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Estimation of Reliability in Multicomponent Stress-strength Based on Generalized Exponential Distribution

Estimación de confiabilidad en la resistencia al estrés de multicomponentes basado en la distribución exponencial generalizada

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Abstract

A multicomponent system of k components having strengths following k-independently and identically distributed random variables $X_1, X_2, \ldots, X_k$ and each component experiencing a random stress Y is considered. The system is regarded as alive only if at least s out of k $(s < k)$ strengths exceed the stress. The reliability of such a system is obtained when strength and stress variates are given by generalized exponential distribution with different shape parameters. The reliability is estimated using ML method of estimation in samples drawn from strength and stress distributions. The reliability estimators are compared asymptotically. The small sample comparison of the reliability estimates is made through Monte Carlo simulation. Using real data sets we illustrate the procedure.

***Key words*:** Asymptotic confidence interval, Maximum likelihood estimation, Reliability, Stress-strength model.

Resumen

Se considera un sistema de k multicomponentes que tiene resistencias que se distribuyen como k variables aleatorias independientes e idénticamente distribuidas $X_1, X_2, \ldots, X_k$ y cada componente experimenta un estrés aleatorio Y. El sistema se considera como vivo si y solo si por lo menos s de $k(s < k)$ resistencias exceden el estrés. La confiabilidad de este sistema se obtiene cuando las resistencias y el estrés se distribuyen como una distribución exponencial generalizada con diferentes parámetros de forma. La confiabilidad es estimada usando el método ML de estimación en muestras extraídas tanto para distribuciones de resistencia como de estrés. Los estimadores de confiabilidad son comparados asintóticamente. La comparación

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para muestras pequeñas de los estimadores de confiabilidad se hace a través de simulaciones Monte Carlo. El procedimiento también se ilustra mediante una aplicación con datos reales.

***Palabras clave*:** confiabilidad, estimación máximo verosímil, intervalos de confianza asintóticos, modelo de resistencia-estrés.

1. Introduction

The two-parameter generalized exponential distribution (GE) has been introduced and studied quite extensively by Gupta & Kundu (1999, 2001, 2002). The two-parameter GE distribution is an alternative to the well known two-parameter gamma, two-parameter Weibull or two parameter log-normal distributions. The two-parameter GE distribution has the following density function and the distribution function, respectively

$$f(x;\alpha,\lambda)=\alpha\lambda e^{-x\lambda}(1-e^{-x\lambda})^{\alpha-1};\quad \text{for } x>0 \tag{1}$$

$$F(x;\alpha,\lambda)=(1-e^{-x\lambda})^{\alpha-1};\quad \text{for } x>0 \tag{2}$$

Here α and λ are the shape and scale parameters, respectively. Now onwards GE distribution with the shape parameter α and scale parameter λ will be denoted by GE(α, λ).

The purpose of this paper is to study the reliability in a multicomponent stress-strength based on X, Y being two independent random variables, where $X\sim$ GE(α, λ) and $Y\sim$ GE(β, λ).

Let the random samples $Y, X_1, X_2, \dots, X_k$ being independent, $G(y)$ be the continuous distribution function of Y and $F(x)$ be the common continuous distribution function of $X_1, X_2, \dots, X_k$. The reliability in a multicomponent stress-strength model developed by Bhattacharyya & Johnson (1974) is given by

$$\begin{aligned}R_{s,k}&=P[\text{at least } s \text{ of the } X_1,X_2,\dots,X_k \quad \text{exceed} \quad Y]\\&=\sum_{i=s}^{k}\binom{k}{i}\int_{-\infty}^{\infty}[1-F(y)]^i\;[F(y)]^{(k-i)}\;dG(y)\end{aligned} \tag{3}$$

Where $X_1, X_2, \dots, X_k$ are independently identically distributed (iid) with common distribution function $F(x)$, this system is subjected to common random stress Y. The probability in (3) is called reliability in a multicomponent stress-strength model (Bhattacharyya & Johnson 1974). The survival probability of a single component stress-strength version has been considered by several authors assuming various lifetime distributions for the stress-strength random variates, e.g. Enis & Geisser (1971), Downtown (1973), Awad & Gharraf (1986), McCool (1991), Nandi & Aich (1994), Surles & Padgett (1998), Raqab & Kundu (2005), Kundu & Gupta (2005), Kundu & Gupta (2006), Raqab, Modi & Kundu (2008), Kundu & Raqab (2009). The reliability in a multicomponent stress-strength was developed by Bhattacharyya & Johnson (1974), Pandey & Uddin (1985), and the references therein cover the study of estimating in many standard distributions assigned to one or both stress, strength variates. Recently, Rao & Kantam (2010) studied estimation of reliability in multicomponent stress-strength for the log-logistic distribution.

