61e SÉMINAIRE LOTHARINGIEN DE COMBINATOIRE
Grande Hotel da Curia, Anadia (Portugal)
21-24 septembre 2008

OPÉRATEURS DIFFÉRENTIELS COMBINATOIRES GÉNÉRAUX

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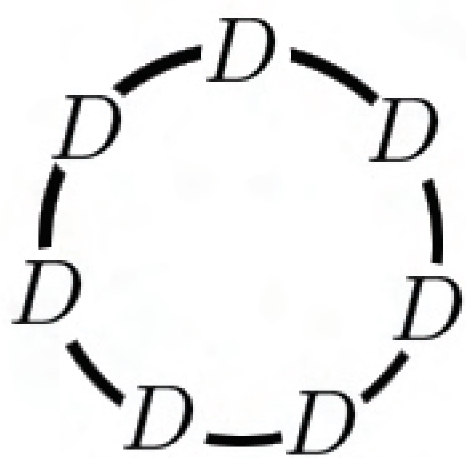
LaCIM - UQÀM

À la mémoire de Pierre Leroux (1942-2008)

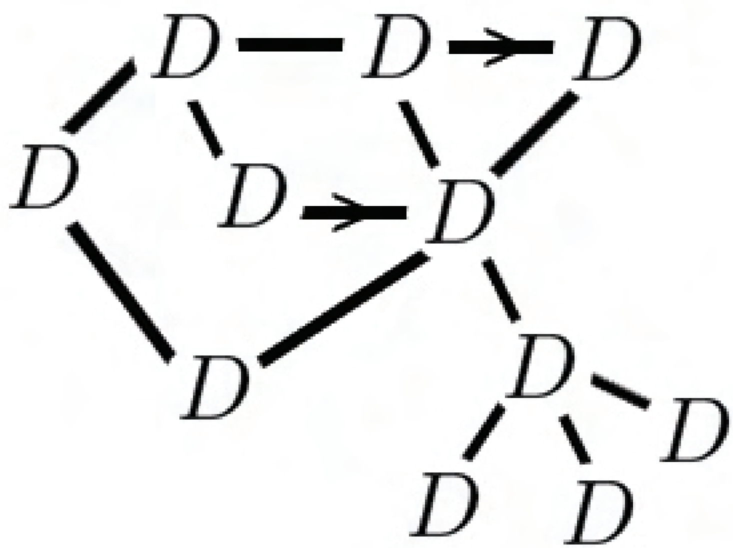
$$D = \mathrm{d}/dX$$

$$D^n = DD \cdots D = \frac{d^n}{dX^n}$$



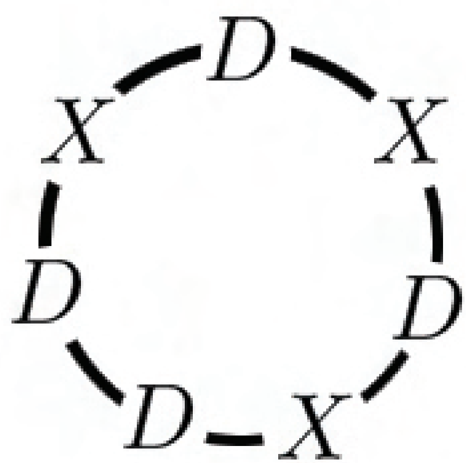
$= ?$

$$D = d/dX$$



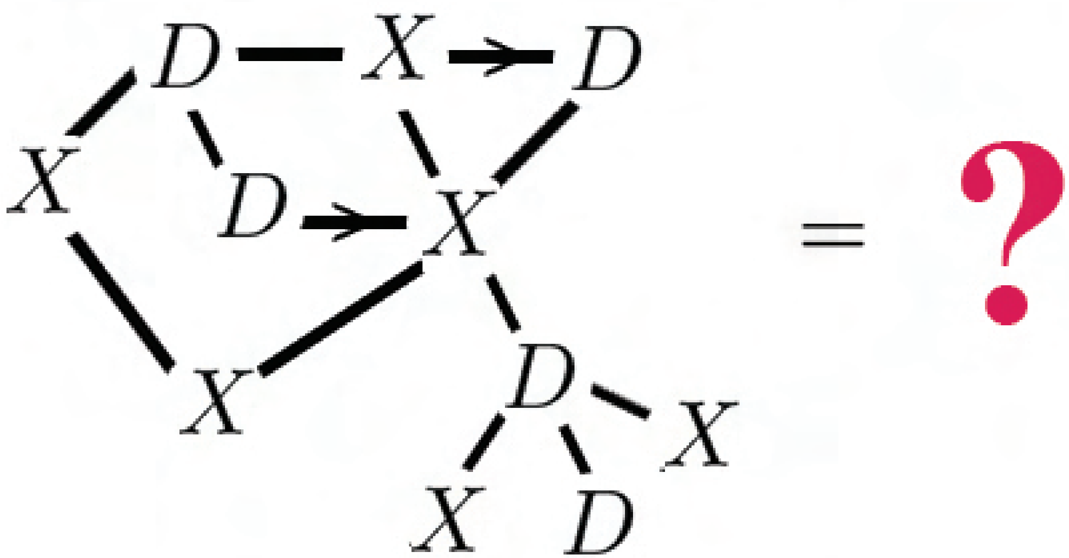
$= ?$

$$D = d/dX$$



$= ?$

$$D = d/dX$$

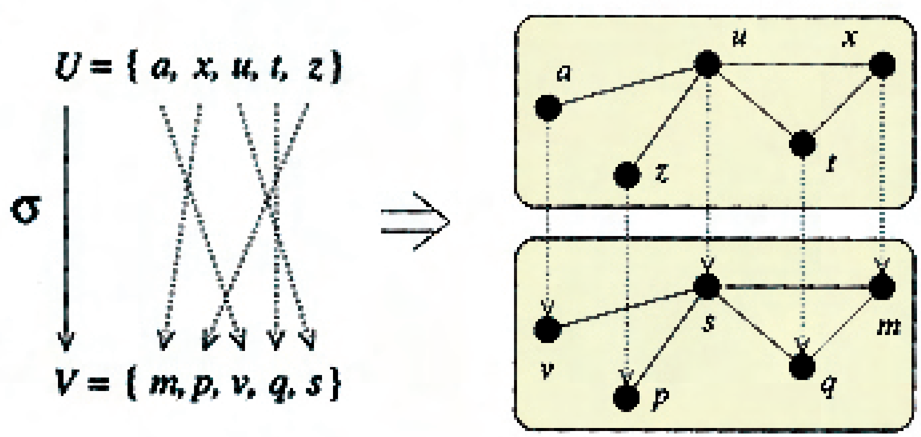


Soit F une classe de structures sur les ensembles finis.

Si F est fermée sous les isomorphismes, on dit que F est une *espèce de structures*.

Une structure dans F est appelée une F-structure.

