Abstract. Let G be a finite group and k a field of characteristic p dividing $|G|$. Then Greenlees has developed a spectral sequence whose E_2 term is the local cohomology of $H^*(G,k)$ with respect to the maximal ideal, and which converges to $H_*(G,k)$. Greenlees and Lyubeznik have used Grothendieck's dual localization to provide a localized form of this spectral sequence with respect to a homogeneous prime ideal $\mathfrak{p}$ in $H^*(G,k)$, and converging to the injective hull $I_\mathfrak{p}$ of $H^*(G,k)/\mathfrak{p}$.

The purpose of this paper is give a representation theoretic interpretation of these local cohomology spectral sequences. We construct a double complex based on Rickard's idempotent kG-modules, and agreeing with the Greenlees spectral sequence from the E_2 page onwards. We do the same for the Greenlees-Lyubeznik spectral sequence, except that we can only prove that the E_2 pages are isomorphic, not that the spectral sequences are. Ours converges to the Tate cohomology of the certain modules $\kappa_\mathfrak{p}$ introduced in a paper of Benson, Carlson and Rickard. This leads us to conjecture that $\hat{H}^*(G,\kappa_\mathfrak{p}) \cong I_\mathfrak{p}$, after a suitable shift in degree. We draw some consequences of this conjecture, including the statement that $\kappa_\mathfrak{p}$ is a pure injective module. We are able to prove the conjecture in some cases, including the case where $H^*(G,k)_\mathfrak{p}$ is Cohen–Macaulay.