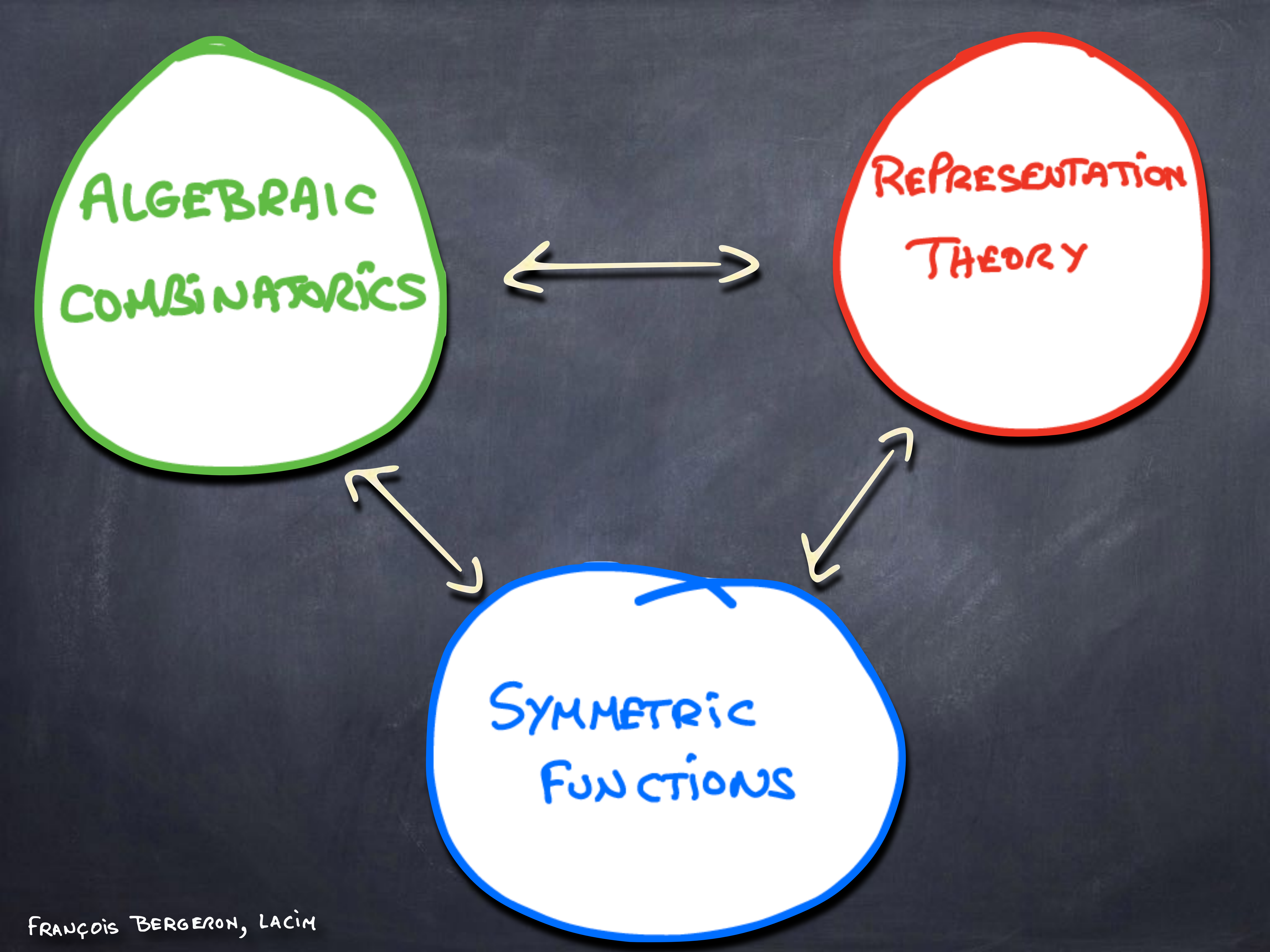
STRUCTURE OF GENERIC MODULES OF DIAGONAL HARMONIC POLYNOMIALS

$$\mathcal{E}_{m,n}$$

FRANÇOIS BERGERON, LACIM


ALGEBRAIC COMBINATORICS
REPRESENTATION THEORY
SYMMETRIC FUNCTIONS
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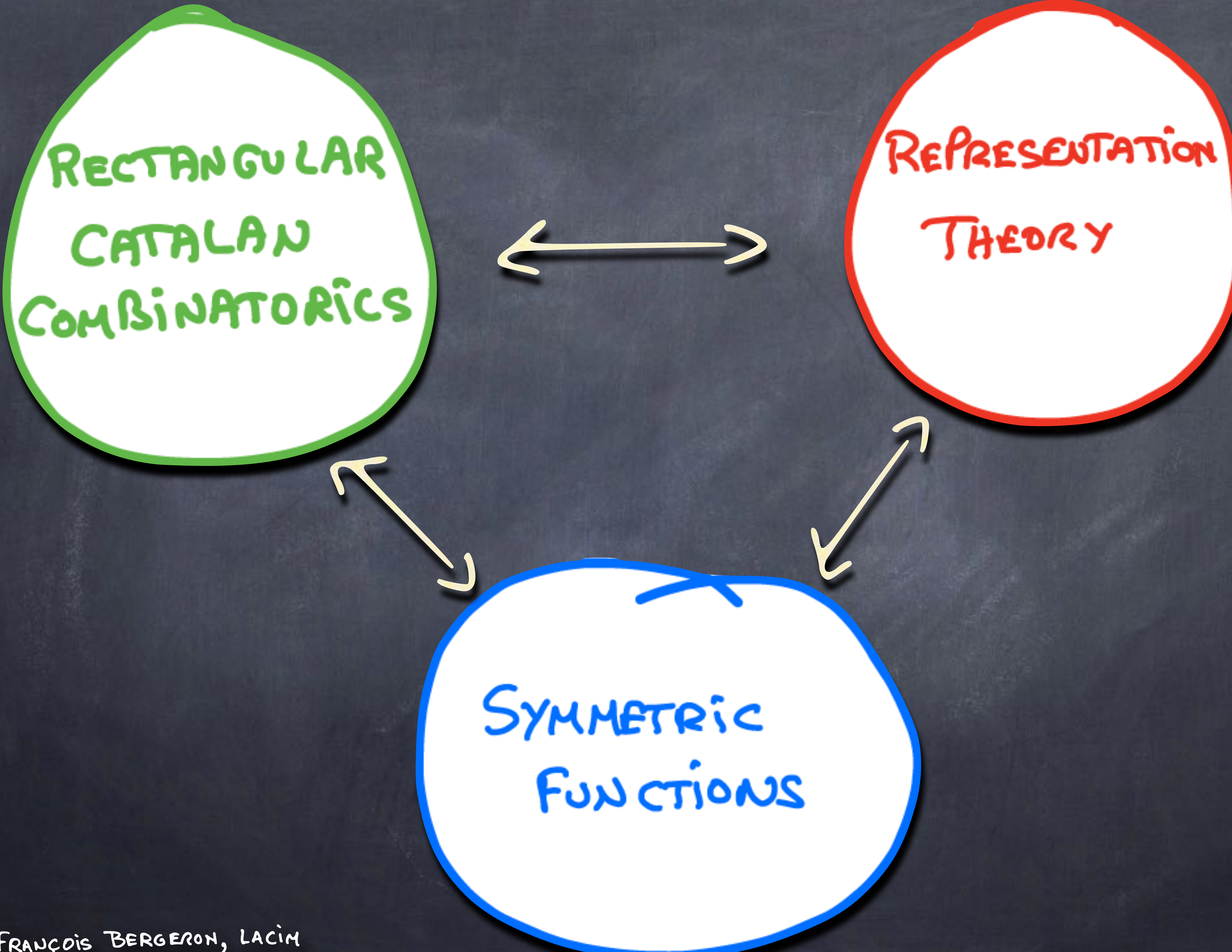