Suppose that a system, with k identical components, functions if $s(1 \leq s \leq k)$ or more of the components simultaneously operate. In this operating environment, the system is subjected to a stress Y which is a random variable with distribution function $G(.)$. The strengths of the components, that is the minimum stress to cause failure, are independent and identically distributed random variables with distribution function $F(.)$. Then, the system reliability, which is the probability that the system does not fail, is the function $R_{s,k}$ given in (3). The estimation of the survival probability in a multicomponent stress-strength system when the stress follows a two-parameter GE distribution has not received much attention in the literature. Therefore, an attempt is made here to study the estimation of reliability in multicomponent stress-strength model with reference to the two-parameter GE probability distribution. In Section 2, we derive the expression for $R_{s,k}$ and develop a procedure for estimating it. More specifically, we obtain the maximum likelihood estimates of the parameters. The Maximum Likelihood Estimators (MLEs) are employed to obtain the asymptotic distribution and confidence intervals for $R_{s,k}$. The small sample comparisons are made through Monte Carlo simulations in Section 3. Also, using real data, we illustrate the estimation process. Finally, some conclusion and comments are provided in Section 4.

2. Maximum Likelihood Estimator of $R_{s,k}$

Let $X \sim \text{GE}(\alpha,\, \lambda)$ and $Y \sim \text{GE}(\beta,\, \lambda)$ with unknown shape parameters α and β and common scale parameter λ, where X and Y are independently distributed. The reliability in multicomponent stress-strength for two-parameter GE distribution using (3) is

$$
\begin{aligned}
R_{s,k} &= \sum_{i=s}^{k} \binom{k}{i} \int_0^\infty [1-(1-e^{-y\lambda})^\alpha]^i [(1-e^{-y\lambda})^\alpha]^{(k-i)}\ \beta\lambda e^{-y\lambda}(1-e^{-y\lambda})^{\beta-1}\ dy \\
&= \sum_{i=s}^{k} \binom{k}{i} \int_0^1\ [1-t^\nu]^i\ [t^\nu]^{(k-i)}\ dt, \quad \text{where} \quad t=(1-e^{-y\lambda})^\beta \quad \text{and} \quad \nu=\frac{\alpha}{\beta} \\
&= \frac{1}{\nu}\sum_{i=s}^{k} \binom{k}{i} \int_0^1\ [1-z]^i\ [z]^{(k-i+\frac{1}{\nu}-1)}\ dz \quad \text{if} \quad z=t^\nu \\
&= \frac{1}{\nu}\sum_{i=s}^{k} \beta(k-i+\frac{1}{\nu}, i+1)
\end{aligned}
$$

After the simplification we get

$$R_{s,k} = \frac{1}{\nu}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}(k+\frac{1}{\nu}-j)\right]^{-1}, \quad \text{since } k \text{ and } i \text{ are integers} \tag{4}$$

The probability in (4) is called reliability in a multicomponent stress-strength model. If α and β are not known, it is necessary to estimate α and β to estimate $R_{s,k}$. In this paper we estimate α and β by the ML method. Once MLEs are obtained then $R_{s,k}$ can be computed using equation (4).