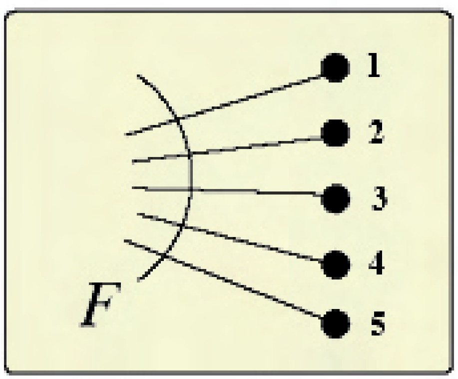
Exemple. La classe G de tous les graphes finis.



SÉRIES ASSOCIÉES À UNE ESPÈCE *F*

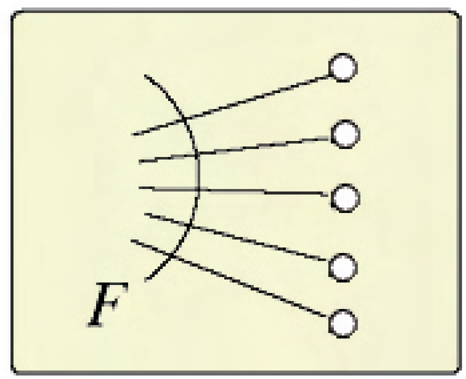
$$F(x) = \sum_{n \geq 0} f_n \frac{x^n}{n!}$$

f_n = nombre de *F*-structures sur { 1, 2, ..., *n* }

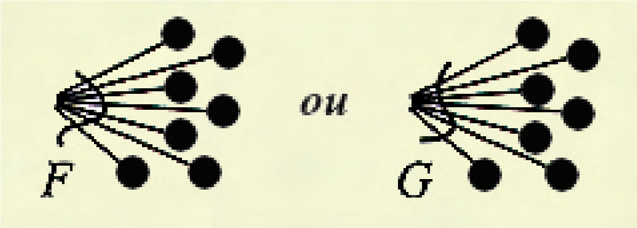
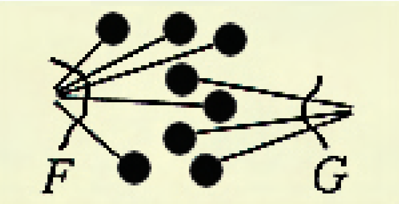
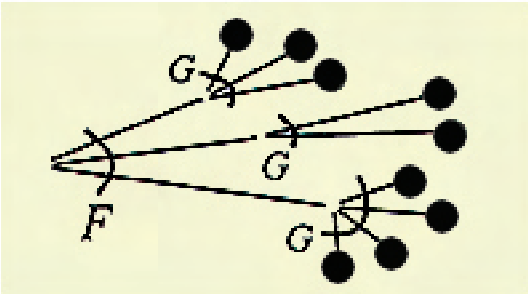
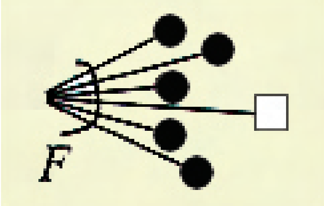


$$\widetilde{F}(x) = \sum_{n \geq 0} \tilde{f}_n \, x^n$$

$\tilde{f}_n$ = nombre de *F*-structures sur *n* éléments *identiques*



PRINCIPALES OPÉRATIONS SUR LES ESPÈCES

ADDITION	$(F+G)$-*structure* :	
MULTIPLICATION	$(F{\cdot}G)$-*structure* :	
SUBSTITUTION	$(F{\circ}G)$-*structure* :	
DÉRIVATION	F'-*structure* :	