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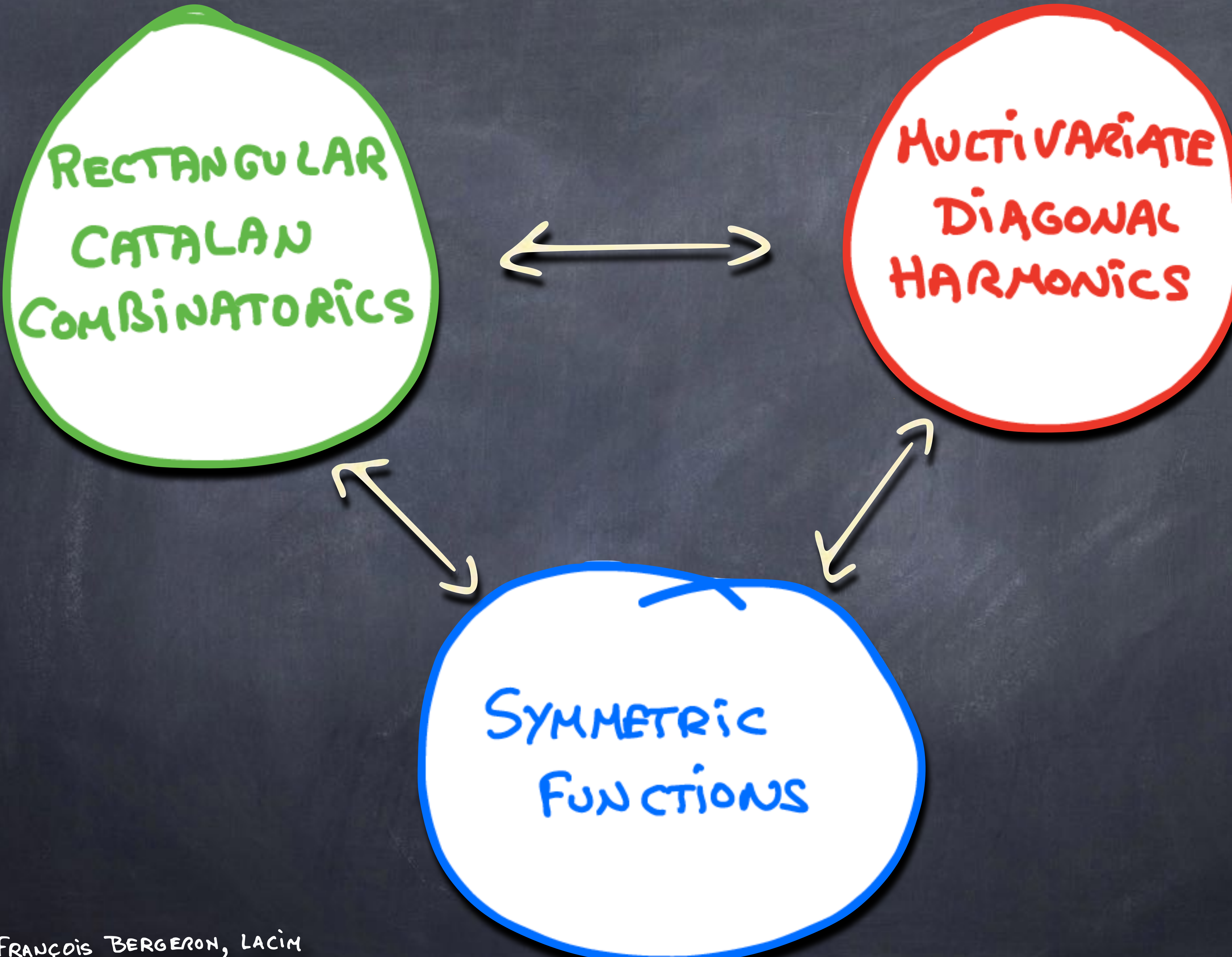

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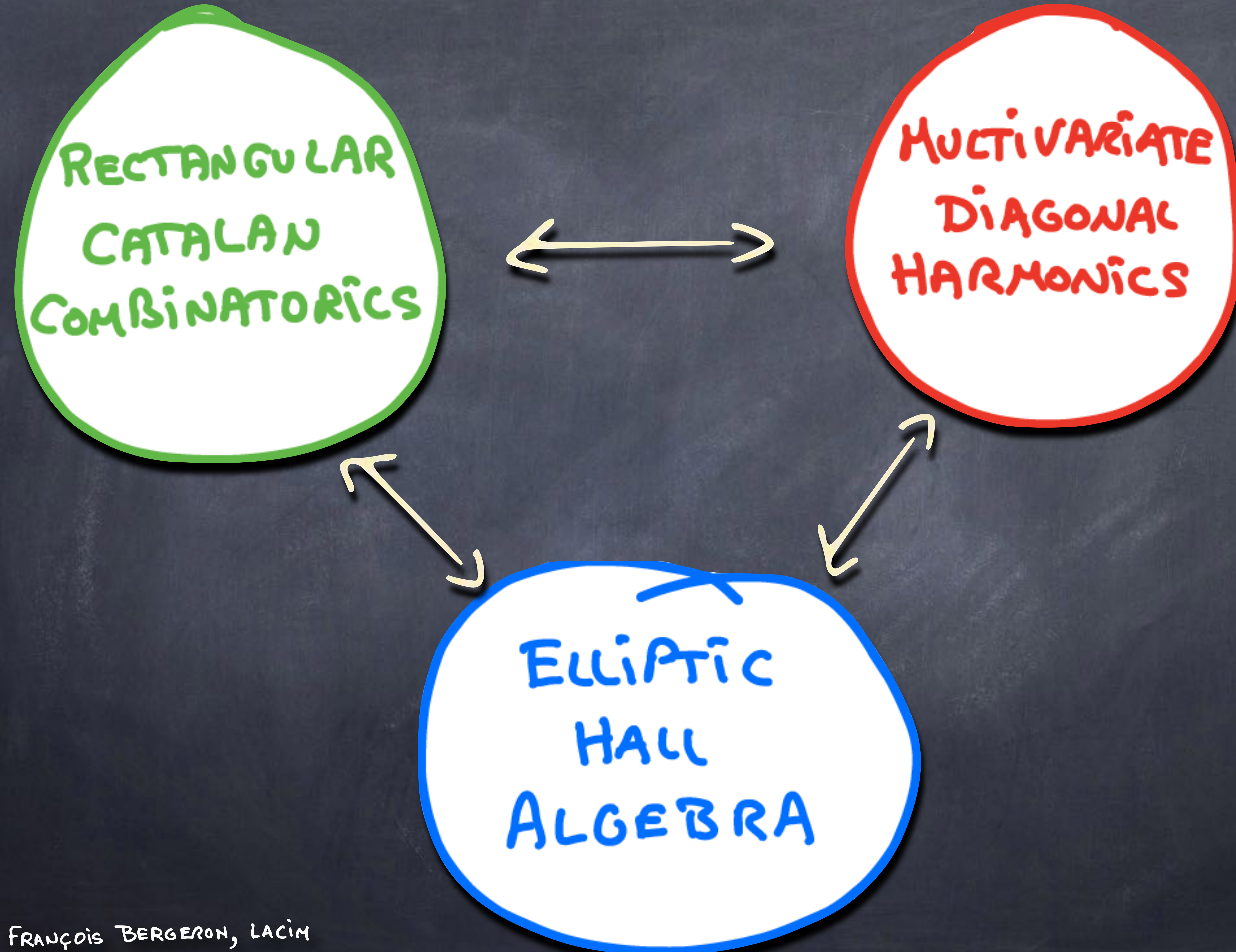
RECTANGULAR CATALAN COMBINATORICS
MULTIVARIATE DIAGONAL HARMONICS
ELLIPTIC HALL ALGEBRA
FRANÇOIS BERGERON, LACIM

Multivariate Diagonal Harmonics

Elliptic Hall Algebra

$\mathcal{E}_{m,n}$

ELLIPTIC
HALL
ALGEBRA

Most Cited Publications

Citations	Publication
206	**MR1701596 (2002i:05013)** Krattenthaler, C. Advanced determinant calculus. The Andrews Festschrift (Maratea, 1998). *Sém. Lothar. Combin.* 42 (1999), Art. B42q, 67 pp. (Reviewer: George E. Andrews) 05A19 (15A15 33C99)
105	**MR1868978 (2002k:05005)** Krattenthaler, C. Permutations with restricted patterns and Dyck paths. Special issue in honor of Dominique Foata's 65th birthday (Philadelphia, PA, 2000). *Adv. in Appl. Math.* 27 (2001), no. 2-3, 510–530. (Reviewer: Timothy Y. Chow) 05A05 (05A15 05E35 30B70 33C45 42C05)
104	**MR2178686 (2006g:05022)** Krattenthaler, C. Advanced determinant calculus: a complement. *Linear Algebra Appl.* 411 (2005), 68–166. (Reviewer: George E. Andrews) 05A19 (05A15 15A15)
53	**MR2261181 (2007h:05011)** Krattenthaler, C. Growth diagrams, and increasing and decreasing chains in fillings of Ferrers shapes. *Adv. in Appl. Math.* 37 (2006), no. 3, 404–431. (Reviewer: Marni Mishna) 05A15 (05A17 05E10)
42	**MR1291781 (96d:15004)** Krattenthaler, C. A new matrix inverse. *Proc. Amer. Math. Soc.* 124 (1996), no. 1, 47–59. (Reviewer: Jaroslav Zemánek) 15A09 (33D20)
40	**MR1801472 (2001m:82041)** Krattenthaler, Christian; Guttmann, Anthony J.; Viennot, Xavier G. Vicious walkers, friendly walkers and Young tableaux. II. With a wall. *J. Phys. A* 33 (2000), no. 48, 8835–8866. (Reviewer: Jesper Lykke Jacobsen) 82B41 (05E10)
29	**MR1254150 (95i:05109)** Krattenthaler, C. The major counting of nonintersecting lattice paths and generating functions for tableaux. *Mem. Amer. Math. Soc.* 115 (1995), no. 552, vi+109 pp. (Reviewer: Kevin W. J. Kadell) 05E10 (05A15)
28	**MR1389777 (97e:05014)** Gessel, Ira M.; Krattenthaler, C. Cylindric partitions. *Trans. Amer. Math. Soc.* 349 (1997), no. 2, 429–479. (Reviewer: Miklós Bóna) 05A15 (05A17 05A30 33D20 33D80)
27	**MR2200854 (2007d:16082)** Brouder, Christian; Frabetti, Alessandra; Krattenthaler, Christian Non-commutative Hopf algebra of formal diffeomorphisms. *Adv. Math.* 200 (2006), no. 2, 479–524. (Reviewer: María Ofelia Ronco) 16W30 (81T15)
26	**MR2295224 (2008a:11078)** Krattenthaler, C.; Rivoal, T. Hypergéométrie et fonction zêta de Riemann. (French) [Hypergeometry and Riemann zeta functions] *Mem. Amer. Math. Soc.* 186 (2007), no. 875, x+87 pp. (Reviewer: Yann Bugeaud) 11J72 (11J82 33C20)

MR1028034 (91e:05005) 05A15 05A30

Krattenthaler, Christian [**Krattenthaler, Christian F.**] (A-WIEN)

Counting lattice paths with a linear boundary. I. (German summary)

Österreich. Akad. Wiss. Math.-Natur. Kl. Sitzungsber. II **198** (1989), *no. 1-3,* 87–107.