Let $X_1, X_2, \dots, X_n$; $Y_1, Y_2, \dots, Y_m$ be two ordered random samples of size n, m, respectively, on strength, stress variates following a GE distribution with shape parameters α and β and a common scale parameter λ. The log-likelihood function of the observed sample is

$$\begin{aligned} L(\alpha,\beta,\lambda) = (m+n)\ln\lambda + n\ln\alpha + m\ln\beta - \lambda\left[\sum_{i=1}^{n} x_i - \sum_{j=1}^{m} y_j\right] + \\ (\alpha-1)\sum_{i=1}^{n}\ln(1-e^{-x_i\lambda}) + (\beta-1)\sum_{j=1}^{m}\ln(1-e^{-y_j\lambda}) \end{aligned} \tag{5}$$

The MLEs of α, β and λ, say $\widehat{\alpha}$, $\widehat{\beta}$ and $\widehat{\lambda}$, respectively, can be obtained as the solution of

$$\widehat{\alpha} = \frac{-n}{\sum\limits_{i=1}^{n} ln(1-e^{-x_i\lambda})} \tag{6}$$

$$\widehat{\beta} = \frac{-m}{\sum\limits_{j=1}^{m} ln(1-e^{-y_j\lambda})} \tag{7}$$

$$\begin{aligned} g(\lambda) = 0 \Rightarrow \frac{m+n}{\lambda} - \frac{n\sum\limits_{i=1}^{n}\frac{x_i\ e^{-x_i\lambda}}{1-e^{-x_i\lambda}}}{\sum\limits_{k=1}^{n} ln(1-e^{-x_k\lambda})} - \frac{m\sum\limits_{j=1}^{m}\frac{y_j\ e^{-y_j\lambda}}{1-e^{-y_j\lambda}}}{\sum\limits_{k=1}^{m} ln(1-e^{-y_k\lambda})} \\ -\sum_{i=1}^{n}\frac{x_i}{1-e^{-x_i\lambda}} - \sum_{j=1}^{m}\frac{y_j}{1-e^{-y_j\lambda}} \end{aligned} \tag{8}$$

Therefore, $\widehat{\lambda}$ is a simple iterative solution of the non-linear equation $g(\lambda) = 0$. Once we obtain $\widehat{\lambda}$; $\widehat{\alpha}$ and $\widehat{\beta}$ can be obtained from (6) and (7), respectively. Therefore, the MLE of $R_{s,k}$ becomes

$$\widehat{R}_{s,k} = \frac{1}{\widehat{\nu}}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}(k+\frac{1}{\widehat{\nu}}-j)\right]^{-1}, \quad \text{where} \quad \widehat{\nu} = \frac{\widehat{\alpha}}{\widehat{\beta}} \tag{9}$$

To obtain the asymptotic confidence interval for $R_{s,k}$, we proceed as below: The asymptotic variance of the MLE is given by

$$V(\widehat{\alpha}) = \left[E(-\frac{\partial^2 L}{\partial \alpha^2})\right] = \frac{\alpha^2}{n} \quad \text{and} \quad V(\widehat{\beta}) = \left[E(-\frac{\partial^2 L}{\partial \beta^2})\right] = \frac{\beta^2}{n} \tag{10}$$

The asymptotic variance (AV) of an estimate of $R_{s,k}$ which is a function of two independent statistics $\widehat{\alpha}$ and $\widehat{\beta}$ is given by Rao (1973).

$$AV(\widehat{R}_{s,k}) = V(\widehat{\alpha}) \left[\frac{\partial R_{s,k}}{\partial \alpha}\right]^2 + V(\widehat{\beta}) \left[\frac{\partial R_{s,k}}{\partial \beta}\right]^2 \tag{11}$$

From the asymptotic optimum properties of MLEs (Kendall & Stuart 1979) and of linear unbiased estimators (David 1981), we know that MLEs are asymptotically equally efficient having the Cramer-Rao lower bound as their asymptotic variance, as given in (10). Thus, from equation (11), the asymptotic variance of $\widehat{R}_{s,k}$ can be obtained. To avoid the difficulty of the derivation of the $R_{s,k}$, we obtain the derivatives of $R_{s,k}$ for (s,k)=(1,3) and (2,4) separately and they are given by

$$\frac{\partial R_{1,3}}{\partial \alpha} = \frac{3}{\beta\left(3\widehat{\nu}+1\right)^2} \quad \text{and} \quad \frac{\partial R_{1,3}}{\partial \beta} = \frac{-3\widehat{\nu}}{\beta\left(3\widehat{\nu}+1\right)^2}$$

$$\frac{\partial R_{2,4}}{\partial \alpha} = \frac{12\widehat{\nu}(7\widehat{\nu}+2)}{\beta\left[(3\widehat{\nu}+1)(4\widehat{\nu}+1)\right]^2} \quad \text{and} \quad \frac{\partial R_{2,4}}{\partial \beta} = \frac{-12\widehat{\nu}^2(7\widehat{\nu}+2)}{\beta\left[(3\widehat{\nu}+1)(4\widehat{\nu}+1)\right]^2}$$