COMPORTEMENT AU NIVEAU DES SÉRIES

$$(F+G)(x) = F(x)+G(x), \quad (F\cdot G)(x) = F(x)\cdot G(x)$$

$$(F\circ G)(x) = F(G(x)), \quad F'(x) = \frac{d}{dx}F(x)$$

$$(\widetilde{F+G})(x) = \widetilde{F}(x)+\widetilde{G}(x), \quad (\widetilde{F\cdot G})(x) = \widetilde{F}(x)\cdot\widetilde{G}(x)$$

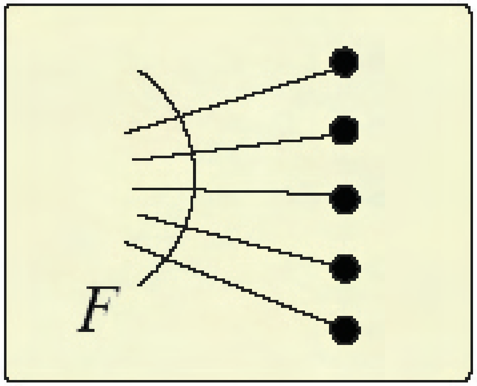
$$(\widetilde{F\circ G})(x) \neq \widetilde{F}(\widetilde{G}(x)), \quad \widetilde{F}'(x) \neq \frac{d}{dx}\widetilde{F}(x)$$

SÉRIE INDICATRICE

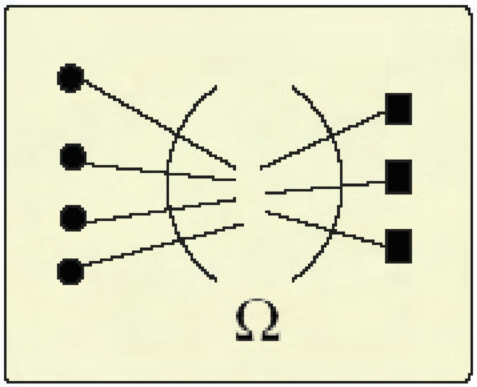
$$Z_F(x_1, x_2, x_3, \ldots) = \sum_{n \geq 0} \frac{1}{n!} \sum_{\sigma \in \mathbb{S}_n} \mathrm{fix} F[\sigma]\, x_1^{\sigma_1} x_2^{\sigma_2} x_3^{\sigma_3} \ldots$$

$$F(x) = Z_F(x, 0, 0, \ldots)$$

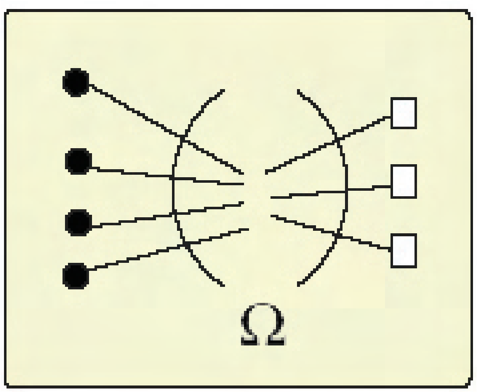
$$\widetilde{F}(x) = Z_F(x, x^2, x^3, \ldots)$$



Une $F(X)$-structure sur
5 éléments • de sorte X



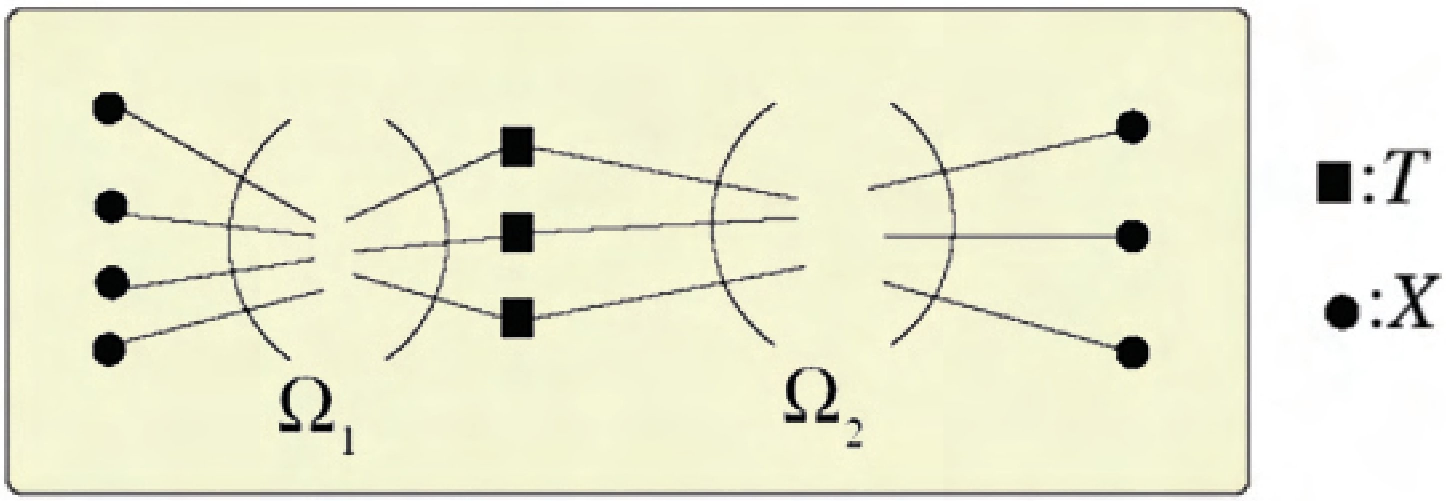
Une $\Omega(X,T)$-structure sur
4 éléments • de sorte X et
3 éléments ■ de sorte T



