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Nonparametric Time Series Analysis of the Conditional Mean and Volatility Functions for the COP/USD Exchange Rate Returns

Análisis de series de tiempo no paramétrico de las funciones de media y varianza condicional de los retornos de la tasa de cambio COP/USD

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Abstract

The modeling and estimation of the conditional volatility associated with a stochastic process usually have been based on parametric ARCH-type and stochastic volatility models. These time series models are very powerful in representing the dynamic stochastic properties of the data generating process only if the parametric functions are correctly specified. The nonparametric approach acquires importance as a complementary and flexible method to explore these properties without imposing particular functional forms on the conditional moments of process. This paper presents an application of nonparametric time series methods to estimate the conditional volatility function of the COP/USD exchange rate returns. Additionally, we estimate the conditional mean function under this approach.

Key words: Nonparametric regression, Local polynomial regression, Nonlinear time series, Variance function estimation, Autoregressive conditional heteroscedasticity, Time series analysis.

Resumen

La modelación y estimación de la volatilidad condicional asociada a un proceso estocástico ha estado basada en los modelos paramétricos tipo ARCH y de volatilidad estocástica. Estos modelos son muy poderosos para representar las propiedades dinámicas estocásticas del proceso generador de

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datos solo si las funciones paramétricas están correctamente especificadas. En este sentido, el enfoque no paramétrico adquiere importancia como un método complementario y flexible para explorar dichas propiedades al no imponer formas funcionales particulares en los momentos condicionales del proceso. Este documento presenta una aplicación de los métodos no paramétricos de series de tiempo para estimar la función de volatilidad condicional de los retornos de la tasa de cambio COP/USD. Además, se estima la función de media condicional bajo este enfoque.

Palabras clave: regresión no paramétrica, regresión polinomial local, series de tiempo no lineales, estimación de la función de varianza, heterocedasticidad condicional autorregresiva, análisis de series de tiempo.

1. Introduction

In numerous publications researchers have written about the important role that associated volatility plays in a stochastic process, particularly in economics and finance. For example, the estimation of a conditional volatility measure that approximates its principal empirical features such as cluster volatility, asymmetries, leverage effects, and long memory, among others, is crucial for different issues in finance, like financial risk management, asset pricing, and efficient portfolio allocation. Subsequently, the development of models to adequately approximate the volatility process has concentrated the attention of researchers in the past two decades (Andersen, Bollerslev & Diebold 2009, Straumann 2005).

In this way, most volatility models have concentrated the attention on the parametric approach assuming an explicit functional form to the volatility process. This being said since Engle's (1982) Autoregressive Conditional Heteroscedasticity -ARCH- specification, where he explicitly expresses conditional volatility as a linear function of past squared innovations of the process, there has been an exponential growth of different parametric specifications. A short list of these specifications: Bollerslev's (1986) Generalized ARCH -GARCH- model, Engle & Bollerslev's (1986) Integrated GARCH -IGARCH- model, Nelson's (1991) Exponential GARCH -EGARCH- model, Ding et al. (1993) Asymmetric Power ARCH -APARCH- model, Baillie et al. (1996) Fractionally Integrated GARCH -FIGARCH- model, and Davidson's (2004) Hyperbolic GARCH -HYGARCH- model, among others. For a complete review of ARCH-type models, see Bollerslev et al. (1992), Bollerslev et al. (1994), and Andersen, Davis, Kreiß & Mikosch (2009). The estimation of ARCH-type models is commonly done by maximum likelihood under different distribution functions such as the usual Gaussian distribution, the Student-t distribution, the Generalized Error distribution (GED), and the skewed-Student distribution.

Jointly with ARCH-type models are also the Stochastic Volatility -SV- models. This class of parametric models presents, unlike ARCH models, an alternative approach to the specification of the volatility function where the standard specification contains an unobserved variance component (latent state variable) which is modeled directly as a linear stochastic process, such as an autoregression (Harvey et al. 1994). See Ghysels et al. (1996), Shephard (2005), and Andersen, Davis, Kreiß & Mikosch (2009) for a complete overview about SV models. The estimation of SV models covers a wide range of estimation procedures, for instance quasi-maximum likelihood, applying the Kalman filter, Bayesian estimation, generalized method of moments, and efficient method of moments.

However, as it is well-known, the parametric time series models are very powerful in representing the stochastic dynamical properties of the data generating process if the parametric functions are correctly specified (Hardle & Linton 1994, Fan & Yao 2005), and searching for a parametric functional form is critical and not always is a simple task, especially when the process has nonlinear characteristics, as is the case of financial time series variables. Thus the nonparametric approach gains importance as a way of searching more flexible models without imposing particular functional forms of the conditional moments such as a mean, variance, or density function of process. The nonparametric estimates may be used as an end product or, perhaps more importantly, as a guide to identifying a parametric model to be used in a subsequent stage or to validate an existing one (Masry & Tjostheim 1995). Additionally, the estimation of nonparametric regression functions is not always complicated; on the contrary, it usually takes much less time estimation with respect to some more complicated parametric models where convergence problems are commonly found in their estimation algorithms.