Thus $AV(\widehat{R}_{1,3}) = \frac{9\widehat{\nu}^2}{(3\widehat{\nu}+1)^4}\left(\frac{1}{n}+\frac{1}{m}\right)$

$$AV(\widehat{R}_{2,4}) = \frac{144\widehat{\nu}^4(7\widehat{\nu}+2)^2}{\left[(3\widehat{\nu}+1)(4\widehat{\nu}+1)\right]^4}\left(\frac{1}{n}+\frac{1}{m}\right)$$

as $n \to \infty, m \to \infty$, $\frac{\widehat{R}_{s,k}-R_{s,k}}{AV(\widehat{R_{s,k}})} \xrightarrow{d} N(0,1)$ and the asymptotic confidence 95% confidence interval for $R_{s,k}$ is given by

$$\widehat{R}_{s,k} \pm 1.96\sqrt{AV(\widehat{R}_{s,k})}$$

The asymptotic confidence 95% confidence interval for $R_{1,3}$ is given by

$$\widehat{R}_{1,3} \pm 1.96\frac{3\widehat{\nu}}{\left(3\widehat{\nu}+1\right)^2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)}, \quad \text{where} \quad \widehat{\nu} = \frac{\widehat{\alpha}}{\widehat{\beta}}$$

The asymptotic confidence 95% confidence interval for $R_{2,4}$ is given by

$$\widehat{R}_{2,4} \pm 1.96\frac{12\widehat{\nu}^2(7\widehat{\nu}+2)}{\left[(3\widehat{\nu}+1)(4\widehat{\nu}+1)\right]^2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)}, \quad \text{where} \quad \widehat{\nu} = \frac{\widehat{\alpha}}{\widehat{\beta}}$$

3. Simulation Study and Data Analysis

3.1. Simulation Study

In this subsection we present some results based on Monte Carlo simulations to compare the performance of the $R_{s,k}$ using different sample sizes. 3,000 random samples of size 10(5)35 each from stress population, strength population are generated for $(\alpha, \beta) = (3.0, 1.5)$, (2.5,1.5), (2.0,1.5), (1.5,1.5), (1.5,2.0),(1.5,2.5) and (1.5,3.0) in line with Bhattacharyya & Johnson (1974). The MLE of scale parameter λ is estimated by the iterative method, and the using λ the shape parameters α and β are estimated from (6) and (7). These ML estimators of α and β are then substituted in ν to get the reliability in a multicomponent reliability for $(s, k) = (1, 3), (2, 4)$. The average bias and average mean square error (MSE) of the reliability estimates over the 3000 replications are given in Tables 1 and 2. Average confidence length and coverage probability of the simulated 95% confidence intervals of $R_{s,k}$ are given in Tables 3 and 4. The true values of reliability in multicomponent stress-strength with the given combinations for $(s, k) = (1, 3)$ are 0.857, 0.833, 0.800, 0.750, 0.692, 0.643, 0.600, and for $(s, k) = (2, 4)$ are 0.762, 0.725, 0.674, 0.600, 0.519, 0.454, and 0.400. Thus, the true value of reliability in multicomponent stress-strength model decreases as β increases for a fixed α whereas reliability in multicomponent stress-strength increases as increases for a fixed β in both the cases (s, k). Therefore, the true value of reliability decreases as ν decreases, and *vice versa*. The average bias and average MSE decrease as sample size increases for both methods of estimation in reliability. Also the bias is negative in both situations of (s, k). It verifies the consistency property of the MLE of $R_{s,k}$. Whereas, among the parameters the absolute bias and MSE decrease as α increases for a fixed β in both cases of (s, k) and the absolute bias and MSE increase as β increases for a fixed α in both the cases of (s, k). The length of the confidence interval also decreases as the sample size increases. The coverage probability is close to the nominal value in all cases but slightly less than 0.95. Overall, the performance of the confidence interval is quite good for all combinations of parameters. Whereas, among the parameters we observed the same phenomenon for average length and average coverage probability that we observed in the case of average bias and MSE.