Une $\Omega(X,1)$-structure sur
4 éléments • de sorte X et
3 éléments □ *identiques*
de sorte T

PRODUIT CARTÉSIEN PARTIEL

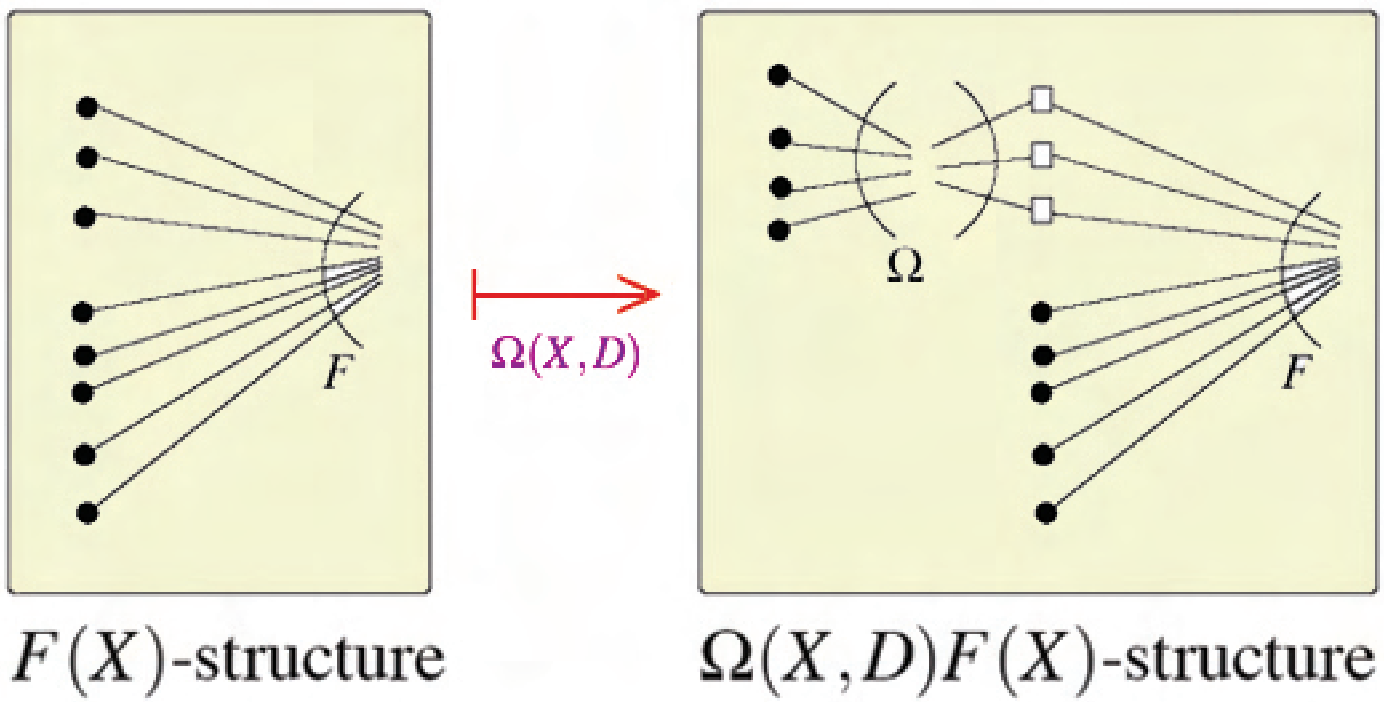
$$\Omega_1(X,T) \times_T \Omega_2(X,T)$$