Although the use of nonparametric methods in time series analysis has a long tradition, it has obtained popularity with modern nonparametric techniques, particularly in the analysis of nonlinear time series, due to the existence of large data sets and computational advances (Hardle et al. 1997). Some references about the development of nonparametric time series theory and its applications are: books by Hardle (1990), Fan & Gijbels (1996), and Fan & Yao (2005), and the articles by Robinson (1983), Hardle et al. (1997), Tjostheim (1999), and references therein. This approach is applied to a vast range of areas in economics, and has come to obtain great popularity in financial econometrics, for example, in modeling the drift and diffusion process underlying asset returns, among other issues in empirical finance. See, for example, Pagan & Ullah (1988), Diabold & Nason (1990), Mizrach (1992), Bossaerts et al. (1995), Bossaerts et al. (1996), Ait-Sahalia (1996*a*), Ait-Sahalia (1996*b*), Ait-Sahalia & Lo (1998), and Ait-Sahalia & Lo (2000). Particular studies using the nonparametric approach to estimate the conditional volatility function are Engle & Gonzalez-Rivera (1991), Bossaerts et al. (1995), Bossaerts et al. (1996), Masry & Tjostheim (1995), Hardle & Tsybakov (1997), Fan & Yao (1998), and Ziegelmann (2002).

In this paper, we apply nonparametric time series methods to estimate the conditional mean and volatility functions for the Colombian Peso/US Dollar - COP/USD- exchange rate returns. The fundamental reason for studying this variable is that the Colombian exchange rate has had significant variation episodes generating great uncertainty, and with large and severe costs on various economic sectors. Additionally, international asset pricing theories and international portfolio management depend on the expected foreign exchange rate movements (Bollerslev et al. 1992); therefore this paper can be a contribution to properly understand the foreign exchange rate dynamics using the advantages of the nonparametric time series methods. The reason being that in Colombia, almost all analyses about foreign exchange rate has been concentrated on parametric models. The only nonparametric study for the COP/USD exchange rate is by Julio et al. (2005). For international nonlinear analysis on exchange rates using nonparametric procedures, see the studies by Meese & Rose (1990), Diabold & Nason (1990), LeBaron (1990), Bossaerts et al. (1995), Bossaerts et al. (1996), and Hardle & Tsybakov (1997).

The paper is organized as follows: Section 2 makes a short description of the nonparametric time series model and its different estimation methods. Section 3 applies the nonparametric model to estimate both the conditional mean and volatility functions for the returns of the COP/USD exchange rate process. Finally, Section 4 concludes.

2. The Nonparametric Time Series Model

The starting point of the data generating process of a strictly stationary discrete-time stochastic process $\{X_t\}$ defined on some probability space $(\Omega, \mathcal{F}, P)$ is the general univariate nonlinear stochastic regression model given by

$$X_t = m(X_{t-1}, \dots, X_{t-p}) + \sigma(X_{t-1}, \dots, X_{t-p})\varepsilon_t, \quad t = 1, \dots, T \tag{1}$$

where $m(x_{t-1}, \dots, x_{t-p}) = \mathbb{E}(X_t \mid X_{t-1} = x_1, \dots, X_{t-p} = x_p)$ is the nonlinear autoregressive conditional mean (smooth) function, $\sigma^2(x_{t-1}, \dots, x_{t-p}) = \mathrm{Var}(X_t \mid X_{t-1} = x_1, \dots, X_{t-p} = x_p)$ represents the nonlinear autoregressive conditional variance (smooth) function, and $\{\varepsilon_t\}$ is an independent and identically distributed (i.i.d.) sequence of random variables with $\mathbb{E}(\varepsilon_t \mid X_{t-1}, \dots, X_{t-p}) = 0$, $\mathrm{Var}(\varepsilon_t \mid X_{t-1}, \dots, X_{t-p}) = 1$, and independent of $\{X_{t-1}, X_{t-2}, \dots\}$.

The model (1) is known as the Conditional Heteroscedastic Autoregressive Nonlinear -CHARN- model; see Bossaerts et al. (1996), or the Nonparametric Autoregressive Conditional Heteroscedastic -NARCH- model; see Fan & Yao (2005).

This model is the most flexible nonparametric time series model because it does not impose any (parametric) particular form on the conditional mean and volatility functions. However, due to the well-known "curse of dimensionality" problem, the estimation of equation (1) is complicated.[1] As a consequence, it is necessary to assume a certain level of structure on the conditional functions $m(\cdot)$ and $\sigma(\cdot)$.[2]

[1] Nonparametric regression estimators are very flexible, but their statistical accuracy decreases greatly if there are several explicatory variables in the model (Hardle et al. 2004). Additionally, their estimation is difficult unless the sample size is excessively large (Fan & Yao 2005), and (Fan & Gijbels 1996).

