ABSTRACT. Let (X_n) be a stationary sequence. We prove the following

(i) If the variables (X_n) are iid and $\mathbf{E}(|X_1|) < \infty$ then

$$\lim_{p\to 1^+}\left((p-1)\left(\sum_{n=1}^{\infty}\frac{|X_n(x)|^p}{n^p}\right)\right)^{1/p} = \mathbf{E}(|X_1|), a.e.$$

(ii) If $X_n(x) = f(T^n x)$ where (X, F, μ, T) is an ergodic dynamical system, then

$$\lim_{p\to 1^+}\left((p-1)\left(\sum_{n=1}^{\infty}\left(\frac{f(T^n x)}{n}\right)^p\right)\right)^{1/p} = \int f d\mu \qquad \text{a.e. for } f \geq 0,\ f \in L \log L.$$

Furthermore the maximal function,

$$\sup_{1<p<\infty}(p-1)^{1/p}\left(\sum_{n=1}^{\infty}\left(\frac{f(T^n x)}{n}\right)^p\right)^{1/p} \text{ is integrable for functions}, f \geq 0,\ f \in L \log L.$$

These limits are linked to the maximal function $N^*(x) = \|(\frac{X_n(x)}{n})\|_{1,\infty}$.