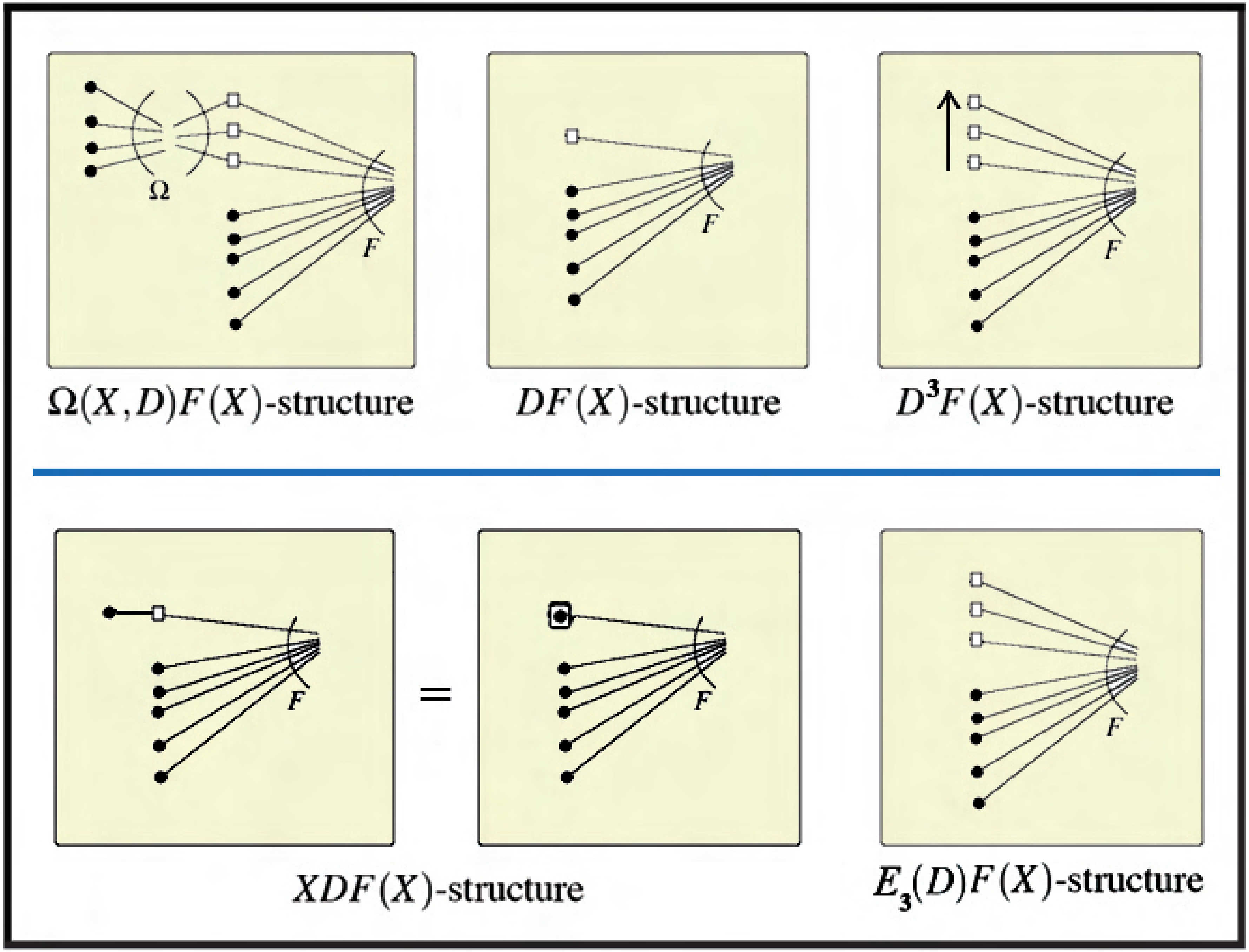


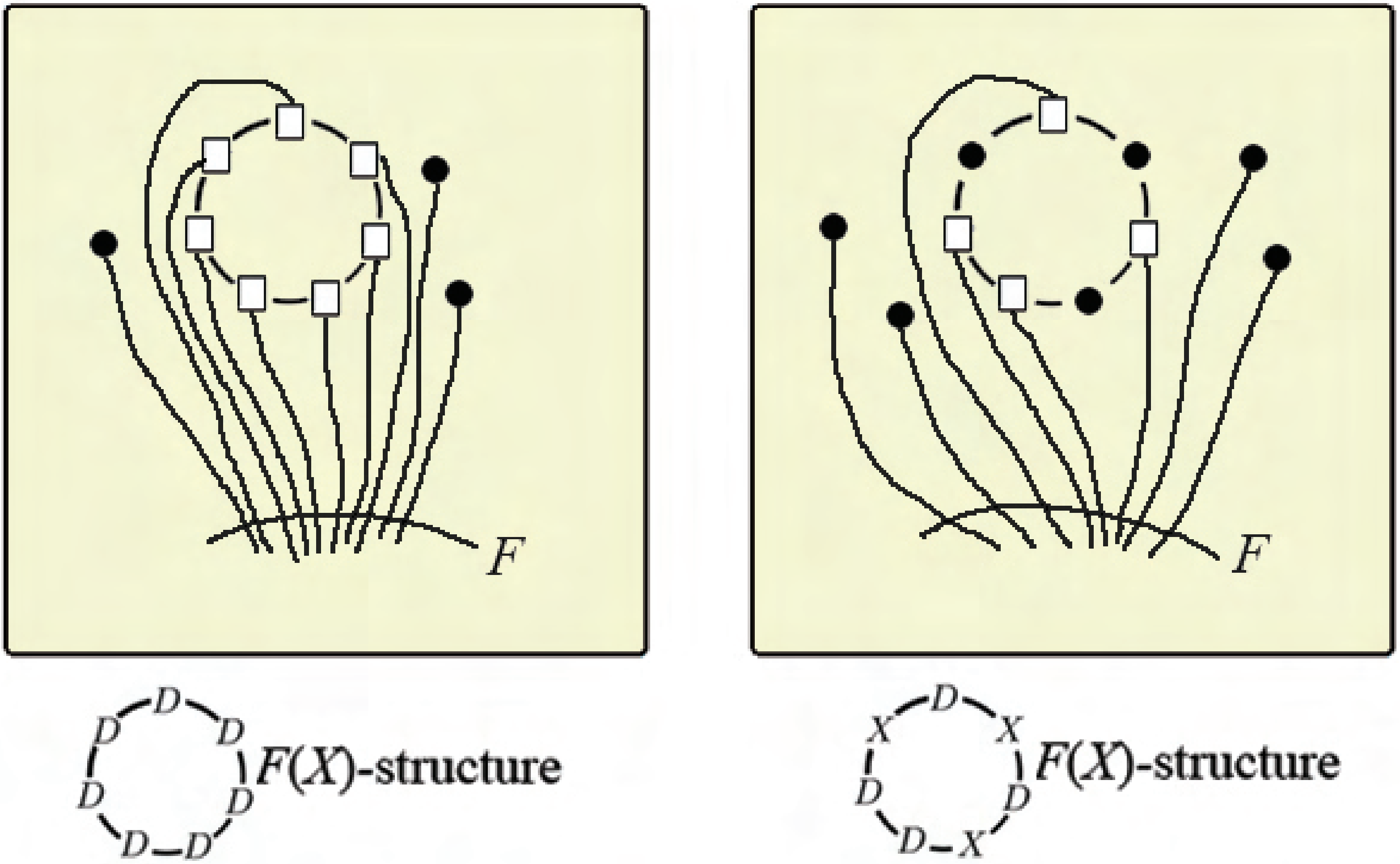
$\Omega_1(X,T) \times_T \Omega_2(X,T)$-structure

OPÉRATEURS DIFFÉRENTIELS COMBINATOIRES GÉNÉRAUX

$$\Omega(X,D)F(X) := \Omega(X,T) \times_T F(X+T)|_{T:=1}$$




Ω
F