[2] A very popular nonparametric model is the Functional-Coefficient Autoregressive -FAR- model (Chen & Tsay 1993), where the conditional mean and volatility functions are specified as

$$m(X_{t-1}, \dots, X_{t-p}) = a_1(X_{t-d})X_1 + \cdots + a_p(X_{t-d})X_{t-p}$$
$$\sigma^2(X_{t-1}, \dots, X_{t-p}) = b_1(X_{t-d})X_1^2 + \cdots + b_p(X_{t-d})X_{t-p}^2$$

The usual assumption is: suppose $p = 1$ such that the model (1) becomes

$$X_t = m(X_{t-1}) + \sigma(X_{t-1})\varepsilon_t \tag{2}$$

Following Hardle & Tsybakov (1997), and Fan & Yao (1998) if $\{X_t\}$ is a stationary process, the conditional variance function can be decomposed as

$$\begin{aligned}\sigma^2(x) &= \mathbb{E}(X_t^2 \mid X_{t-1} = x) - \{\mathbb{E}(X_t \mid X_{t-1} = x)\}^2 \\ &= g(x) - \{m(x)\}^2\end{aligned} \tag{3}$$

such that the conditional variance estimate is based on the nonparametric estimation of $g(x)$ and $m(x)$ given by $\widehat{\sigma}_T^2(x) = \widehat{g}_T(x) - \{\widehat{m}_T(x)\}^2$.

2.1. Nonparametric Kernel Estimation

A way to obtain estimates of functions $m(x)$ and $g(x)$ is by applying the popular Nadaraya-Watson estimator given by:

$$\begin{aligned}\widehat{m}_T(X_{t-1}) &= \frac{\sum_{t=2}^{T} K\left((X_{t-1} - x)/h_T\right) X_t}{\sum_{t=2}^{T} K\left((X_{t-1} - x)/h_T\right)} \\ \widehat{g}_T(X_{t-1}) &= \frac{\sum_{t=2}^{T} K\left((X_{t-1} - x)/h_T\right) X_t^2}{\sum_{t=2}^{T} K\left((X_{t-1} - x)/h_T\right)}\end{aligned} \tag{4}$$

where $K(\cdot) : \mathbb{R} \to \mathbb{R}$ is the (continuous, bordered, symmetric, and integrating to one) Kernel function and $h_T > 0$ is the bandwidth parameter (also smoothing parameter), $h_T \to 0$ as $T \to \infty$. The Nadaraya−Watson estimator is a special case of the local polynomial estimation explained below. The Kernel functions most commonly used are the Gaussian, Quartic, and Epanechnikov Kernels.

These estimators are strongly consistent and asymptotically normal for α-mixing observations;[3] see Robinson (1983), and Masry & Tjostheim (1995).

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with $a_i(\cdot)$ and $b_i(\cdot)$, $i = 1, \ldots, p$ one-dimensional unknown functions, and X_{t-d} is the model-dependent variable. Another common model is the Additive Autoregressive -*AAR*- model (Jones 1978) which assumes an additive structure for conditional mean and variance,

$$\begin{aligned}m(X_{t-1}, \ldots, X_{t-p}) &= m_1(X_{t-1}) + \cdots + m_p(X_{t-p}) \\ \sigma^2(X_{t-1}, \ldots, X_{t-p}) &= \sigma_1(X_{t-1}^2) + \cdots + \sigma_p(X_{t-p}^2)\end{aligned}$$

where $m_i(\cdot)$ and $\sigma_i(\cdot)$, $i = 1, \ldots, p$ are univariate unknown functions. For other nonparametric models such as Partially Linear models and Single-Index Models, see Hardle & Tsybakov (1997), Hardle et al. (2004), Fan & Yao (2005), and Gao (2007).

[3] A sequence is said to be α-mixing if $\alpha(n) \to 0$ when $n \to \infty$, with $\alpha(n)$ defined as

$$\alpha(n) = \sup_{A \in \mathcal{F}_{-\infty}^{k}, B \in \mathcal{F}_{k+n}^{\infty}} \mid P(A \cap B) - P(A)P(B) \mid, \quad n = 1, 2, \ldots,$$

where $\mathcal{F}_i^j$ is the σ-field generated by $X_i, \ldots, X_j$. See Robinson (1983), and Fan & Yao (2005).