The lattice paths studied in this paper consist of unit horizontal and vertical steps in the positive directions. For nonnegative integers n, r, and t, let $L_+(n,r,t)$ be the set of paths from the origin to the point $(n, rn+t)$ not touching the line $y = rx + t$ except at the final point. It is well known that

$$(1) \qquad |L_+(n,r,t)| = \frac{t}{t+(r+1)n}\binom{t+(r+1)n}{n}.$$

This paper is concerned with q-extensions of (1) and related results. Note that for $r = 1$, $|L_+(n,r,t)|$ is a ballot number, and for $r = t = 1$, it is a Catalan number.

The statistics on paths studied here are most easily defined by first converting each path to a binary word: each horizontal step is converted to a 0 and each vertical step to a 1. Given a binary word $w = w_1 \cdots w_n$, we define its "down-set" to be $D(w) = \{i : w_i > w_{i+1},\ 1 \le i \le n-1\}$, and we define the three statistics $\operatorname{des} w = |D(w)|$, $\alpha(w) = \sum_{i\in D(w)} |\{j \le i : w_j = 0\}|$, $\beta(w) = \sum_{i\in D(w)} |\{j \le i : w_j = 1\}|$. Thus $\alpha(w) + \beta(w)$ is the major index of w.

It is well known that

$$(2) \qquad \sum_{n=0}^{\infty} |L_+(n,r,t)| \frac{z^n}{(1+z)^{(r+1)n+t}} = 1,$$

which is related by Lagrange inversion to (1). One of the author's main results is the following generalization of (2): Let

$$G_n(r,t,x,a,b) = \sum_{w\in L_+(n,r,t)} x^{\operatorname{des} w} a^{\alpha(w)} b^{\beta(w)}.$$

Then for $t \ge 1$,

$$(3) \qquad \sum_{n=0}^{\infty} G_n(r,t,x,a,b) \frac{a^n z^n}{(1+z)\cdots(a^n+z)(1+bxz)\cdots(1+b^{rn+t-1}xz)} = 1.$$

The author also gives an analogue of (3) for a "dual" of G_n and derives recurrences and convolution identities for G_n and its dual. *Ira Gessel*

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PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 90, Number 2, February 1984

A NEW q-LAGRANGE FORMULA AND SOME APPLICATIONS

CHRISTIAN KRATTENTHALER

ABSTRACT. A new q-extension of the Lagrange-Bürmann expansion and related formulas are proved. Finally we give a method to find q-generalizations of Riordan's inverse relations.

TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 305, Number 2, February 1988

OPERATOR METHODS AND LAGRANGE INVERSION: A UNIFIED APPROACH TO LAGRANGE FORMULAS

CH. KRATTENTHALER

ABSTRACT. We present a general method of proving Lagrange inversion formulas and give new proofs of the s-variable Lagrange-Good formula [**13**] and the q-Lagrange formulas of Garsia [**7**], Gessel [**10**], Gessel and Stanton [**11, 12**] and the author [**18**]. We also give some q-analogues of the Lagrange formula in several variables.

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- RECTANGULAR CATALAN COMBINATORICS
- $\boxed{\text{MACDONALD POLYNOMIALS AND OPERATORS}}$
- DIAGONAL HARMONICS
- DIAGONAL COINVARIANT SPACE
- HILBERT SCHEMES OF POINTS IN THE PLANE
- CHEREDNIK HECKE ALGEBRAS.
- REFINED KNOT INVARIANTS
- ELLIPTIC HALL ALGEBRA.

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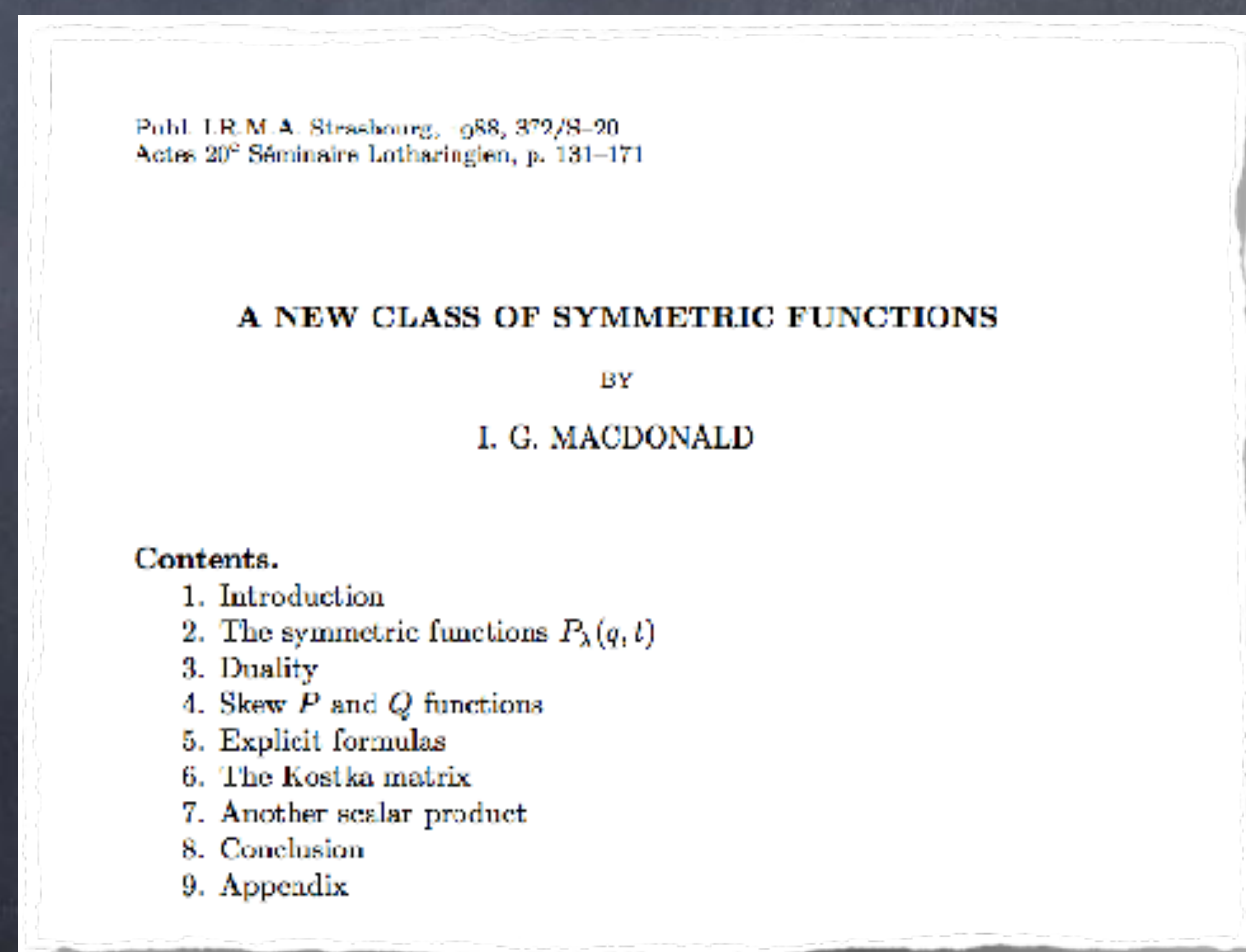
AN G. MACDONALD



Publ. I.R.M.A. Strasbourg, 1988, 372/S-20
Actes 20^e Séminaire Lotharingien, p. 131–171

A NEW CLASS OF SYMMETRIC FUNCTIONS

BY

I. G. MACDONALD

Contents.

1. Introduction
2. The symmetric functions $P_\lambda(q,t)$
3. Duality
4. Skew P and Q functions
5. Explicit formulas
6. The Kostka matrix
7. Another scalar product
8. Conclusion
9. Appendix

1988

Séminaire Lotharingien de Combinatoire

Issue 20

24 sept. - 1 oct. 1988, Alghero

Preface

[B20a] I.G. Macdonald,
A new class of symmetric functions (41 pp)
[B20b] Marie Pierre Delest, Serge Dulucq and Luc Favreau,
An Analogue to Robinson-Schensted Correspondence for Oscillating Tableaux (14 pp)
[B20c] Jacques Désarménien et Dominique Foata,
Statistiques d'ordre sur les permutations colorées (18 pp)
[B20d] Jiang Zeng,
La β-extension de la formule d'inversion de Lagrange à plusieurs variables
[B20e] Dina Ghinelli,
A rational congruence for a standard orbit decomposition
[B20f] Arnold Richard Kräuter,
Maximale Permanenten von (1,-1)-Matrizen mit beliebigem Rang (7 pp)
[B20g] Volker Strehl
A Combinatorial Proof of Louck's Conjecture (7 pp)
[B20h] Alain Lascoux and Marcel-Paul Schützenberger
Arêtes et Tableaux (10 pp)
[B20i] Andreas Dress and Christian Siebeneicher
Ein Lemma über Perlenketten (9 pp)
[B20j] Pierre Duchet
n^{n-2} (6 pp)
[B20k] Peter Hoffman
Littlewood-Richardson without Algorithmically Defined Bijections (6 pp)

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MACDONALD POLYNOMIALS occur AS EIGENFUNCTIONS in THE SOLUTION OF AN instance OF SCHRÖDINGER EQUATION

