

## ON ANTI-INVERSE RINGS

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Following B. Cerović [1], a ring  $R (\neq 0)$  is called an anti-inverse ring if every element  $x$  in  $R$  has an anti-inverse  $x^*$  :  $x^*xx^* = x$  and  $xx^*x = x$ . If  $R$  is an anti-inverse ring, then so is every non-zero homomorphic image of  $R$ , and  $x^2 = x^{*2} = (xx^*)^2 = (x^*x)^2$  and  $x = x^{*2}xx^{*2} = x^5$  for any  $x \in R$ ; in particular,  $R$  is a strongly regular ring.

The present objective is to prove neatly the following proposition which covers all the results in [1].

PROPOSITION. *The following are equivalent:*

- (1)  $R$  is an anti-inverse ring.
- (2)  $R$  is a subdirect sum of  $GF(2)$ 's and  $GF(3)$ 's
- (3)  $R$  satisfies the polynomial identity  $x^3 - x = 0$ .

*Proof.* Obviously, (2)  $\Rightarrow$  (3)  $\Rightarrow$  (1). It remains therefore to prove that (1) implies (2). Without loss of generality, we may assume that  $R$  is subdirectly irreducible. Then, we can easily see that the strongly regular ring  $R$  is a division ring. Now, let  $x$  be an arbitrary non-zero element of  $R$ . Then,  $x^2 = 1$  and  $0 = (xx^* - x^*x)^2 = 2(x^2 - x^4) = 2(x^2 - 1)$ . Hence, if  $R$  is not of characteristic 2 then  $x = \pm 1$ , and so  $R = GF(3)$ . On the other hand, if  $R$  is of characteristic 2 then  $0 = x^4 - 1 = (x - 1)^2$  implies  $x = 1$ , and so  $R = GF(2)$ .

## REFERENCES

- [1] B. Cerović, *Anti-inverse rings*, Publ. Inst. Math. (Beograd) (N.S.) **29 (43)** (1981), 45-48.

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