2.2. Local Polynomial Regression

Another nonparametric technique used to estimate the functions $m(x)$ and $g(x)$ is proposed by Hardle & Tsybakov (1997), who applied the local polynomial regression method. The estimates for $m(x)$ and $g(x)$ functions are derived through the solution of the following weighted least-squares problems:

$$\begin{aligned} c_T(x) &= \arg\min_{c\in\mathbb{R}^l} \sum_{t=1}^{T} (X_t - c' U_{tT})^2 K\left((X_{t-1}-x)/h_T\right) \\ \overline{c}_T(x) &= \arg\min_{\overline{c}\in\mathbb{R}^l} \sum_{t=1}^{T} (X_t^2 - \overline{c}' U_{tT})^2 K\left((X_{t-1}-x)/h_T\right) \end{aligned} \tag{5}$$

where $K(\cdot)$ and $h_T > 0$ are again the Kernel function and bandwidth parameter, respectively, and $U_{tT} = F(u_{tT})$ with $F(u) = (1, u, \dots, u^{l-1}/(l-1)!)'$ and $u_{tT} = (X_{t-1}-x)/h_T$ (the symbol $'$ denotes the transpose of a row vector). Note that when $l = 1$, the local polynomial fit is reduce to the Nadaraya-Watson estimator.

The estimators of $m(x)$ and $g(x)$ are given by $\widehat{m}_T(x) = \widehat{c}_T(x)' F(0)$ and $\widehat{g}_T(x) = \widehat{\overline{c}}_T(x)' F(0)$, respectively; such that the estimator of the conditional variance functions is defined as

$$\widehat{\sigma}_T^2(x) = \widehat{\overline{c}}_T(x)' F(0) - \{\widehat{c}_T(x)' F(0)\}^2 \tag{6}$$

Hardle & Tsybakov (1997) establish the asymptotic normality of local polynomial estimators for conditional mean and variance.

In the application of the local polynomial nonparametric regression method to estimate the volatility function to DM/USD and YEN/USD foreign exchange rates Hardle & Tsybakov (1997) use a local linear approximation ($l = 2$), such that

$$\begin{aligned} c_T(x) &= \arg\min_{c\in\mathbb{R}^2} \sum_{t=1}^{T} \left(X_t - c_1 - c_2(X_{t-1}-x)\right)^2 K\left((X_{t-1}-x)/h_T\right) \\ \overline{c}_T(x) &= \arg\min_{\overline{c}\in\mathbb{R}^2} \sum_{t=1}^{T} \left(X_t^2 - \overline{c}_1 - \overline{c}_2(X_{t-1}-x)\right)^2 K\left((X_{t-1}-x)/h_T\right) \end{aligned} \tag{7}$$

Note that in the minimization problems in (5), employed to obtain local approximation estimates of $m(x)$ and $g(x)$, the Kernel function and bandwidth parameter are commons in both equations. This strategy is used by Hardle & Tsybakov (1997) to avoid nonnegative estimators of $\sigma^2(x)$ and to reduce bias. Therefore, Fan & Yao (1998) propose a residual-based estimator to conditional variance based on local linear regression.

From (2) we have that $r_t^2 = \{X_t - m(X_{t-1})\}^2 = \sigma^2(X_{t-1})\varepsilon_t^2$, such that its conditional expectation is $\mathbb{E}(r_t^2 \mid X_{t-1} = x) = \sigma^2(x)$. This therefore shows that it is natural to estimate $\sigma^2(x)$ using the estimated residuals. Consequently the estimates of $m(x)$ and $\sigma^2(x)$ are derived from the solutions of the following minimization problems:

$$
\begin{aligned}
\widehat{a}_T(x) &= \arg\min_{a\in\mathbb{R}^2} \sum_{t=1}^{T} \{X_t - a_1 - a_2(X_{t-1}-x)\}^2 K\left((X_{t-1}-x)/h_{1T}\right) \\
\widehat{b}_T(x) &= \arg\min_{b\in\mathbb{R}^2} \sum_{t=1}^{T} \{\widehat{r}_t^2 - b_1 - b_2(X_{t-1}-x)\}^2 W\left((X_{t-1}-x)/h_{2T}\right)
\end{aligned}
\tag{8}
$$

where $K(\cdot)$ and $W(\cdot)$ are the Kernel functions, $h_{1T} > 0$ and $h_{2T} > 0$ the bandwidth parameters, and $\widehat{r}_t^2 = \{X_t - \widehat{m}_T(x)\}^2$ the estimated residuals.

The estimate of $m(x)$ is given by $\widehat{m}_T(x) = \widehat{a}_T(x)'e = \widehat{a}_1$ where $e = (1,0)'$ such that the residuals are $\widehat{r}_t^2 = \{X_{t-1} - \widehat{a}_1\}^2$ which are used in the above second minimization problem to obtain the estimator of $\sigma^2(x)$ given by

$$
\widehat{\sigma}_T^2(x) = \widehat{b}_T(x)'e = \widehat{b}_1 \tag{9}
$$

Fan & Yao (1998) demonstrate the asymptotic normality and efficiency of $\widehat{\sigma}_T^2(x)$, and apply their method to estimate the conditional mean and volatility functions to yields of the three-month Treasury Bill.