Ω(X, D)F(X)-structure
F
DF(X)-structure
F
D³F(X)-structure
F
=
F
XDF(X)-structure
F
E₃(D)F(X)-structure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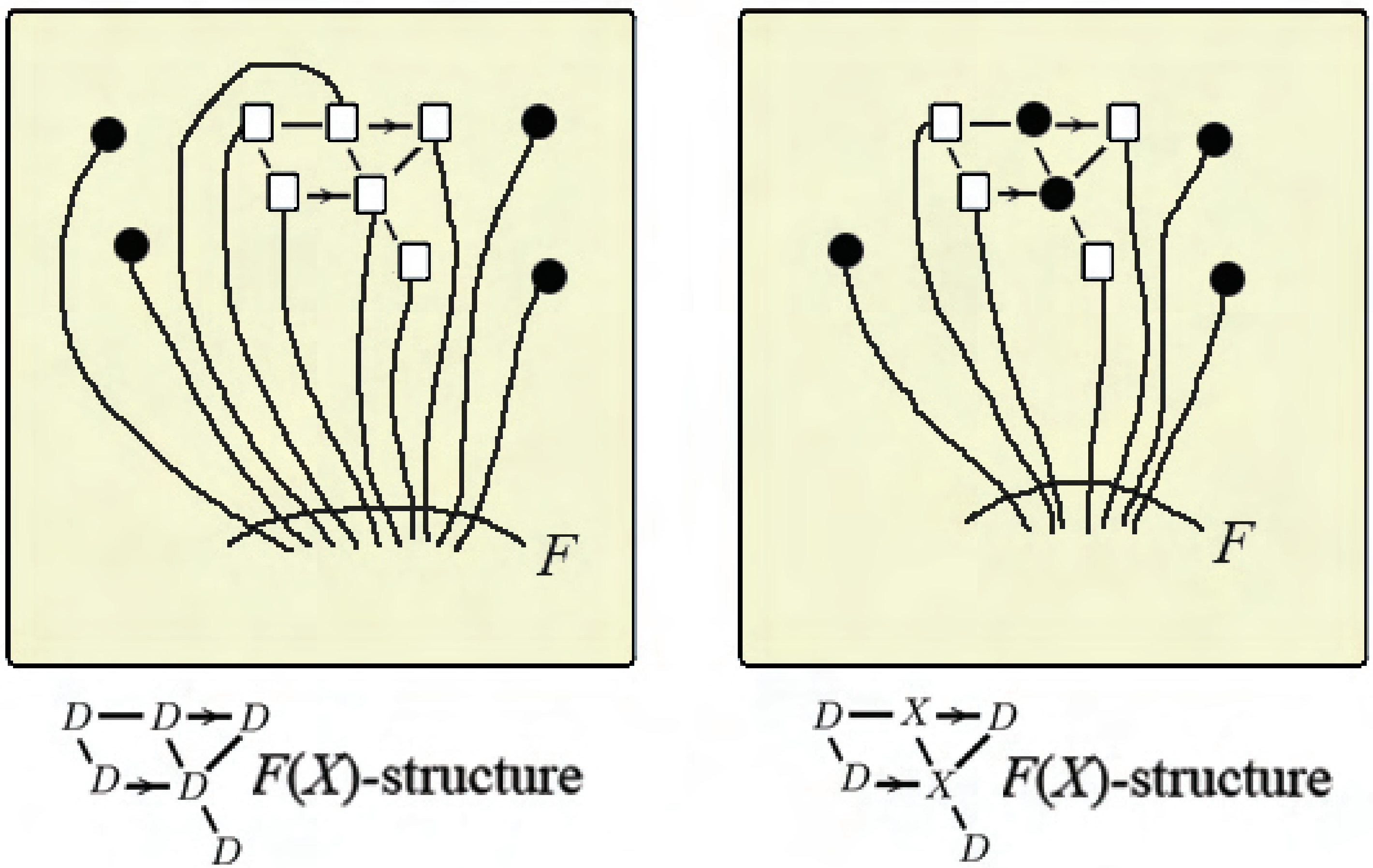
F
D D D D D D D D
F(X)-structure
D X X D D X D D
F(X)-structure