3.2. Data Analysis

In this subsection we analyze two real data sets and demonstrate how the proposed methods can be used in practice. The first data set is reported by Lawless (1982) and the second one is given by Linhardt & Zucchini (1986). Both are analyzed and fitted for various lifetime distributions. We fit the generalized exponential distribution to the two data sets separately. The first data set (Lawless 1982, p. 228) presented here arose in tests on endurance of deep groove ball bearings. The data presented are the number of million revolutions before failure for each of the 23 ball bearings in the life test, and they are: 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, and 173.40. Gupta & Kundu (2001) studied the validity of the model and they compute the Kolmogorov-Smirnov (KS) distance between the empirical distribution function and the fitted distribution functions of generalized exponential distribution which is 0.1058 with a corresponding p-value of 0.9592.

Table 1: Average bias of the simulated estimates of $R_{s,k}$.

		(α,β)						
(s,k)	(n,m)	(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	−0.0029	−0.0047	−0.0072	−0.0109	−0.0150	−0.0183	−0.0207
	(15,15)	−0.0021	−0.0042	−0.0058	−0.0081	−0.0105	−0.0123	−0.0137
	(20,20)	−0.0018	−0.0027	−0.0039	−0.0058	−0.0079	−0.0096	−0.0109
	(25,25)	−0.0012	−0.0020	−0.0030	−0.0046	−0.0064	−0.0078	−0.0089
	(30,30)	−0.0011	−0.0019	−0.0028	−0.0041	−0.0055	−0.0066	−0.0075
	(35,35)	−0.0002	−0.0006	−0.0012	−0.0021	−0.0031	−0.0040	−0.0047
(2,4)	(10,10)	−0.0029	−0.0039	−0.0063	−0.0092	−0.0116	−0.0128	−0.0131
	(15,15)	−0.0022	−0.0034	−0.0059	−0.0075	−0.0087	−0.0092	−0.0091
	(20,20)	−0.0017	−0.0027	−0.0040	−0.0056	−0.0070	−0.0077	−0.0080
	(25,25)	−0.0010	−0.0019	−0.0030	−0.0044	−0.0056	−0.0063	−0.0065
	(30,30)	−0.0009	−0.0011	−0.0030	−0.0041	−0.0051	−0.0057	−0.0059
	(35,35)	−0.0003	−0.0002	−0.0008	−0.0016	−0.0023	−0.0027	−0.0029

Table 2: Average MSE of the simulated estimates of $R_{s,k}$.

		(α,β)						
(s,k)	(n,m)	(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	0.0041	0.0052	0.0068	0.0092	0.0119	0.0139	0.0153
	(15,15)	0.0026	0.0033	0.0043	0.0058	0.0075	0.0087	0.0096
	(20,20)	0.0017	0.0022	0.0029	0.0040	0.0052	0.0061	0.0068
	(25,25)	0.0014	0.0018	0.0024	0.0032	0.0042	0.0050	0.0055
	(30,30)	0.0011	0.0014	0.0018	0.0025	0.0032	0.0038	0.0043
	(35,35)	0.0009	0.0011	0.0015	0.0021	0.0027	0.0032	0.0036
(2,4)	(10,10)	0.0096	0.0115	0.0141	0.0171	0.0193	0.0199	0.0196
	(15,15)	0.0062	0.0075	0.0091	0.0111	0.0125	0.0130	0.0128
	(20,20)	0.0042	0.0051	0.0063	0.0078	0.0090	0.0094	0.0094
	(25,25)	0.0035	0.0043	0.0052	0.0065	0.0074	0.0078	0.0078
	(30,30)	0.0028	0.0033	0.0041	0.0050	0.0058	0.0060	0.0060
	(35,35)	0.0022	0.0027	0.0034	0.0042	0.0049	0.0052	0.0052

The second data set (from Linhardt & Zucchini 1986, p. 69) represents the failure times of the air conditioning system of an airplane (in hours): 23, 261, 87, 7, 120, 14, 62, 47, 225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14, 71, 11, 14, 11, 16, 90, 1, 16, 52, 95. Gupta & Kundu (2003) studied the validity of the generalized exponential distribution and they compute the Kolmogorov-Smirnov (KS) distance between the empirical distribution function and the fitted distribution functions which is 0.1744 with a corresponding p-value 0.2926. Therefore, it is clear that the generalized exponential model fits quite well to both the data sets.