The reason for using the local polynomial regression, especially the local linear estimator applied in the Hardle & Tsybakov (1997) and Fan & Yao (1998), is because the local polynomial estimator has diverse statistical properties. Among these are: Agreeable nice asymptotic properties such as asymptotic minimax efficiency (Fan 1993), good finite sampling and design-adaptation properties, and it overcomes the drawbacks of the Nadaraya-Watson estimator and other nonparametric estimators such as large biases due to boundary effects. See Fan (1993), Fan & Gijbels (1996), and Fan & Yao (2005) for a complete derivation and description of statistical properties of the local polynomial estimator.

Note that the implementation of the above estimators depends on the appropriate selection of both bandwidth parameter and Kernel function. For example, for local linear estimator, Hardle & Tsybakov (1997) applied the cross-validation method to choose the bandwidth parameter using the Quartic Kernel, and Fan & Yao (1998) applied the data-driven bandwidth selection method using the Epanechnikov Kernel. See Fan & Yao (2005) for a complete description of bandwidth parameter selection methods for dependent processes.

3. Empirical Application for the COP/USD Exchange Rate Returns

In this section we apply the previously described different nonparametric estimators to estimate both the conditional mean and variance functions for the COP/USD exchange rate returns, S_t, defined as $X_t = \log(S_t/S_{t-1})$. The data consists of 3515 commercial daily observations (from 2 January 1995-20 June

2008). There are two reasons for choosing this period. Firstly, the applied work on COP/USD exchange rate usually studies this series without the exchange rate band regime, and due to the nonparametric time series methods are very effective in modeling potential nonlinearities (for example, due to interventions), we consider that is important to use the whole period. Secondly, the estimation and bandwidth selection methods are difficult unless the sample size is large (Fan & Yao 2005, Fan & Gijbels 1996). The statistical source of the database is the Central Bank of Colombia.

The graphs of the COP/USD exchange rate, its returns and respectively squared returns are presented in Figure 1. As we can see, the COP/USD exchange rate returns present the well-known cluster volatility phenomena. This is related to the excess kurtosis as is ilustrated by the Kernel density estimation for the COP/USD exchange rate returns in Figure 1.

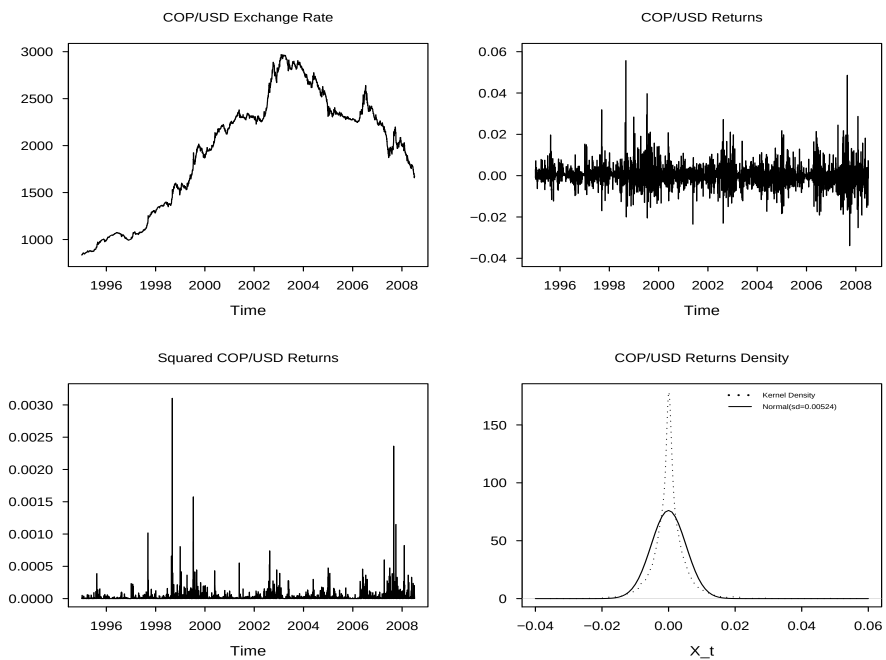

Figure 1: COP/USD exchange rate, COP/USD exchange rate returns, squared COP/USD exchange rate returns, and Kernel density estimation for COP/USD exchange rate returns compared with a mean zero normal density with standard deviation, $\widehat{\sigma} = 0.00524$.

To test for the existence of nonlinearity in the COP/USD returns and its squares, we applied the popular BDS test (see Brock et al. 1996), which can be considered a misspecification test in time series analysis. This test has power against a wide range of linear and nonlinear alternatives. The results displayed in Table 1 show that the null hypothesis of i.i.d. is rejected for most combinations of m and ϵ for both variables. Since there appears to be no discernible linear structure in the returns and its squares, the results suggest that there may be a nonlinear structure.

TABLE 1: BDS test statistics for nonlinearity of X_t and X_t^2.