F
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F(X)-structure
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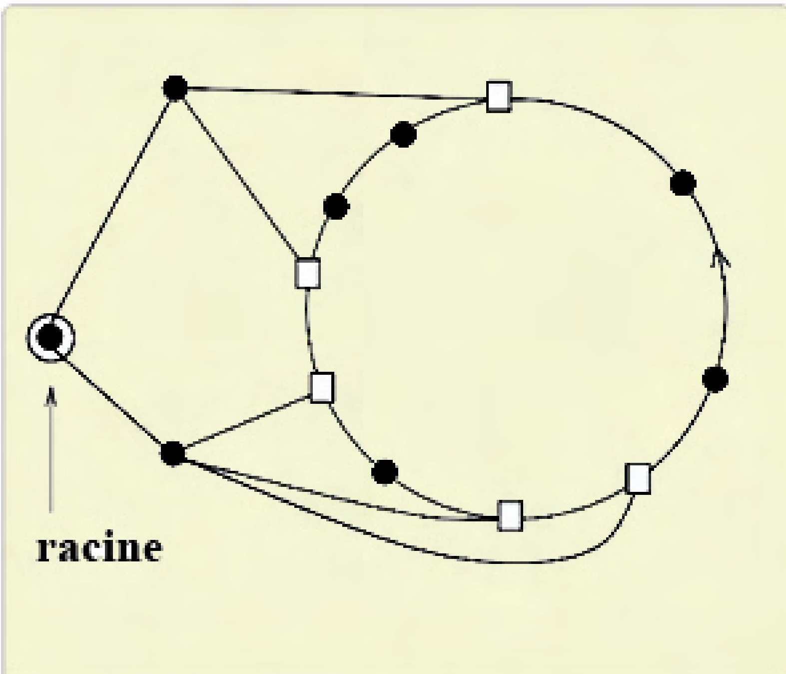
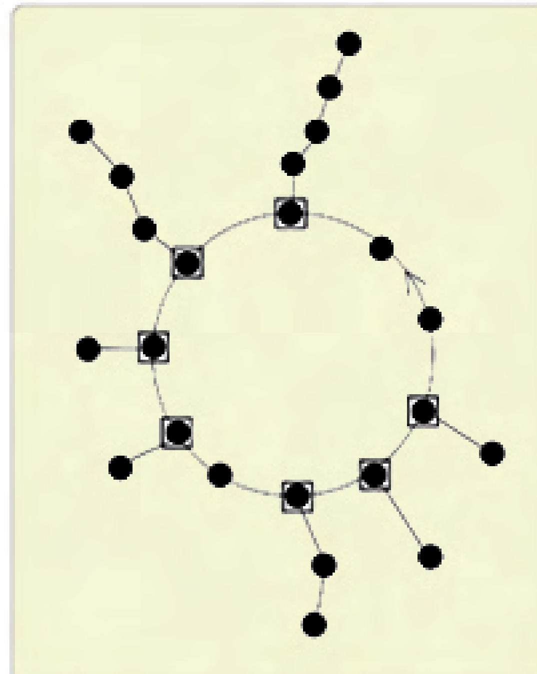
F(X)-structure



