COMMENT ON 'A DECOMPOSITION OF SCHUR FUNCTIONS AND AN ANALOGUE OF THE ROBINSON-SCHENSTED-KNUTH ALGORITHM'

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The purpose of this comment is to clarify the connections between Demazure characters and the objects studied in this work. The nonsymmetric Macdonald polynomials introduced by Macdonald [7] and studied by Cherednick [1] are denoted by $E_\alpha(X;q,t)$, where α is a weak composition and $X = (x_1, x_2, \ldots)$. The Demazure characters introduced by Demazure in [2] and studied by Ion [3], Joseph [4], and Sanderson [10] are the specializations $E_\alpha(X;0,0)$.

Marshall [8] works with a variation of the above nonsymmetric polynomials obtained by reversing the indexing composition, reversing the variables, and replacing q and t by q^{-1} and t^{-1} respectively. These nonsymmetric polynomials, denoted $\hat{E}_\alpha(X;q,t)$, can therefore be written as $\hat{E}_\alpha(x_1, x_2, \ldots; q, t) = E_{\text{reverse}(\alpha)}(\ldots, x_2, x_1; q^{-1}, t^{-1})$. It is these polynomials that we specialize to obtain the polynomials explored in this paper. In fact, the specializations of the $\hat{E}_\alpha(X;q,t)$ to $q = t = 0$ are equivalent to the second family of Demazure characters, often called "standard bases" or "Demazure atoms", introduced by Lascoux and Schützenberger in [5] and studied by Lascoux in [6]. Please see [9] for a combinatorial proof of this equivalence.

We provide the following short table for the partition $\lambda = (2,1,0)$ to illustrate the distinction between $E_\alpha(X;0,0)$ and $\hat{E}_\alpha(X;0,0)$.

Composition α	$E_\alpha(X;0,0)$	$\hat{E}_\alpha(X;0,0)$
$(2,1,0)$	$x_1^2x_2$	$x_1^2x_2$
$(2,0,1)$	$x_1^2x_2 + x_1^2x_3$	$x_1^2x_3$
$(1,2,0)$	$x_1^2x_2 + x_1x_2^2$	$x_1x_2^2$
$(1,0,2)$	$x_1^2x_2 + x_1x_2^2 + x_1^2x_3 + x_1x_2x_3 + x_1x_3^2$	$x_1x_2x_3 + x_1x_3^2$
$(0,2,1)$	$x_1^2x_2 + x_1x_2^2 + x_1^2x_3 + x_1x_2x_3 + x_2^2x_3$	$x_1x_2x_3 + x_2^2x_3$
$(0,1,2)$	$x_1^2x_2 + x_1x_2^2 + x_1^2x_3 + 2x_1x_2x_3 + x_2^2x_3 + x_1x_3^2 + x_2x_3^2$	$x_2x_3^2$

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