ \;& 2\{\Omega(Y,Z)\eta(X)\eta(W) - \Omega(X,Z)\eta(Y)\eta(W)\} - g(\phi R(X,Y)W,Z)\big]\xi \\
+ \;& \eta(X)[\Omega(W,Z)Y - g(Y,Z)\phi W - R(Y,\phi W)Z] \\
- \;& \eta(Y)[\Omega(W,Z)X - g(X,Z)\phi W - R(X,\phi W)Z] \\
- \;& \eta(Z)[2\{\Omega(Y,W)X - \Omega(X,W)Y\} - \phi R(X,Y)W - g(Y,W)\phi X \\
+ \;& g(X,W)\phi Y] + 2\{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z)\eta(W).
\end{aligned}
$$

holds for arbitrary vector fields X, Y, Z, $W \in \chi(M)$.

Now we take a locally ϕ-symmetric LP-Sasakian manifold with respect to a semi-symmetric metric connection. Then the relation (3.4) holds for any horizontal vector fields X, Y, Z, W.

Let X, Y, Z, W be arbitrary vector fields of $\chi(M)$. We now compute $(\widetilde{\nabla}_{\phi^2 W}\widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z$ in two different ways. Firstly, by the virtue of (2.1), it follows from (3.4) that

$$
\begin{aligned}
(3.9) \qquad & (\widetilde{\nabla}_{\phi^2 W}\widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\
= \;& \big[g(R(\phi^2 X, \phi^2 Y)\phi^2 W, \phi^2 Z) - g(R(\phi^2 X, \phi^2 Y)\phi^3 W, \phi^2 Z) \\
+ \;& \alpha(\phi^2 X, \phi^2 W)\{g(Y,Z) + \eta(Y)\eta(Z)\} \\
- \;& \alpha(\phi^2 Y, \phi^2 W)\{g(X,Z) + \eta(X)\eta(Z)\} \\
+ \;& \alpha(\phi^2 Y, \phi^2 Z)\{g(X,W) + \eta(X)\eta(W)\} \\
- \;& \alpha(\phi^2 X, \phi^2 Z)\{g(Y,W) + \eta(Y)\eta(W)\} \\
+ \;& \alpha(\phi^2 Y, \phi^3 W)\{g(X,Z) + \eta(X)\eta(Z)\} \\
- \;& \alpha(\phi^2 X, \phi^3 W)\{g(Y,Z) + \eta(Y)\eta(Z)\} \\
+ \;& \Omega(Y,W)\alpha(\phi^2 X, \phi^2 Z) - \Omega(X,W)\alpha(\phi^2 Y, \phi^2 Z) \\
+ \;& \{g(X,W) + \eta(X)\eta(W)\}\{g(Y,Z) + \eta(Y)\eta(Z)\} \\
- \;& \{g(Y,W) + \eta(Y)\eta(W)\}\{g(X,Z) + \eta(X)\eta(Z)\} \\
+ \;& \{g(Y,W) + \eta(Y)\eta(W)\}\Omega(X,Z) - \{g(X,W) + \eta(X)\eta(W)\}\Omega(Y,Z) \\
+ \;& \{g(X,Z) + \eta(X)\eta(Z)\}\Omega(Y,W) - \{g(Y,Z) + \eta(Y)\eta(Z)\}\Omega(X,W) \\
+ \;& \Omega(X,W)\Omega(Y,Z) - \Omega(Y,W)\Omega(X,Z)\big]\xi.
\end{aligned}
$$

From (2.4) we have

$$
(3.10) \qquad g(\phi^2 X, \xi) = g(\phi^2 Y, \xi) = g(\phi^2 Z, \xi) = 0
$$

and hence $\phi^2 X$, $\phi^2 Y$, $\phi^2 Z$ are horizontal vector fields of $\chi(M)$. Then by the virtue of (2.1) it follows that

$$
\begin{aligned}
(3.11) \quad R(\phi^2 X, \phi^2 Y)\phi^2 W \;=\; & R(X,Y)W + \{\eta(Y)X - \eta(X)Y\}\eta(W) \\
+ \;& \{g(Y,W)\eta(X) - g(X,W)\eta(Y)\}\xi,
\end{aligned}
$$

$$R(\phi^2 X, \phi^2 Y)\phi^3 W = R(X,Y)\phi W + \{\Omega(Y,W)\eta(X) - \Omega(X,W)\eta(Y)\}\xi, \tag{3.12}$$

$$\alpha(\phi^2 X, \phi^2 W) = \alpha(X,W) + \frac{1}{2}\eta(X)\eta(W). \tag{3.13}$$