	X_t				X_t^2			
$m\backslash\epsilon$	0.0026	0.0052	0.0079	0.0105	0e+00	1e−04	1e−04	2e−04
2	16.508	17.078	15.279	13.424	11.163	9.718	7.513	6.103
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
3	22.301	21.550	19.088	17.442	14.018	12.615	11.318	11.644
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
4	28.976	25.196	21.426	19.114	15.720	13.521	12.439	12.864
	(0)	(0) (0)	(0)	(0)	(0)	(0)	(0)	
5	38.839	28.877	23.283	20.095	16.937	13.770	12.835	13.305
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)

m: embedding dimension, ϵ: close point.
p-values in parenthesis.

The Kernel function used in all estimations for both the conditional mean and volatility functions was the Epanechnikov Kernel, given by $K(u) = \frac{3}{4}(1 - u^2)I(|u| \leq 1)$. In addition, we developed all estimations by employing other kernel functions such as the Quartic Kernel, $K(u) = \frac{15}{18}(1-u^2)^2 I(|u| \leq 1)$, and the Gaussian Kernel, $K(u) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}u^2)$; and by not necessarily using the same kernel function for the conditional mean and volatility, we obtained very similar results. For optimal bandwidth parameter selection, we always choose it by applying the following bandwidth selection methods: cross-validation, rule of thumb, the pre-asymptotic substitution method by Fan & Gijbels (1995), and the plug-in approach. After the empirical comparison of the estimations obtained by means of the different bandwidth selection methods, the optimal bandwidth parameters for the conditional mean were 0.0134 and 0.0140 to Nadaraya-Watson and local linear polynomial estimators, respectively, and for the conditional variance were 0.0134, 0.0127, and 0.0149 to the Nadaraya-Watson, Hardle & Tsybakov (1997), and Fan & Yao (1998) estimators, respectively. All estimations and computations were carried out using the Xplore software version 4.8 and the `locpol` R package developed by Cabrera (2008). We also applied a robust local polynomial regression proposed by Cleveland (1979) to guard against deviant points (outliers) which may have had a distorting effect on the smoothing. As the results were very similar, they have not been illustrated to save space.[4]

Figure 2 shows the scatterplots of X_t against X_{t-1}, and the conditional mean function estimated by (a) Nadaraya-Watson (local constant estimator), and (b) local linear polynomial estimators denoted by $\widehat{m}_T(X_{t-1})$. The dashed lines correspond to pointwise 95% asymptotic confidence intervals.[5] As we can see, the shape of both estimate curves is almost equal, except for on the left edge where there is a boundary effect in the local constant fit thus generating bias problems in the lineal direction on the edges.

[4] The plots are available upon request.
[5] See Fan & Yao (2005) for a complete construction of the confidence intervals for dependent data.

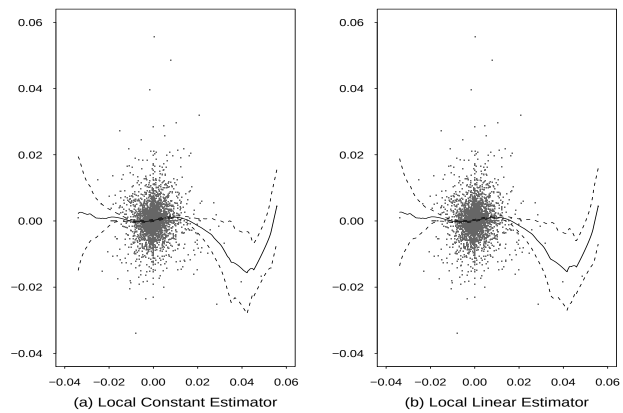

FIGURE 2: (a) X_t against X_{t-1}, and local constant polynomial fit $\widehat{m}_T(X_{t-1})$. (b) X_t against X_{t-1} and local linear polynomial fit $\widehat{m}_T(X_{t-1})$.

The most important segment in the graphs corresponds at the center: between -0.02 and 0.02, because there in that interval are most of the observations, and therefore more efficiency in the estimation of the conditional mean function. As we can see in that segment, the slope is almost zero which that possibly means that an efficient exchange rate market exists. The results found in the literature on exchange rate markets are diverse. Some show evidence of low negative slopes (mean reversion) (Hardle & Tsybakov 1997) and others low positive slopes (Julio et al. 2005).

The graphs of the estimated residuals, $\widehat{r}_t = \{X_t - \widehat{m}_T(x)\}$, where $\widehat{m}_T(x)$ is the estimated conditional mean function obtained from the local linear polynomial estimation, and its squares, $\widehat{r}_t^2$, are illustrated in Figure 3. The latter is employed in the estimation for the conditional variance function using the local linear estimator of Fan & Yao (1998). Additionally the scatterplot between $\widehat{r}_t$ against X_{t-1}, including the estimated conditional mean curve, and the Kernel density estimation for residuals compared with a mean zero normal density with standard deviation, $\widehat{\sigma} = 0.00524$ are shown in Figure 3.