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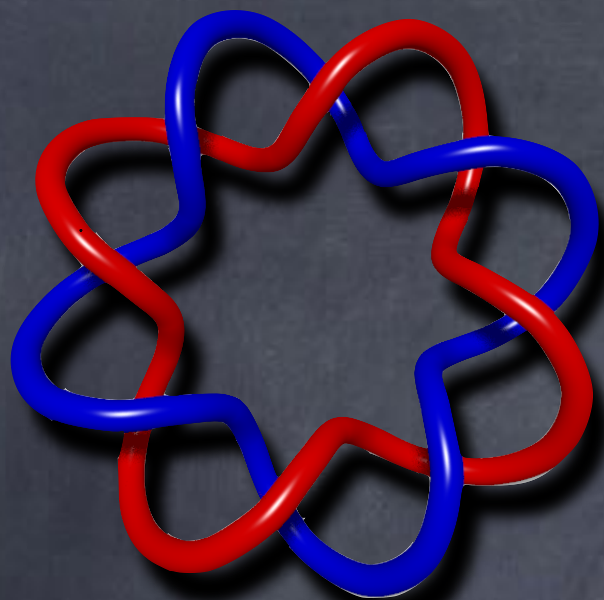
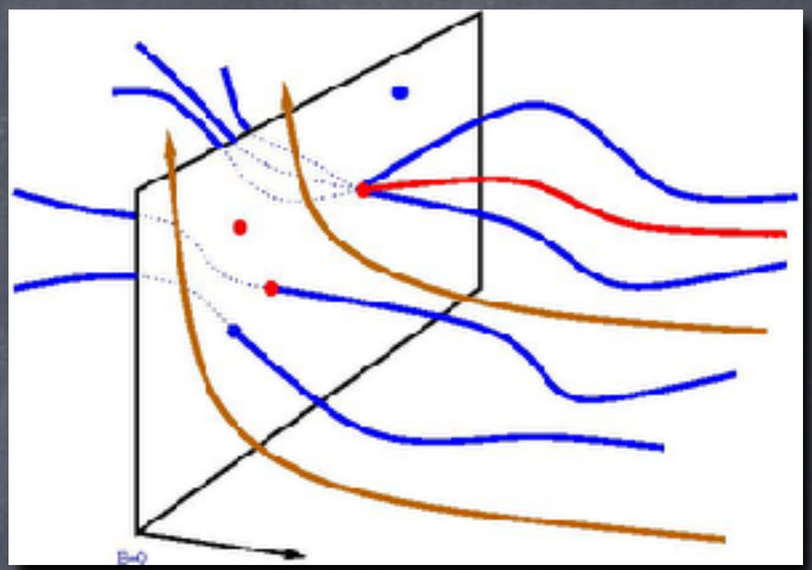
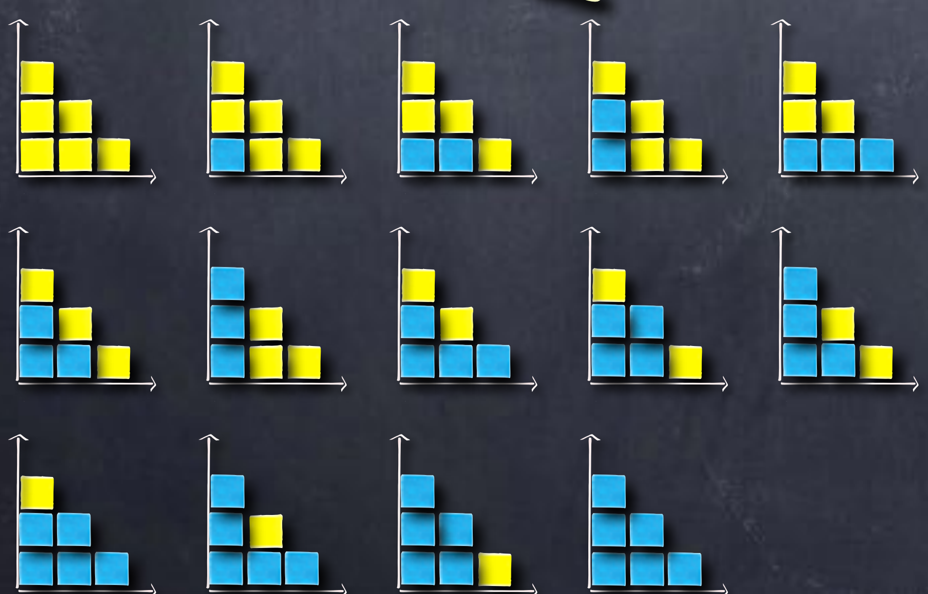
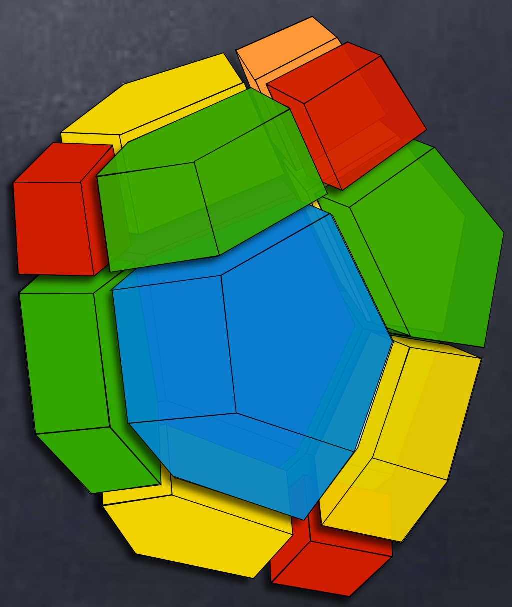


$\nabla(e_n)$



$\mathcal{E}_{m,n}$





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Multivariate Lift of the Shuffle Conjecture

François Bergeron, LaCIM

$$\xi_{m,n}(q,t;X) = \sum_{\lambda \subseteq \delta_{m,n}} t^{\mathrm{AREA}(\lambda)} \sum_{\substack{\pi \text{ OF} \\ \text{SHAPE } (\lambda+1^m)/\lambda}} q^{\mathrm{DINV}(\pi)} X_\pi$$

$$\Downarrow$$

$$\xi_{m,n}(1,t;X) = \sum_{\lambda \subseteq \delta_{m,n}} t^{|\delta_{m,n}|-|\lambda|} S_{(\lambda+1^m/\lambda)}(X)$$

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TRI-VARIATE LIFT

$$\left\langle \mathcal{E}_{rn,n}(1,1,1;\mathbf{x}),\, p_1^n \right\rangle = (r+1)^n (rn+1)^{n-2}$$

$$\mathcal{E}_{rn,n}(1,1,1;\mathbf{x}) =$$

$$\sum_{\mu \vdash n} (-1)^{n-\ell(\mu)} \frac{(rn+1)^{\ell(\mu)-2}}{z_\mu} \prod_{k\in\mu} h_k(\mathbf{x}) \binom{(r+1)k}{k}$$

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Multivariate Lift

$$s_\lambda(q, t, r, \ldots)$$

$$\xi_{m,n} = \sum_{\mu \vdash n} \sum_{\lambda} c_{\lambda\mu} \, (s_\lambda \otimes s_\mu)$$

$$s_\mu(x_1, x_2, x_3, \ldots)$$

$$\xi_{3,3} = 1 \otimes s_3 + (s_1 + s_2) \otimes s_{21} + (s_{11} + s_3) \otimes s_{111}$$

$$(s_{11} + s_3)(q,t) = qt + q^3 + q^2t + qt^2 + t^3$$

(q,t)– CATALAN

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Cat$_{mn}$ Symmetric Functions

$$\begin{pmatrix} \boxed{s_1} & s_1 & s_2 & s_2 & \cdots \\ s_1 & \boxed{s_{11}+s_3} & s_{11}+s_3 & s_{21}+s_4 & \cdots \\ s_2 & s_{11}+s_3 & \boxed{s_{31}+s_{41}+s_6} & s_{31}+s_{41}+s_6 & \cdots \\ s_2 & s_{21}+s_4 & s_{31}+s_{41}+s_6 & \boxed{s_{42}+s_{43}+s_{61}+s_{62}+s_{71}+s_{81}+s_{(10)}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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$$\xi_{3,3} = 1 \otimes \Delta_3 + (\Delta_1 + \Delta_2) \otimes \Delta_{21} + (\Delta_{11} + \Delta_3) \otimes \Delta_{111}$$

$$\begin{aligned}
\xi_{4,4} = 1 \otimes \Delta_4 &+ (\Delta_1 + \Delta_2 + \Delta_3) \otimes \Delta_{31} \\
&+ (\Delta_{21} + \Delta_2 + \Delta_4) \otimes \Delta_{22} \\
&+ (\Delta_{11} + \Delta_{21} + \Delta_{31} + \Delta_3 + \Delta_4 + \Delta_5) \otimes \Delta_{211} \\
&+ (\Delta_{111} + \Delta_{31} + \Delta_{41} + \Delta_6) \otimes \Delta_{1111}
\end{aligned}$$

Tri-variate Lift

$$
\begin{aligned}
&s_4\\
&+(q^3+q^2r+q^2t+qr^2+qrt+qt^2+r^3+r^2t+rt^2+t^3\\
&\qquad +q^2+qr+qt+r^2+rt+t^2+q+r+t)s_{31}\\
&+(q^4+q^3r+q^3t+q^2r^2+q^2rt+q^2t^2+qr^3+qr^2t+qrt^2+qt^3+r^4+r^3t+r^2t^2+rt^3+t^4\\
&\qquad +q^2r+q^2t+qr^2+2\,qrt+qt^2+r^2t+rt^2+q^2+qr+qt+r^2+rt+t^2)s_{22}\\
&+(q^5+q^4r+q^4t+q^3r^2+q^3rt+q^3t^2+q^2r^3+q^2r^2t+q^2rt^2+q^2t^3+qr^4\\
&\qquad +qr^3t+qr^2t^2+qrt^3+qt^4+r^5+r^4t+r^3t^2+r^2t^3+rt^4+t^5\\
&\qquad +q^4+2\,q^3r+2\,q^3t+2\,q^2r^2+3\,q^2rt+2\,q^2t^2+2\,qr^3+3\,qr^2t+3\,qrt^2\\
&\qquad +2\,qt^3+r^4+2\,r^3t+2\,r^2t^2+2\,rt^3+t^4+q^3+2\,q^2r+2\,q^2t+2\,qr^2\\
&\qquad +3\,qrt+2\,qt^2+r^3+2\,r^2t+2\,rt^2+t^3+qr+qt+rt)s_{211}\\
&+(q^6+q^5r+q^5t+q^4r^2+q^4rt+q^4t^2+q^3r^3+q^3r^2t+q^3rt^2+q^3t^3+q^2r^4\\
&\qquad +q^2r^3t+q^2r^2t^2+q^2rt^3+q^2t^4+qr^5+qr^4t+qr^3t^2+qr^2t^3+qrt^4+qt^5\\
&\qquad +r^6+r^5t+r^4t^2+r^3t^3+r^2t^4+rt^5+t^6+q^4r+q^4t+q^3r^2+2\,q^3rt+q^3t^2\\
&\qquad +q^2r^3+2\,q^2r^2t+2\,q^2rt^2+q^2t^3+qr^4+2\,qr^3t+2\,qr^2t^2+2\,qrt^3\\
&\qquad +qt^4+r^4t+r^3t^2+r^2t^3+rt^4+q^3r+q^3t+q^2r^2\\
&\qquad +2\,q^2rt+q^2t^2+qr^3+2\,qr^2t+2\,qrt^2+qt^3+r^3t\\
&\qquad +r^2t^2+rt^3+qrt)s_{1111}
\end{aligned}
$$