In view of (3.11) - (3.13), (3.9) yields

$$\begin{aligned}
&(3.14) \qquad (\widetilde{\nabla}_{\phi^2 W}\widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\
&= \big[g(R(X,Y)W,Z) - g(R(X,Y)\phi W, Z) + \alpha(X,W)\{g(Y,Z) + \eta(Y)\eta(Z)\} \\
&- \alpha(Y,W)\{g(X,Z) + \eta(X)\eta(Z)\} + \frac{1}{2}\{\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\}\eta(W) \\
&+ \alpha(Y,Z)g(X,W) - \alpha(X,Z)g(Y,W) + \frac{1}{2}\{\eta(Y)g(X,W) - \eta(X)g(Y,W)\}\eta(Z) \\
&+ \{\eta(X)\alpha(Y,Z) - \eta(Y)\alpha(X,Z)\}\eta(W) + \alpha(Y,\phi W)g(X,Z) - \alpha(X,\phi W)g(Y,Z) \\
&+ \{\eta(X)\alpha(Y,\phi W) - \eta(Y)\alpha(X,\phi W)\}\eta(Z) + \Omega(Y,W)\alpha(X,Z) - \Omega(X,W)\alpha(Y,Z) \\
&+ \frac{1}{2}\{\eta(X)\Omega(Y,W) - \eta(Y)\Omega(X,W)\}\eta(Z) + g(X,W)g(Y,Z) - g(Y,W)g(X,Z) \\
&+ g(Y,W)\Omega(X,Z) - g(X,W)\Omega(Y,Z) + \{\eta(Y)\Omega(X,Z) - \eta(X)\Omega(Y,Z)\}\eta(W) \\
&+ \Omega(Y,W)g(X,Z) - \Omega(X,W)g(Y,Z) + \{\eta(X)\Omega(Y,W) - \eta(Y)\Omega(X,W)\}\eta(Z) \\
&+ \Omega(X,W)\Omega(Y,Z) - \Omega(Y,W)\Omega(X,Z)\big]\xi.
\end{aligned}$$

By the virtue of (2.1) we have

$$\begin{aligned}
(3.15) \qquad (\widetilde{\nabla}_{\phi^2 W}\widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z &= (\widetilde{\nabla}_W \widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\
&+ \eta(W)(\widetilde{\nabla}_\xi \widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z.
\end{aligned}$$

Now, for any horizontal vector fields X, Y and Z, we have from (3.4) that

$$(\widetilde{\nabla}_\xi \widetilde{R})(X,Y)Z = 0, \tag{3.16}$$

which implies that

$$(\widetilde{\nabla}_\xi \widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z = 0. \tag{3.17}$$

Using (3.17) in (3.15) we obtain

$$(\widetilde{\nabla}_{\phi^2 W}\widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z = (\widetilde{\nabla}_W \widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z. \tag{3.18}$$

In view of (2.1), we have

$$\begin{aligned}
(3.19) \qquad & (\widetilde{\nabla}_W \widetilde{R})(\phi^2 X, \phi^2 Y)\phi^2 Z \\
= \;& (\widetilde{\nabla}_W \widetilde{R})(X,Y)Z + \eta(Z)(\widetilde{\nabla}_W \widetilde{R})(X,Y)\xi \\
+ \;& \eta(Y)(\widetilde{\nabla}_W \widetilde{R})(X,\xi)Z + \eta(Y)\eta(Z)(\widetilde{\nabla}_W \widetilde{R})(X,\xi)\xi \\
+ \;& \eta(X)(\widetilde{\nabla}_W \widetilde{R})(\xi,Y)Z + \eta(X)\eta(Z)(\widetilde{\nabla}_W \widetilde{R})(\xi,Y)Z.
\end{aligned}$$