Figures 4(a) and 4(b) depict the scatterplots of the squared returns, X_t^2, against X_{t-1}, and the estimated regression curve of the conditional variance, denoted by $\widehat{\sigma}_T^2(X_{t-1})$, estimated by the Nadaraya-Watson and the local linear estimator by Hardle & Tsybakov (1997). Figure 4(c) shows the scatterplot of the squared residuals, $\widehat{r}_t$, against X_{t-1}, and the local linear estimator of Fan & Yao (1998).

Figure 5 shows the volatility functions, $\widehat{\sigma}_T(X_{t-1})$, estimated by (a) the Nadaraya-Watson, (b) the Hardle & Tsybakov (1997), and (c) the Fan & Yao (1998) estimators. The results show that the conditional volatility function estimated by means of the Fan and Yao residual-based local linear estimator is smoother than the local constant and linear estimators using the squared returns.

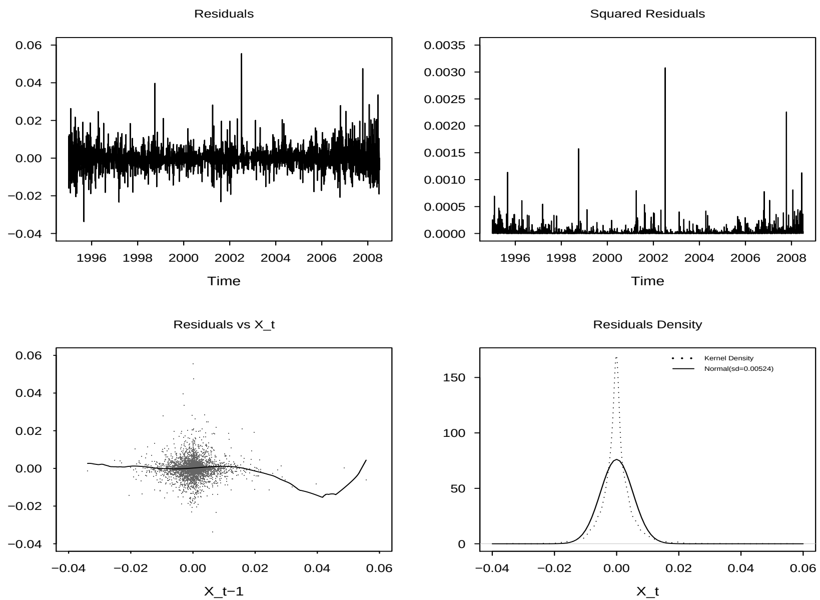

FIGURE 3: Residuals, squared residuals, residuals against X_{t-1}, and Kernel density estimation for residuals compared to a mean zero normal density with standard deviation, $\widehat{\sigma} = 0.00524$.

Furthermore, the results do not to show evidence of volatility asymmetries. On the contrary, the well-known U-shape in the conditional volatility function (except to the Fan and Yao's estimator) is present. This result concurs with findings in other studies that employ the parametric approach (Castaño et al. 2008, Maya & Gómez 2008). This indicates, for example, that it is proper to carry out an option evaluation on the COP/USD exchange rate with the symmetric volatility "smile". However, this symmetric U-shape is particularly clear in the central area of the graphs where the majority of observations are, this is, in the segment between -0.02 and 0.02 (see Figure 3). As expected, this symmetry in volatility is broken due to boundary effects on the right and left edges where there are few observations for correct smoothing (Hardle 1990, Hardle & Tsybakov 1997, Fan & Gijbels 1996, Hardle et al. 2004).

Finally, the results are along the lines of the findings by Julio et al. (2005), who applied the local lineal polynomial regression.[6] The goal of their study was to determine the features of the "volatility U-shape" and mean response functions, and the market effect of central bank interventions on those functions. They found that "discretional interventions" tend to change the concavity of the "volatility U-shape". However, that change was moderated and it never produced "volatility skew".

[6] In their study they used a different sample, from September 27th 1999-March 31st 2005. Additionally, they fit a local linear approximation to conditional mean and local quadratic approximation to conditional variance.