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MODULES OF DIAGONAL HARMONIC POLYNOMIALS

FRANÇOIS BERGERON, LACIM

Action of $GL_\infty \times \mathbb{S}_n$ on Polynomials in the variables

$$
GL_\infty \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ \vdots & \vdots & \ddots & \vdots \\ z_1 & z_2 & \cdots & z_n \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix} \mathbb{S}_n
$$

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$f(x) \longmapsto f(x\cdot\sigma)$ ACTION OF $\mathbb{S}_n$

$$X = \begin{pmatrix} x_2 & x_1 & \cdots & x_n \\ y_2 & y_1 & \cdots & y_n \\ \vdots & \vdots & \ddots & \vdots \\ z_2 & z_1 & \cdots & z_n \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

Action of GL_∞

$$f(X) \longmapsto f(T\cdot X)$$

$$T \circlearrowright \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ \vdots & \vdots & \ddots & \vdots \\ z_1 & z_2 & \cdots & z_n \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

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GL_∞-CHARACTER = MULTIVARIATE HILBERT SERIES

$$\begin{array}{c}\left(\begin{array}{cccc} x_1 & x_2 & \cdots & x_m \\ y_1 & y_2 & \cdots & y_m \\ \vdots & \vdots & \ddots & \vdots \\ z_1 & z_2 & \cdots & z_m \\ \vdots & \vdots & \ddots & \vdots \end{array}\right)\end{array} \begin{array}{l} \rightarrow q_1 \\ \rightarrow q_2 \\ \quad\vdots \\ \rightarrow q_k \\ \quad\vdots \end{array}$$

SYMMETRIC IN THE q_i

AND

SCHUR POSITIVE

THE MODULE $\mathcal{E}_{m,n}$

$\mathcal{P}_{m,n}$ SMALLEST MODULE CONTAINING

$$\Delta_{m,n} := \det\left(x_i^a\,\theta_i^b\right)_{\substack{1\le i\le n\\(a,b)\in\gamma_{m,n}}}$$

CLOSED UNDER

- PARTIAL DERIVATIVES
- POLARIZATION

$\boxed{\theta_i \quad \text{INERT VARIABLES (DEGREE 0)}}$

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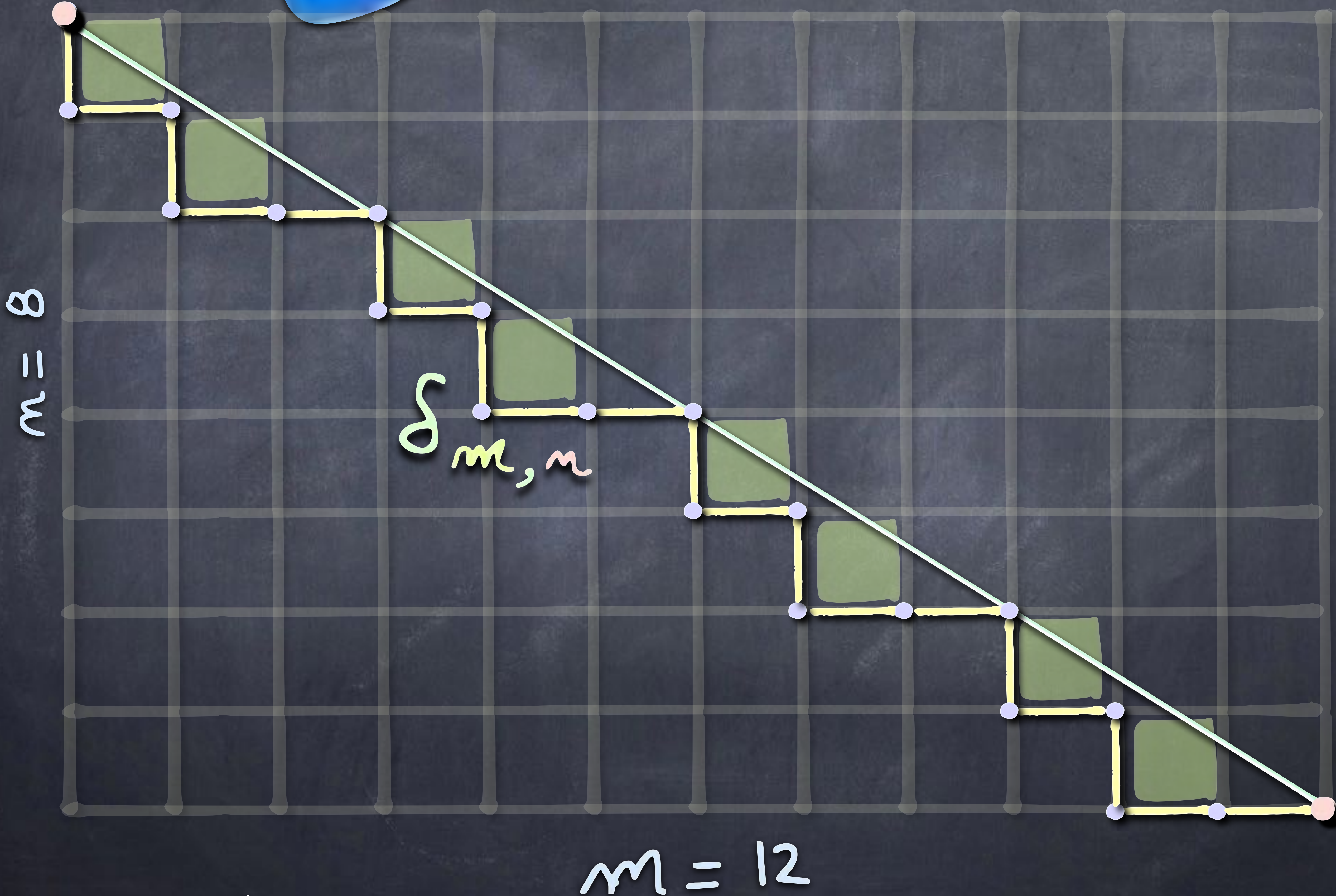
$\gamma_{m,n}$:= LIST OF COORDINATES ASSOCIATED TO PATH



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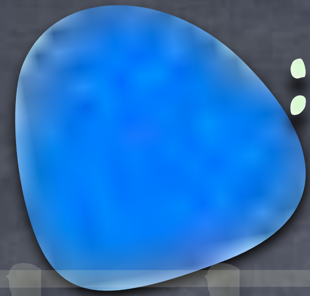
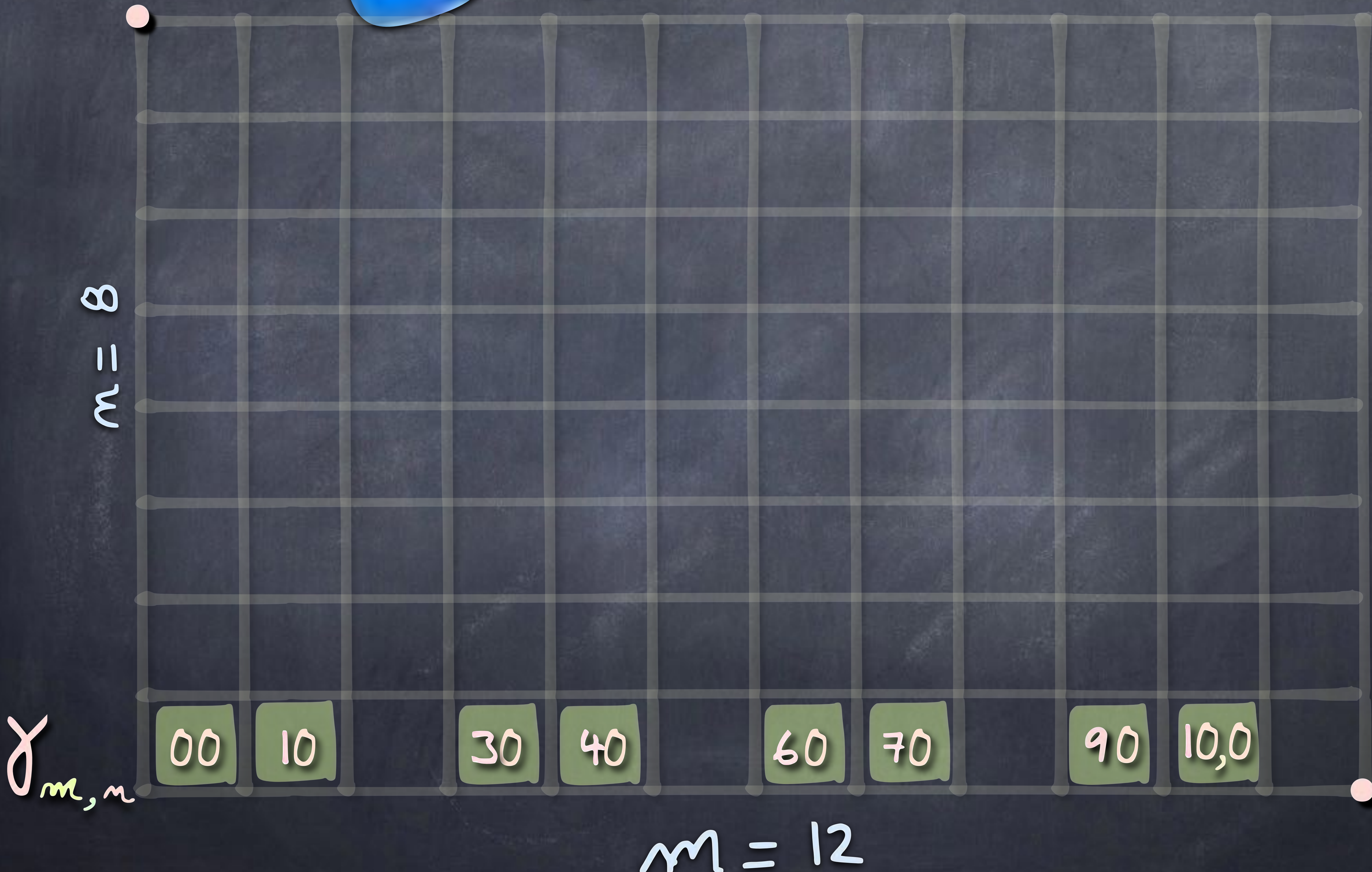