Using (2.36) in (3.19) we get

$$(3.20)\ (\widetilde{\nabla}_W\widetilde{R})(\phi^2X,\phi^2Y)\phi^2Z$$
$$= (\widetilde{\nabla}_W\widetilde{R})(X,Y)Z - \eta(Z)R(X,Y)\phi W - \eta(Y)R(X,\phi W)Z + \eta(X)R(Y,\phi W)Z$$
$$+ \frac{1}{2}\big[\eta(Z)\{\Omega(Y,W)X - \Omega(X,W)Y\} + \eta(Y)\Omega(W,Z)X - \eta(X)\Omega(W,Z)Y\big]$$
$$- \eta(Z)\{\alpha(Y,\phi W)X - \alpha(X,\phi W)Y\} + \eta(Y)\alpha(Z,\phi W)X - \eta(X)\alpha(Z,\phi W)Y$$
$$- \eta(X)\{\alpha(Y,Z)W - \eta(W)\alpha(Y,Z)\xi\} + \eta(Y)\eta(Z)\{\alpha(X,W)\xi - \alpha(X,\phi W)\xi\}$$
$$- \eta(X)\eta(Z)\{\alpha(Y,W)\xi - \alpha(Y,\phi W)\xi\} - \frac{1}{2}\eta(X)\{g(Y,Z)W - \eta(W)g(Y,Z)\xi\}$$
$$+ \frac{1}{2}\eta(Y)\{g(X,Z)W - \eta(W)g(X,Z)\xi\}.$$

From (3.18) and (3.20) we get

$$(3.21)\ (\widetilde{\nabla}_{\phi^2W}\widetilde{R})(\phi^2X,\phi^2Y)\phi^2Z$$
$$= (\widetilde{\nabla}_W\widetilde{R})(X,Y)Z - \eta(Z)R(X,Y)\phi W - \eta(Y)R(X,\phi W)Z + \eta(X)R(Y,\phi W)Z$$
$$+ \frac{1}{2}\big[\eta(Z)\{\Omega(Y,W)X - \Omega(X,W)Y\} + \eta(Y)\Omega(W,Z)X - \eta(X)\Omega(W,Z)Y\big]$$
$$- \eta(Z)\{\alpha(Y,\phi W)X - \alpha(X,\phi W)Y\} + \eta(Y)\alpha(Z,\phi W)X - \eta(X)\alpha(Z,\phi W)Y$$
$$- \eta(X)\{\alpha(Y,Z)W - \eta(W)\alpha(Y,Z)\xi\} + \eta(Y)\eta(Z)\{\alpha(X,W)\xi - \alpha(X,\phi W)\xi\}$$
$$- \eta(X)\eta(Z)\{\alpha(Y,W)\xi - \alpha(Y,\phi W)\xi\} - \frac{1}{2}\eta(X)\{g(Y,Z)W - \eta(W)g(Y,Z)\xi\}$$
$$+ \frac{1}{2}\eta(Y)\{g(X,Z)W - \eta(W)g(X,Z)\xi\}.$$

From (3.14) and (3.21) we obtain

$$(3.22)\ (\widetilde{\nabla}_W\widetilde{R})(X,Y)Z$$
$$\begin{aligned}
= &\ \Big[g(R(X,Y)W,Z) - g(R(X,Y)\phi W,Z)\\
+ &\ \alpha(X,W)g(Y,Z) - \alpha(Y,W)g(X,Z)\\
+ &\ \alpha(Y,Z)g(X,W) - \alpha(X,Z)g(Y,W)\\
+ &\ \frac{1}{2}\{\eta(Y)g(X,W) - \eta(X)g(Y,W)\}\eta(Z)\\
- &\ \eta(Y)\eta(W)\alpha(X,Z) + \alpha(Y,\phi W)g(X,Z) - \alpha(X,\phi W)g(Y,Z)\\
+ &\ \Omega(Y,W)\alpha(X,Z) - \Omega(X,W)\alpha(Y,Z)\\
+ &\ \frac{1}{2}\{\eta(X)\Omega(Y,W) - \eta(Y)\Omega(X,W)\}\eta(Z)\\
+ &\ g(X,W)g(Y,Z) - g(Y,W)g(X,Z) + g(Y,W)\Omega(X,Z) - g(X,W)\Omega(Y,Z)\\
+ &\ \{\eta(Y)\Omega(X,Z) - \eta(X)\Omega(Y,Z)\}\eta(W)\\
+ &\ \Omega(Y,W)g(X,Z) - \Omega(X,W)g(Y,Z)
\end{aligned}$$

$$\begin{aligned}
&+ \quad \{\eta(X)\Omega(Y,W) - \eta(Y)\Omega(X,W)\}\eta(Z) \\
&+ \quad \Omega(X,W)\Omega(Y,Z) - \Omega(Y,W)\Omega(X,Z)\Big]\xi \\
&+ \quad \eta(Z)R(X,Y)\phi W + \eta(Y)R(X,\phi W)Z - \eta(X)R(Y,\phi W)Z \\
&- \quad \frac{1}{2}\big[\eta(Z)\{\Omega(Y,W)X - \Omega(X,W)Y\} \\
&+ \quad \eta(Y)\Omega(W,Z)X - \eta(X)\Omega(W,Z)Y\big] \\
&+ \quad \eta(Z)\{\alpha(Y,\phi W)X - \alpha(X,\phi W)Y\} \\
&- \quad \eta(Y)\alpha(Z,\phi W)X + \eta(X)\alpha(Z,\phi W)Y \\
&+ \quad \eta(X)\alpha(Y,Z)W + \frac{1}{2}\{\eta(X)g(Y,Z)W - \eta(Y)g(X,Z)W\}.
\end{aligned}$$