We use the iterative procedure to obtain the MLE of λ using (8), and MLEs of α and β are obtained by substituting MLE of λ in (6) and (7). The final estimates are $\widehat{\lambda} = 2.80609$, $\widehat{\alpha} = 1.00667$ and $\widehat{\beta} = 0.02098$. Based on the estimates of α and

β, the MLE of $R_{s,k}$ becomes $\widehat{R}_{1,3} = 0.893191$ and $\widehat{R}_{2,4} = 0.819677$. The 95% confidence intervals for $R_{1,3}$ become (0.841368, 0.945014) and for $R_{2,4}$ become (0.735472, 0.903882).

Table 3: Average confidence length of the simulated 95% confidence intervals of $R_{s,k}$.

		(α,β)						
(s,k)	(n,m)	(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	0.2112	0.2399	0.2762	0.3221	0.3627	0.3873	0.4012
	(15,15)	0.1747	0.1981	0.2279	0.2659	0.3000	0.3212	0.3337
	(20,20)	0.1512	0.1716	0.1977	0.2311	0.2614	0.2804	0.2918
	(25,25)	0.1351	0.1534	0.1768	0.2069	0.2342	0.2515	0.2619
	(30,30)	0.1238	0.1404	0.1618	0.1893	0.2145	0.2304	0.2401
	(35,35)	0.1140	0.1295	0.1492	0.1748	0.1982	0.2132	0.2224
(2,4)	(10,10)	0.3267	0.3628	0.4045	0.4485	0.4744	0.4782	0.4697
	(15,15)	0.2721	0.3020	0.3368	0.3742	0.3973	0.4020	0.3962
	(20,20)	0.2366	0.2630	0.2939	0.3274	0.3486	0.3533	0.3486
	(25,25)	0.2119	0.2356	0.2635	0.2939	0.3134	0.3180	0.3141
	(30,30)	0.1943	0.2161	0.2416	0.2697	0.2878	0.2923	0.2890
	(35,35)	0.1794	0.1996	0.2234	0.2497	0.2669	0.2716	0.2688

Table 4: Average coverage probability of the simulated 95% confidence intervals of $R_{s,k}$.

		(α,β)						
(s,k)	(n,m)	(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	0.9230	0.9247	0.9277	0.9220	0.9140	0.9070	0.9053
	(15,15)	0.9327	0.9330	0.9357	0.9323	0.9303	0.9280	0.9243
	(20,20)	0.9373	0.9387	0.9397	0.9400	0.9360	0.9293	0.9243
	(25,25)	0.9287	0.9323	0.9347	0.9360	0.9340	0.9293	0.9247
	(30,30)	0.9347	0.9360	0.9393	0.9403	0.9420	0.9427	0.9363
	(35,35)	0.9453	0.9480	0.9497	0.9477	0.9450	0.9417	0.9347
(2,4)	(10,10)	0.9197	0.9213	0.9230	0.9177	0.9133	0.9133	0.9097
	(15,15)	0.9320	0.9323	0.9340	0.9333	0.9307	0.9277	0.9237
	(20,20)	0.9353	0.9373	0.9390	0.9387	0.9327	0.9310	0.9260
	(25,25)	0.9287	0.9320	0.9333	0.9383	0.9333	0.9300	0.9263
	(30,30)	0.9353	0.9380	0.9410	0.9397	0.9390	0.9393	0.9363
	(35,35)	0.9453	0.9490	0.9490	0.9453	0.9433	0.9380	0.9360

4. Conclusions

In this paper, we have studied the multicomponent stress-strength reliability for generalized exponential distribution when both stress, strength variates follow the same population. Also, we have estimated asymptotic confidence interval for the multicomponent stress-strength reliability. The simulation results indicate that the average bias and average the MSE decrease as sample size increases for both situations of (s,k). Among the parameters the absolute bias and MSE decrease (increase) as α increases (β increases) in both the cases of (s,k). The length of the confidence interval also decreases as the sample size increases and the coverage probability is close to the nominal value in all sets of parameters considered here. Using real data, we illustrate the estimation process.