:= LIST OF COORDINATES ASSOCIATED TO PATH



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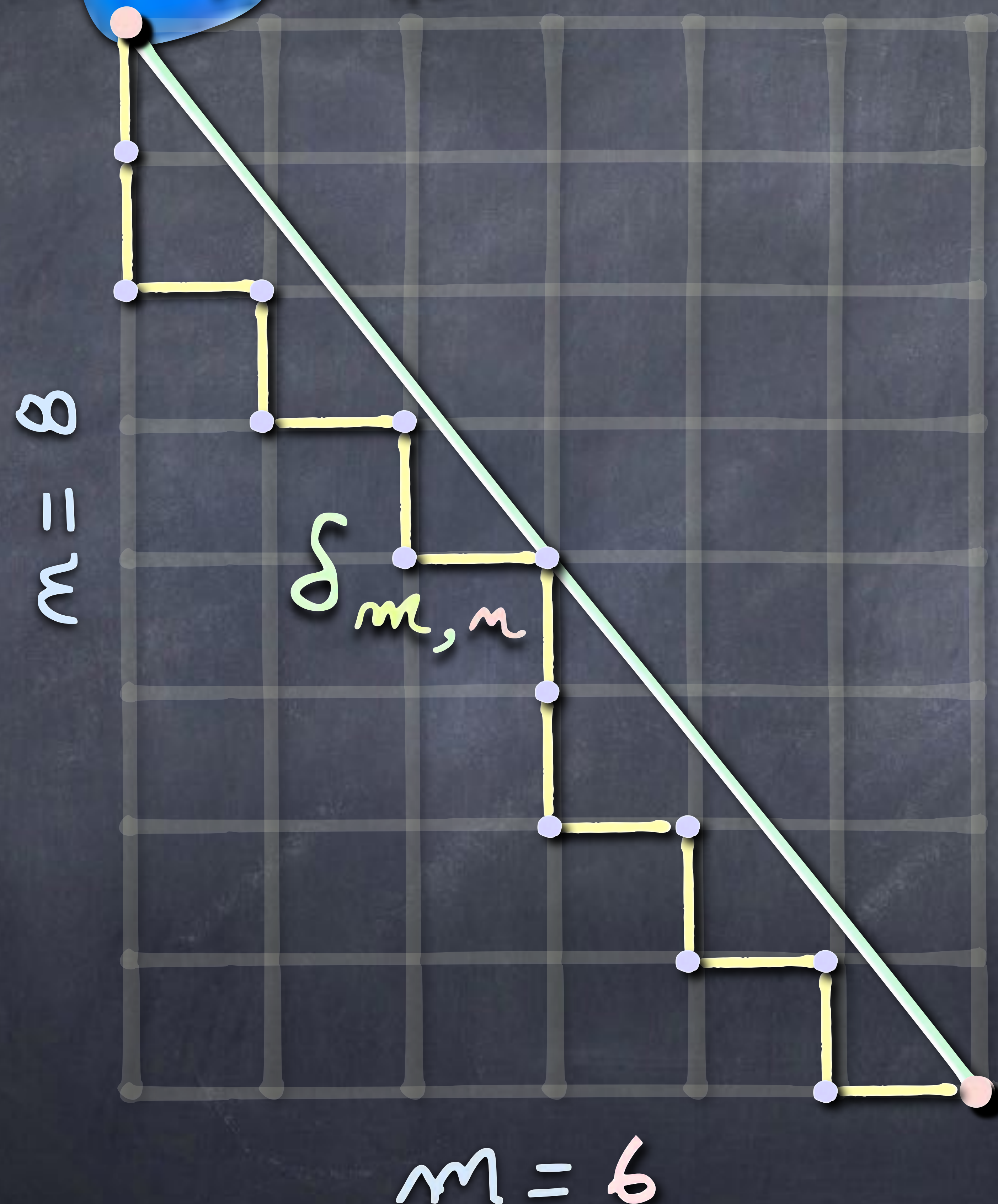
$\gamma_{m,n}$:= LIST OF COORDINATES ASSOCIATED TO PATH



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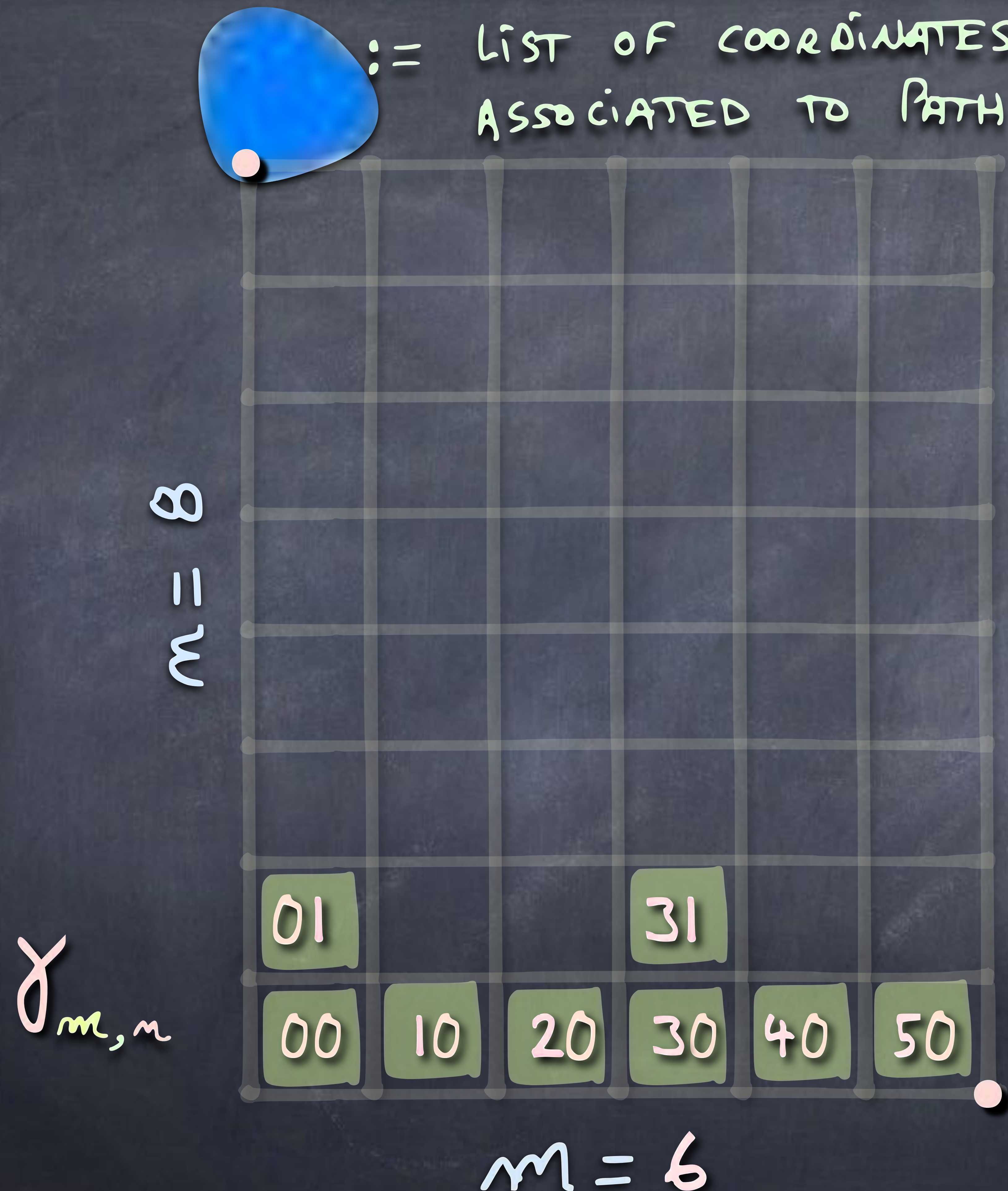
:= LIST OF COORDINATES ASSOCIATED TO PATH
n = 8
$\gamma_{m,n}$
01
31
00
10
20
30
40
50
m = 6