Thus in a locally ϕ-symmetric LP-Sasakian manifold with respect to a semi-symmetric metric connection, the relation (3.22) holds for any X, Y, Z, $W \in \chi(M)$.

Next, if the relation (3.22) holds in an LP-Sasakian manifold with respect to semi-symmetric metric connection then for any horizontal vector fields X, Y, Z, W, we obtain the relation (3.4) and hence the manifold is locally ϕ-symmetric with respect to semi-symmetric metric connection. Thus we can state the following:

Theorem 3.4. *An LP-Sasakian manifold (M^n, g) is locally ϕ-symmetric with respect to a semi-symmetric metric connection if and only if the relation* (3.22) *holds for any vector fields X, Y, Z, $W \in \chi(M)$.*

In view of (2.36), (3.22) yields

$$\begin{aligned}
(3.23) \qquad & (\nabla_W R)(X,Y)Z \\
= \quad & [\Omega(W,Y) - g(W,Y)]\eta(Z)X + [\Omega(W,Z) - g(W,Z)]\eta(Y)X \\
+ \quad & 2\eta(Z)\eta(W)[\eta(X)Y - \eta(Y)X] + [\alpha(Y,Z)\eta(X) - \alpha(X,Z)\eta(Y)]W \\
+ \quad & [g(W,X) - \Omega(W,X)]\eta(Z)Y + [g(W,Z) - \Omega(W,Z)]\eta(X)Y \\
+ \quad & g(Y,Z)\eta(X)[\frac{1}{2}W - \phi W] - g(X,Z)\eta(Y)[\frac{1}{2}W - \phi W] \\
+ \quad & \eta(Z)R(X,Y)\phi W + \eta(Y)R(X,\phi W)Z - \eta(X)R(Y,\phi W)Z \\
+ \quad & \frac{1}{2}\big[\eta(X)\Omega(W,Z)Y - \eta(Y)\Omega(W,Z)X \\
- \quad & \eta(Z)\{\Omega(Y,W)X - \Omega(X,W)Y\}\big] \\
+ \quad & \eta(Z)\{\alpha(Y,\phi W)X - \alpha(X,\phi W)Y\} \\
- \quad & \eta(Y)\alpha(Z,\phi W)X + \eta(X)\alpha(Z,\phi W)Y \\
+ \quad & \eta(X)\alpha(Y,Z)W + \frac{1}{2}\{\eta(X)g(Y,Z)W - \eta(Y)g(X,Z)W\} \\
+ \quad & \Big[2\{\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\}\eta(Y) - g(R(X,Y)\phi W,Z)
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{1}{2}\{\eta(Y)g(X,W) - \eta(X)g(Y,W)\}\eta(Z) \\
&- \quad \eta(Y)\eta(W)\alpha(X,Z) + \alpha(Y,\phi W)g(X,Z) - \alpha(X,\phi W)g(Y,Z) \\
&+ \quad \Omega(Y,W)\alpha(X,Z) - \Omega(X,W)\alpha(Y,Z) + \frac{3}{2}\{\eta(X)\Omega(Y,W) \\
&- \quad \eta(Y)\Omega(X,W)\}\eta(Z) + g(Y,W)\Omega(X,Z) - g(X,W)\Omega(Y,Z) \\
&+ \quad \{\eta(Y)\Omega(X,Z) - \eta(X)\Omega(Y,Z)\}\eta(W) + 2\{\Omega(Y,W)g(X,Z) \\
&- \quad \Omega(X,W)g(Y,Z)\} + \Omega(X,W)\Omega(Y,Z) - \Omega(Y,W)\Omega(X,Z)\Big]\xi.
\end{aligned}
$$

This leads to the following:

Theorem 3.5. *In a locally ϕ-symmetric LP-Sasakian manifold with respect to asemi-symmetric metric connection, the relation* (3.23) *holds for any vector fields X, Y, Z, $W \in \chi(M)$.*