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References

Awad, M. & Gharraf, K. (1986), 'Estimation of $P(Y < X)$ in Burr case: A comparative study', *Communications in Statistics-Simulation and Computation* **15**, 389–403.

Bhattacharyya, G. K. & Johnson, R. A. (1974), 'Estimation of reliability in multicomponent stress-strength model', *Journal of the American Statistical Association* **69**, 966–970.

David, H. A. (1981), *Order Statistics*, John Wiley and Sons, New York.

Downtown, F. (1973), 'The estimation of $P(X > Y)$ in the normal case', *Technometrics* **15**, 551–558.

Enis, P. & Geisser, S. (1971), 'Estimation of the probability that $Y < X$', *Journal of the American Statistical Association* **66**, 162–168.

Gupta, R. D. & Kundu, D. (1999), 'Generalized exponential distributions', *Australian and New Zealand Journal of Statistics* **41**, 173–188.

Gupta, R. D. & Kundu, D. (2001), 'Generalized exponential distributions; different method of estimations', *Journal of Statistical Computation and Simulation* **69**, 315–338.

Gupta, R. D. & Kundu, D. (2002), 'Generalized exponential distributions; statistical inferences', *Journal of Statistical Theory and Applications* **1**, 101–118.

Gupta, R. D. & Kundu, D. (2003), 'Discriminating between the Weibull and generalized exponential distributions', *Computational Statistics and Data Analysis* **43**, 179–196.

Kendall, M. G. & Stuart, A. (1979), *The Advanced Theory of Statistics*, Vol. 2, Charles Griffin and Company Limited, London.

Kundu, D. & Gupta, R. (2006), 'Estimation of $P(Y < X)$ for Weibull distribution', *IEEE Transactions on Reliability* **55**(2), 270–280.

Kundu, D. & Gupta, R. D. (2005), 'Estimation of $P(Y < X)$ for the generalized exponential distribution', *Metrika* **61**(3), 291–308.

Kundu, D. & Raqab, M. Z. (2009), 'Estimation of $R = P(Y < X)$ for three-parameter Weibull distribution', *Statistics and Probability Letters* **79**, 1839–1846.

Lawless, J. F. (1982), *Statistical Models and Methods for Lifetime Data*, John Wiley & Sons, New York.

Linhardt, H. & Zucchini, W. (1986), *Model Selection*, Wiley Eastern Limited, New York.

McCool, J. I. (1991), 'Inference on $P(Y < X)$ in the Weibull case', *Communications in Statistics-Simulation and Computation* **20**, 129–148.

Nandi, S. B. & Aich, A. B. (1994), 'A note on estimation of $P(X > Y)$ for some distributions useful in life-testing', *IAPQR Transactions* **19**(1), 35–44.

Pandey, M. & Uddin, B. M. (1985), Estimation of reliability in multicomponent stress-strength model following Burr distribution, *in* 'Proceedings of the First Asian congress on Quality and Reliability', New Delhi, India, pp. 307–312.

Rao, C. R. (1973), *Linear Statistical Inference and its Applications*, Wiley Eastern Limited, India.

Rao, G. S. & Kantam, R. R. L. (2010), 'Estimation of reliability in multicomponent stress-strength model: Log-logistic distribution', *Electronic Journal of Applied Statistical Analysis* **3**(2), 75–84.

Raqab, M. Z. & Kundu, D. (2005), 'Comparison of different estimators of $P(Y < X)$ for a scaled Burr type X distribution', *Communications in Statistics-Simulation and Computation* **34**(2), 465–483.

Raqab, M. Z., Modi, M. T. & Kundu, D. (2008), 'Estimation of $P(Y < X)$ for the 3-parameter generalized exponential distribution', *Communications in Statistics-Theory and Methods* **37**(18), 2854–2864.

Surles, J. G. & Padgett, W. J. (1998), 'Inference for $P(Y < X)$ in the Burr type X model', *Communications in Statistics-Theory and Methods* **7**, 225–238.