Polarization

Polarization operators

$$\sum_{i=1}^{n} y_i \frac{\partial^k}{\partial x_i^k}$$

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$$\boxed{\mathcal{E}_{m,n} := \mathcal{P}_{m,n}/\mathcal{Q}_{m,n}}$$

$\mathcal{Q}_{m,n}$ SMALLEST MODULE CONTAINING

$\mathcal{F}\,\Delta_{m,n}$

$\mathcal{F}$ ALL DIAGONAL SYMMETRIC DERIVATION

CLOSED UNDER

- PARTIAL DERIVATIVES
- POLARIZATION

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The Module $\mathcal{E}_{n,m}$

Smallest module containing

$$\Delta_{n,m} := \prod_{1 \le i, j \le m} (x_i - x_j)$$

Closed under

- Partial derivatives
- Polarization

A TOY EXAMPLE $\mathcal{E}_{2,2}$

$$\Delta_2 = x_2 - x_1$$

$\boxed{GL_\infty\text{ - ACTION}}$

$$\begin{array}{cccccccc}
 & 1 & & q_1 & q_2 & & q_k & \\
\mathcal{E}_{2,2} = & \mathbb{Q}\{1\} & \oplus\ \mathbb{Q}[& x_2 - x_1, & y_2 - y_1, & \cdots, & z_2 - z_1, & \cdots] \\
 & \circlearrowleft & & \circlearrowleft & \circlearrowleft & & \circlearrowleft & \\
 & s_2 & & s_{11} & s_{11} & & s_{11} &
\end{array}$$

$\boxed{S_n\text{ - ACTION}}$

$$\mathcal{E}_{2,2} = 1 \otimes s_2 + (q_1 + q_2 + \cdots + q_k + \cdots) \otimes s_{11}$$

$$= 1 \otimes s_2 + s_1 \otimes s_{11}$$

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The structure of

irred. for GL_∞ - action

$$\mathcal{E}_{m,n} = \sum_{\mu \vdash n} \sum_{\lambda} c_{\lambda\mu} (\Delta_\lambda \otimes \Delta_\mu), \qquad c_{\lambda\mu} \in \mathbb{N}$$

irred. for $\mathbb{S}_n$ - action

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$$\xi_{m,n} = \cdots + \langle \xi_{m,n}, \Delta_\mu \rangle \otimes \Delta_\mu + \cdots$$

$$\langle \xi_{m,n}, \Delta_\mu \rangle = \sum_\lambda r_{\lambda\mu} \Delta_\lambda$$

$\mathcal{E}_{n-1,n}$

$$\Delta_{n-1,n} := \det \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} & \theta_1 \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} & \theta_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n & \theta_n \end{pmatrix}$$

$$\mathcal{E}_{n-1,n}$$

$$\Delta_{n-1,n} = \sum_{i=1}^{n} (-1)^{i-1}\, \theta_i\, \Delta_{[n]\setminus\{i\}}$$

$$\Delta_I := \prod_{\substack{i<j \\ i,j\in I}} (x_i - x_j)$$

$$[n] := \{1, 2, \dots, n\}$$

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We deduce that

$$\langle \mathcal{E}_{n-1,n}, \Delta_{1^n} \rangle \simeq \langle \mathcal{E}_{n-1}, \Delta_{1^{n-1}} \rangle$$

A TOY EXAMPLE $\mathcal{E}_{2,3}$

$$\Delta_{2,3}(z) = \det \begin{pmatrix} 1 & z_1 & \theta_1 \\ 1 & z_2 & \theta_2 \\ 1 & z_3 & \theta_3 \end{pmatrix}$$

POLARIZATION

$$\Delta_{2,3} = (x_3 - x_2)\,\theta_1 - (x_3 - x_1)\,\theta_2 + (x_2 - x_1)\,\theta_3$$

$$\partial x_1 \Delta_{2,3} = \theta_2 - \theta_3 \qquad \partial x_2 \Delta_{2,3} = \theta_3 - \theta_1$$

A TOY EXAMPLE $\xi_{2,3}$

$$\Delta_{2,3}(z) = \det \begin{pmatrix} 1 & z_1 & \theta_1 \\ 1 & z_2 & \theta_2 \\ 1 & z_3 & \theta_3 \end{pmatrix}$$

POLARIZATION

$$\xi_{2,3} = \overset{1}{\mathbb{Q}\{\theta_2 - \theta_3, \theta_3 - \theta_1\}}$$

$$\oplus\, \mathbb{Q}\{\underset{q_1}{\Delta_{2,3}(x)}, \underset{q_2}{\Delta_{2,3}(y)}, \ldots, \underset{q_k}{\Delta_{2,3}(z)}, \ldots\}$$

$$\xi_{2,3} = 1 \otimes s_{21} + s_1 \otimes s_{111}$$

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A TOY EXAMPLE $\mathcal{E}_{2,3}$

$$\Delta_{2,3}(z) = \det \begin{pmatrix} 1 & z_1 & \theta_1 \\ 1 & z_2 & \theta_2 \\ 1 & z_3 & \theta_3 \end{pmatrix}$$

POLARIZATION

$$\mathcal{E}_{2,3} = \mathbb{Q}\{\overset{1}{\theta_2 - \theta_3, \theta_3 - \theta_1}\}$$

$$\oplus\ \mathbb{Q}\{\underset{q_1}{\Delta_{2,3}(x)}, \underset{q_2}{\Delta_{2,3}(y)}, \ldots, \underset{q_k}{\Delta_{2,3}(z)}, \ldots\}$$

$$\mathcal{E}_{2,3} = 1 \otimes s_{21} + s_1 \otimes s_{111}$$

$$\mathcal{E}_2 = 1 \otimes s_2 + s_1 \otimes s_{11}$$

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$$\mathcal{E}_{2,n} \qquad n = 2k$$

$$\mathbb{Q}\{\partial x_I \Delta_{2,2k} \mid \#I = \ell\}$$

irred. for $\mathbb{S}_n$-action

$$x_I := \prod_{i \in I} x_i$$

$$\Delta_{2^\ell 1^{2(k-\ell)}}$$

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$$\mathcal{E}_{2,n} \qquad n = 2k$$

$$\mathbb{Q}\{\partial x_I \Delta_{2,2k} \mid \#I = \ell\}$$

+ Polarization

$$\Delta_{k-\ell} \otimes \Delta_{2^\ell 1^{2(k-\ell)}}$$

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$n = 2k$

$$\xi_{2,n} = \sum_{\ell=0}^{k} s_{k-\ell} \otimes s_{2^{\ell} 1^{2(k-\ell)}}$$

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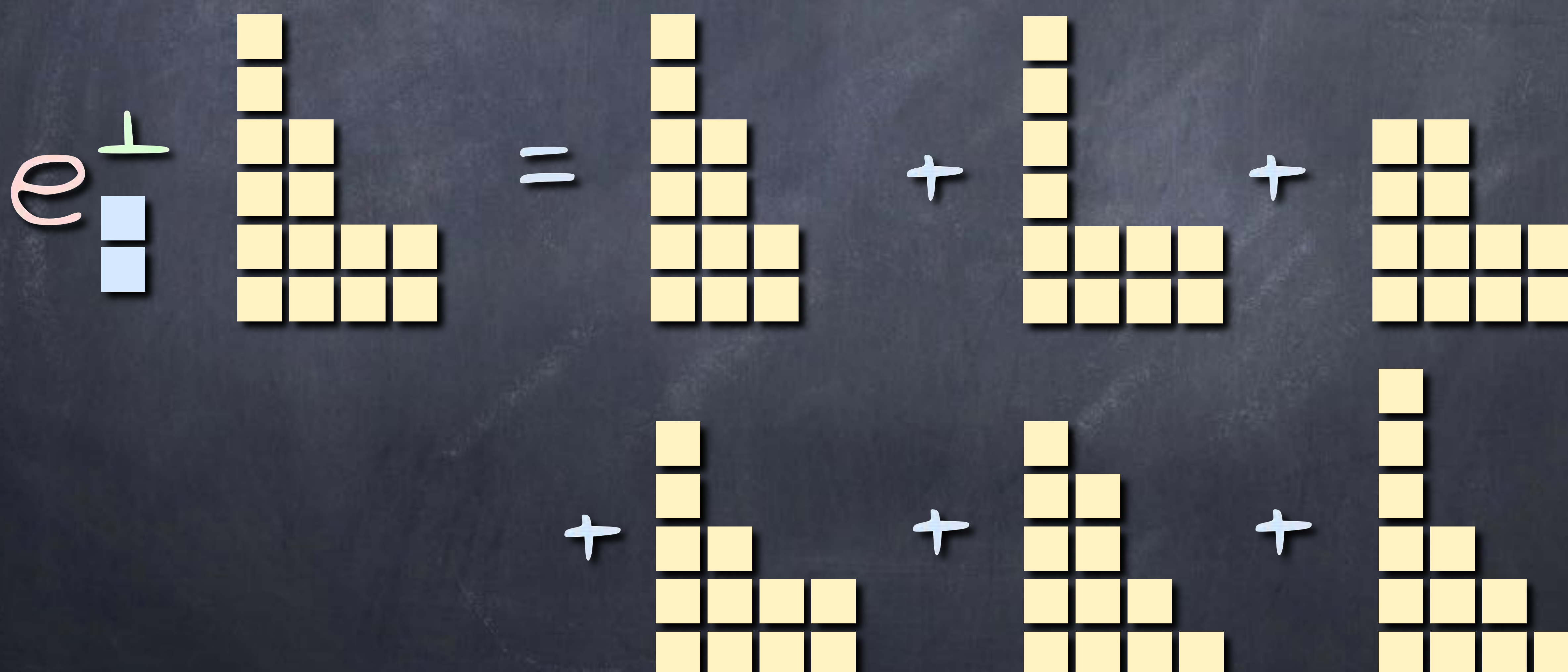
STRUCTURE