From (3.8) and (3.23), we can state the following:

Theorem 3.6. *A locally ϕ-symmetric LP-Sasakian manifold is invariant under a semi-symmetric metric connection if and only if the relation*

$$
\begin{aligned}
&[\Omega(W,Y) - g(W,Y)]\eta(Z)X + [\Omega(W,Z) - g(W,Z)]\eta(Y)X \\
&+ 2\eta(Z)\eta(W)[\eta(X)Y - \eta(Y)X] + [\alpha(Y,Z)\eta(X) - \alpha(X,Z)\eta(Y)]W \\
&+ [g(W,X) - \Omega(W,X)]\eta(Z)Y + [g(W,Z) - \Omega(W,Z)]\eta(X)Y \\
&+ \frac{1}{2}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]W + \eta(Z)[g(Y,W)\phi X - g(X,W)\phi Y] \\
&- \frac{1}{2}\big[\eta(X)\Omega(W,Z)Y - \eta(Y)\Omega(W,Z)X - \eta(Z)\{\Omega(Y,W)X - \Omega(X,W)Y\}\big] \\
&+ \eta(Z)\{\alpha(Y,\phi W)X - \alpha(X,\phi W)Y\} - \eta(Y)\alpha(Z,\phi W)X + \eta(X)\alpha(Z,\phi W)Y \\
&+ \eta(X)\alpha(Y,Z)W + \frac{1}{2}\{\eta(X)g(Y,Z)W - \eta(Y)g(X,Z)W\} \\
&+ \Big[2\{\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\}\eta(Y) + \frac{1}{2}\{\eta(Y)g(X,W) - \eta(X)g(Y,W)\}\eta(Z) \\
&- \eta(Y)\eta(W)\alpha(X,Z) + \alpha(Y,\phi W)g(X,Z) - \alpha(X,\phi W)g(Y,Z) + \Omega(Y,W)\alpha(X,Z) \\
&- \Omega(X,W)\alpha(Y,Z) + \frac{3}{2}\{\eta(X)\Omega(Y,W) - \eta(Y)\Omega(X,W)\}\eta(Z) + g(Y,W)\Omega(X,Z) \\
&- g(X,W)\Omega(Y,Z) + \{\eta(Y)\Omega(X,Z) - \eta(X)\Omega(Y,Z)\}\eta(W) + \Omega(Y,W)g(X,Z) \\
&- \Omega(X,W)g(Y,Z) + \Omega(X,W)\Omega(Y,Z) - \Omega(Y,W)\Omega(X,Z)\Big]\xi = 0
\end{aligned}
$$

holds for arbitrary vector fields X, Y, Z, $W \in \chi(M)$.

4. Locally ϕ-Ricci symmetric LP-Sasakian manifolds admitting semi-symmetric connection

Definition 4.1. An LP-Sasakian manifold M is said to be locally ϕ-Ricci symmetric with respect to the semi-symmetric metric connection if its satisfies the condition

$$\phi^2((\widetilde{\nabla}_X \widetilde{Q})(Y)) = 0 \tag{4.1}$$

for horizontal vector fields X and Y, where $\widetilde{Q}$ is the Ricci-operator with respect to the semi-symmetric metric connection $\widetilde{\nabla}$, i.e. $g(\widetilde{Q}X, Y) = \widetilde{S}(X, Y)$ for all vector fields X, Y.

Let us take an LP-Sasakian manifold, which is ϕ-Ricci symmetric with respect to semi-symmetric metric connection $\widetilde{\nabla}$. Then by the virtue of (2.1) it follows from (4.1) that

$$(\widetilde{\nabla}_X \widetilde{Q})(Y) + \eta((\widetilde{\nabla}_X \widetilde{Q})(Y))\xi = 0$$

from which it follows that

$$(\widetilde{\nabla}_X \widetilde{S})(Y, Z) = 0 \tag{4.2}$$

for all horizontal vector fields X and Y and Z.