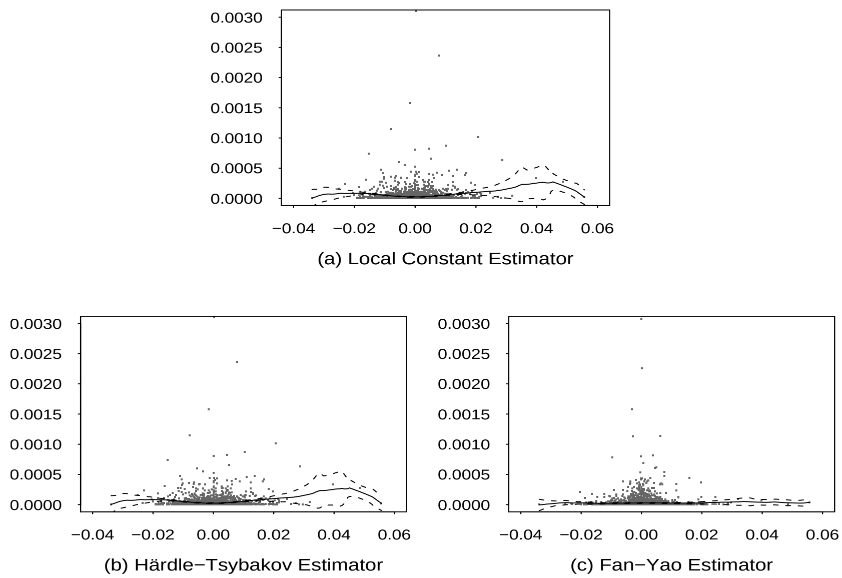

FIGURE 4: (a) X_t^2 against X_{t-1}, and local constant polynomial fit $\widehat{\sigma}_T^2(X_{t-1})$. (b) X_t^2 against X_{t-1}, and local linear polynomial fit $\widehat{\sigma}_T^2(X_{t-1})$. (c) $\widehat{r}_t^2$ against X_{t-1}, and local linear polynomial fit $\widehat{\sigma}_T^2(X_{t-1})$.

4. Conclusions

The role of volatility associated with a stochastic process is well-known, particularly in economics and finance. However, most of the volatility models have focused their attention on the parametric approach to represent the stochastic dynamic properties of the data generating process, assuming explicit functional forms for the mean and variance processes. In this paper a nonparametric time series analysis to the conditional mean and variance functions was carried out on the Colombian Peso/US Dollar -COP/USD- exchange rate returns.

Two nonparametric estimators were applied to estimate the conditional mean function: the local constant polynomial (Nadaraya-Watson) and local linear polynomial estimators; whereas three were used for the conditional variance function: the local constant and linear estimators based on the squared returns and the residual-based local linear estimator. The results show no evidence of asymmetries in the volatility of COP/USD exchange rate. On the contrary, we found the "volatility U-shape". Additionally, the results indicate that there is mean reversion according to the existence of a lineal function relationship to conditional mean.

Finally, as Bossaerts et al. (1995) point out, the nonparametric analysis can be extended considering a less restricted data generating process on the conditional mean function as well as on the conditional variance function including more lags

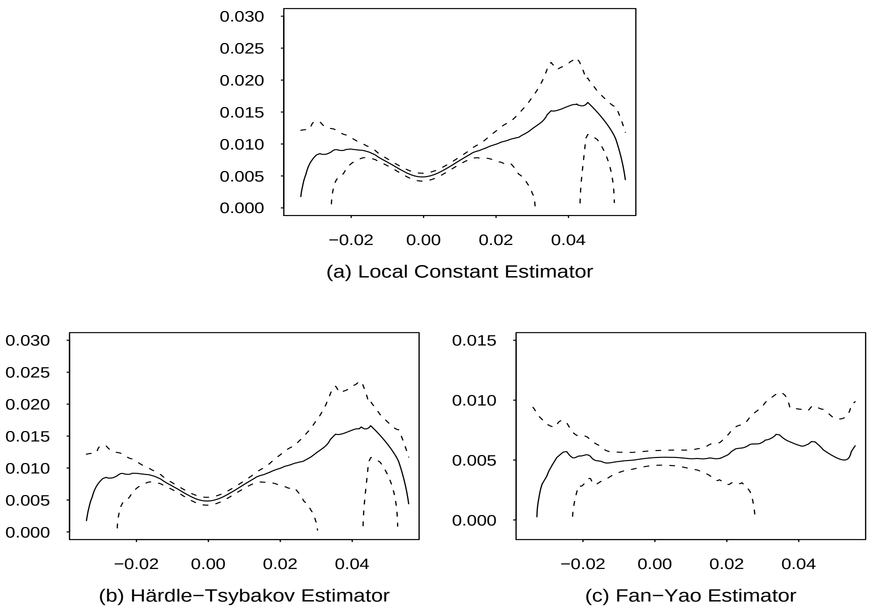

FIGURE 5: (a) $\widehat{\sigma}_T(X_{t-1})$ against X_{t-1} (Nadaraya-Watson). (b) $\widehat{\sigma}_T(X_{t-1})$ against X_{t-1} (Hardle-Tsybakov). (c) $\widehat{\sigma}_T(X_{t-1})$ against X_{t-1} (Fan-Yao).

in both functions. However this implies the well-known "curse of dimensionality" problem. Moreover, other nonparametric models can be attempted; for instance the Functional-Coefficient Autoregressive model, the Additive Autoregressive model, and among others. At this moment this extension is been performed jointly with a multivariate analysis to modeling portfolios of exchange rates and forecast future returns over short horizons.

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