(Dual) Pieri Formula

$$e_k^{\perp} \Delta_\lambda = \sum_{\mu \subset_k \lambda} \Delta_\mu$$



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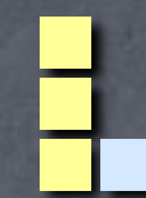
$$(s_{111}+s_{31}+s_{41}+s_{6})$$

$$\downarrow \qquad e_1^{\perp}$$



$$(s_{11}+s_{21}+s_{31}+s_{3}+s_{4}+s_{5})$$

$$\begin{aligned}\xi_4 = 1\otimes s_4 &+ (s_1+s_2+s_3)\otimes s_{31}\\ &+(s_{21}+s_2+s_4)\otimes s_{22}\\ &+(s_{11}+s_{21}+s_{31}+s_3+s_4+s_5)\otimes s_{211}\\ &+(s_{111}+s_{31}+s_{41}+s_6)\otimes s_{1111}\end{aligned}$$

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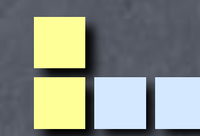
$$(\Delta_{111} + \Delta_{31} + \Delta_{41} + \Delta_{6})$$

$$\downarrow \quad e_2^{\perp}$$



$$(\Delta_{1} + \Delta_{2} + \Delta_{3})$$

$$\begin{aligned}\xi_4 = 1 \otimes \Delta_4 &+ (\Delta_1 + \Delta_2 + \Delta_3) \otimes \Delta_{31} \\ &+ (\Delta_{21} + \Delta_2 + \Delta_4) \otimes \Delta_{22} \\ &+ (\Delta_{11} + \Delta_{21} + \Delta_{31} + \Delta_3 + \Delta_4 + \Delta_5) \otimes \Delta_{211} \\ &+ (\Delta_{111} + \Delta_{31} + \Delta_{41} + \Delta_6) \otimes \Delta_{1111}\end{aligned}$$

$$(\Delta_{111}+\Delta_{31}+\Delta_{41}+\Delta_{6})$$

$$\Big\downarrow \quad e_3^{\perp}$$

$$1$$

$$\begin{aligned}
\xi_4 = 1\otimes\Delta_4 &+ (\Delta_1+\Delta_2+\Delta_3)\otimes\Delta_{31}\\
&+(\Delta_{21}+\Delta_2+\Delta_4)\otimes\Delta_{22}\\
&+(\Delta_{11}+\Delta_{21}+\Delta_{31}+\Delta_3+\Delta_4+\Delta_5)\otimes\Delta_{211}\\
&+(\Delta_{111}+\Delta_{31}+\Delta_{41}+\Delta_6)\otimes\Delta_{1111}
\end{aligned}$$

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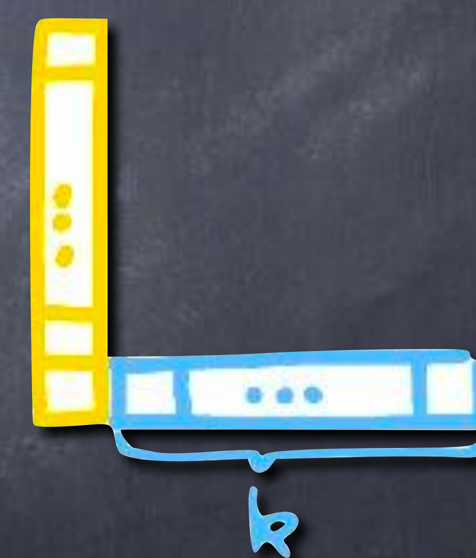
THEOREM

FOR ALL HOOK SHAPES, WE HAVE

$$e_k^{\perp}\langle \mathcal{E}_n, s_{1^n}\rangle = \langle \mathcal{E}_n, s_{(k+1,1^{n-k-1})}\rangle$$

WHERE

$$\mathcal{E}_n = \cdots + \underbrace{\langle \mathcal{E}_n, s_\mu\rangle}_{\boxed{\text{COEFFICIENTS OF } s_\mu}} \otimes s_\mu + \cdots$$



Conjecture

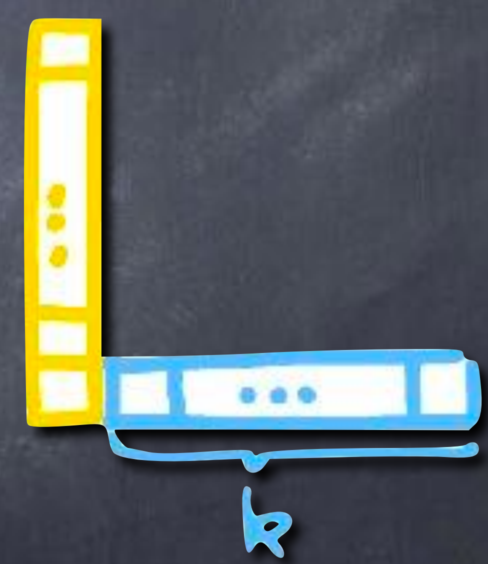
For all hook shapes, we have

$$e_k^{\perp} \langle \xi_{m,n}, s_{1^n} \rangle = \langle \xi_{m,n}, s_{(k+1,1^{n-k-1})} \rangle$$

Where

$$\boxed{\text{coefficients of } s_\mu}$$

$$\xi_{m,n} = \cdots + \langle \xi_{m,n}, s_\mu \rangle \otimes s_\mu + \cdots$$



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Conjecture

For all hook shapes, we have

$$e_k^{\perp}\langle \mathcal{E}_{m,n}, s_{1^n}\rangle = \langle \mathcal{E}_{m,n}, s_{(k+1,1^{n-k-1})}\rangle$$

- $e_k^{\perp}\langle \mathcal{E}_{m,n}, s_{1^n}\rangle = e_k^{\perp}\langle \mathcal{E}_{n,m}, s_{1^m}\rangle$

THE SUPERPOLYNOMIAL OF THE (m,n)-TORUS LINK

$$(1+a)\sum_{k=0}^{n-1} \langle \xi_{m,n}, s_{(k+1,1^{n-k-1})} \rangle a^k$$

$\uparrow$

$\boxed{\text{EVALUATED IN } q,t}$

KHOVANOV-ROZANSKY HOMOLOGY OF (m,n)-TORUS LINKS

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THE SUPERPOLYNOMIAL OF THE (m,n)-TORUS LINK

$$(1+a)\sum_{k=0}^{n-1}\langle \xi_{m,n},\, s_{(k+1,1^{n-k-1})}\rangle\, a^k$$

(m,n)-TORUS LINK $=$ (n,m)-TORUS LINK

KHOVANOV-ROZANSKY HOMOLOGY OF (m,n)-TORUS LINKS

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CONJECTURE

$$\left\langle \mathcal{E}_{m,n},\ s_{(k+1,\,1^{n-k-1})} \right\rangle = \left\langle \mathcal{E}_{n,m},\ s_{(k+1,\,1^{m-k-1})} \right\rangle$$

$$\mathcal{E}_{6,4} = \boxed{s_2} \otimes s_4 + (s_{21} + s_3 + s_{31} + s_4 + s_5) \otimes s_{31}$$
$$+ (s_{111} + s_{22} + s_{31} + s_4 + s_{41} + s_6) \otimes s_{22}$$
$$+ (s_{211} + s_{31} + s_{32} + 2s_{41} + s_5 + s_{51} + s_6 + s_7) \otimes s_{211}$$
$$+ (s_{311} + s_{42} + s_{51} + s_{61} + s_8) \otimes s_{1111}$$

$$\mathcal{E}_{4,6} = s_1 \otimes s_{42} + \boxed{s_2} \otimes s_{411} + s_2 \otimes s_{33}$$
$$+ (s_{11} + s_{21} + s_2 + 2s_3 + s_4) \otimes s_{321}$$
$$+ (s_{21} + s_{31} + s_3 + s_4 + s_5) \otimes s_{3111}$$
$$+ (s_{21} + s_{31} + s_3 + s_5) \otimes s_{222}$$
$$+ (s_{111} + s_{22} + s_{21} + 2s_{31} + s_{41} + 2s_4 + s_5 + s_6) \otimes s_{2211}$$
$$+ (s_{211} + s_{32} + s_{31} + 2s_{41} + s_{51} + s_5 + s_6 + s_7) \otimes s_{21111}$$
$$+ (s_{311} + s_{42} + s_{51} + s_{61} + s_8) \otimes s_{111111}$$

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Theorem For $m = n-1$

$$\langle \xi_{m,n},\, s_{(k+1,\,1^{n-k-1})} \rangle = \langle \xi_{n,m},\, s_{(k+1,\,1^{m-k-1})} \rangle$$

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MANY OTHER PROPERTIES AND IDENTITIES

- Lift of Elliptic Hall Algebra
- Lift of Delta-Conjecture and Rectangular Generalization
- General formula for hook components
- An intriguing general e-positivity property
- Various inclusions

Fin

Christian Krattenthaler's Home Page

I am one of the three editors-in-chief of the Séminaire Lotharingien de Combinatoire

Séminaire Lotharingien de Combinatoire

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Preface

[B14Sa] Francois Bergeron
Une combinatoire du piéthysme
[B14Sb] Arne Dür
The Authomorphism Group of Reed-Solomon Codes
[B14Sc] Joe Gillis, J. Jedwab an Doron Zeilberger
A Combinatorial Interpretation of the Integral of the Product of Legendre Polynomials
[B14Sd] Andreas Dress and Christian Siebeneicher
The Cyclotomic Idenity and the Cyclic Group
[B14Se] Giuseppe Pirillo
On Hales-Jewett's Theorem

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