Let X, Y, Z be arbitrary vector fields of $\chi(M)$. We now compute

$$(\widetilde{\nabla}_{\phi^2 X} \widetilde{S})(\phi^2 Y, \phi^2 Z)$$

in two different ways. Since $\phi^2 X$, $\phi^2 Y$, $\phi^2 Z$ are horizontal vector fields for all X, Y, $Z \in \chi(M)$, from (4.2) we have

$$(\widetilde{\nabla}_{\phi^2 X} \widetilde{S})(\phi^2 Y, \phi^2 Z) = 0 \tag{4.3}$$

for all X, Y, $Z \in \chi(M)$. By the virtue of (2.1) we get

$$(\widetilde{\nabla}_{\phi^2 X} \widetilde{S})(\phi^2 Y, \phi^2 Z) = (\widetilde{\nabla}_X \widetilde{S})(\phi^2 Y, \phi^2 Z) + \eta(X)(\widetilde{\nabla}_\xi \widetilde{S})(\phi^2 Y, \phi^2 Z). \tag{4.4}$$

Now for any horizontal vector fields Y and Z we have from (4.2) that

$$(\widetilde{\nabla}_\xi \widetilde{S})(Y, Z) = 0,$$

which implies that

$$(\widetilde{\nabla}_\xi \widetilde{S})(\phi^2 Y, \phi^2 Z) = 0 \tag{4.5}$$

for arbitrary vector fields Y, $Z \in \chi(M)$.

Using (4.5) in (4.4) we get

$$(\widetilde{\nabla}_{\phi^2 X} \widetilde{S})(\phi^2 Y, \phi^2 Z) = (\widetilde{\nabla}_X \widetilde{S})(\phi^2 Y, \phi^2 Z). \tag{4.6}$$

In view of (2.1), we get

$$\begin{aligned} (\widetilde{\nabla}_X \widetilde{S})(\phi^2 Y, \phi^2 Z) &= (\widetilde{\nabla}_X \widetilde{S})(Y, Z) + \eta(Y)(\widetilde{\nabla}_X \widetilde{S})(Z, \xi) \\ &+ \eta(Z)(\widetilde{\nabla}_X \widetilde{S})(Z, \xi) + \eta(Y)\eta(Z)(\widetilde{\nabla}_X \widetilde{S})(\xi, \xi). \end{aligned} \tag{4.7}$$

Using (2.37) in (4.7) we get

$$(4.8)\ (\widetilde{\nabla}_X\widetilde{S})(\phi^2Y,\phi^2Z) = (\nabla_XS)(Y,Z)-\eta(Z)S(Y,\phi X) + \eta(Y)[S(X,Z)-S(Z,\phi X)]+\eta(Y)\alpha(X,Z) + [(2n-1)\eta(X)-da(X)]\eta(Y)\eta(Z) + (n-1)\eta(Z)\Omega(X,Y)-(n-3)\eta(Y)\Omega(X,Z) + (n-\frac{1}{2})\eta(Y)g(X,Z)-da(X)g(Y,Z).$$

By the virtue of (4.3) and (4.8) we obtain from (4.7) that

$$(4.9)\ (\nabla_XS)(Y,Z) = \eta(Z)S(Y,\phi X)-\eta(Y)[S(X,Z)-S(Z,\phi X)] - \eta(Y)\alpha(X,Z)-[(2n-1)\eta(X)-da(X)]\eta(Y)\eta(Z) - (n-1)\eta(Z)\Omega(X,Y)+(n-3)\eta(Y)\Omega(X,Z) - (n-\frac{1}{2})\eta(Y)g(X,Z)+da(X)g(Y,Z).$$

Thus in a locally ϕ-Ricci symmetric LP-Sasakian manifold with respect to a semi-symmetric metric connection, the relation (4.9) holds for any X, Y, $Z \in \chi(M)$.

Next if the relation (4.9) holds in an LP-Sasakian manifold with respect to a semi-symmetric metric connection then for any horizontal vector fields X, Y, Z with tr.α = constant, we obtain $(\nabla_XS)(Y,Z)=0$ and hence the manifold is locally ϕ-Ricci symmetric with respect to a semi-symmetric metric connection. Thus we can state the following:

Theorem 4.1. *An LP-Sasakian manifold (M^n,g) is locally ϕ-Ricci symmetric with respect to a semi-symmetric metric connection with tr.α = constant if and only if the relation* (4.9) *holds for any vector fields X, Y, $Z \in \chi(M)$.*

Putting $Y=\xi$ in (4.9) and using (2.38), we get

$$(4.10)\quad S(X,Z) = 2(n-2)\Omega(X,Z)-\alpha(X,Z) - (n-\frac{1}{2})g(X,Z)+(2n-1)\eta(X)\eta(Z)$$

for any vector fields X, $Z \in \chi(M)$.
This leads to the following:

Theorem 4.2. *In a locally ϕ-Ricci symmetric LP-Sasakian manifold with respect to a semi-symmetric metric connection, the Ricci tensor is of the form* (4.10).

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