$A(X,D)C_{10}(X)$-structure

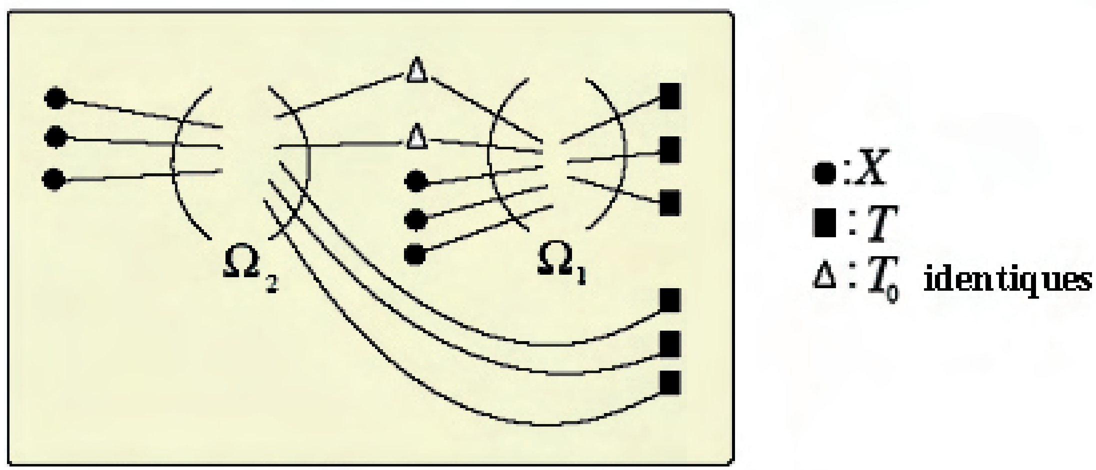


$E(L_{\geq 2}(X)D)C(X)$
-structure
(pieuvre)

COMPOSITION DES OPÉRATEURS DIFFÉRENTIELS COMBINATOIRES

$$\Omega_3(X,D) = \Omega_2(X,D) \odot \Omega_1(X,D)$$

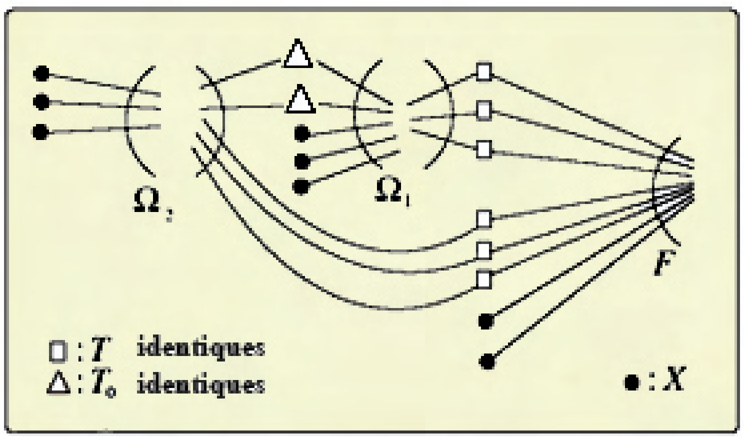
$$\Omega_3(X,T) := \Omega_2(X,T+T_0) \times_{T_0} \Omega_1(X+T_0,T)|_{T_0:=1}$$



$\Omega_3(X,T)$-structure

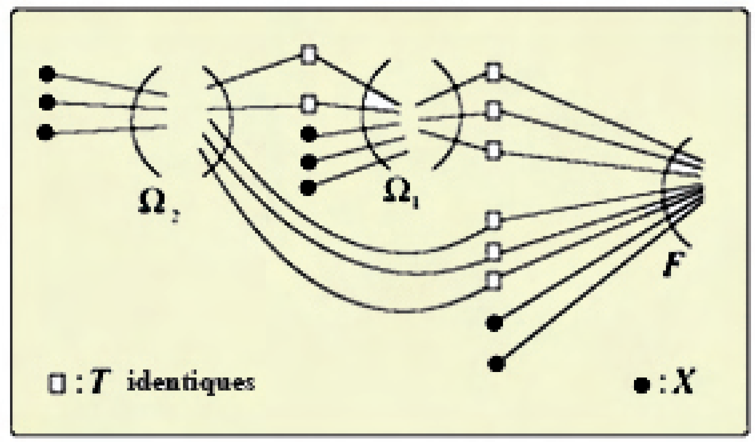
PROPOSITION

$$[\Omega_2(X,T) \odot \Omega_1(X,T)]\, F(X) = \Omega_2(X,D)\, [\Omega_1(X,D) F(X)]$$



[Ω₂(X, T) ⊙ Ω₁(X, T)] F(X)-structure

$\longleftrightarrow$



Ω₂(X, D) [Ω₁(X, D)F(X)]-structure

PROPOSITIONS

$$G(X) = \Omega(X, D)F(X)$$

$$\Downarrow$$

$$Z_G(x_1, x_2, x_3, \ldots) = Z_\Omega(x_1, x_2, x_3, \ldots; \frac{\partial}{\partial x_1}, 2\frac{\partial}{\partial x_2}, 3\frac{\partial}{\partial x_3}, \ldots)Z_F(x_1, x_2, x_3, \ldots)$$

$$Z_{\Omega_2 \odot \Omega_1} = \sum_{n_1, n_2, \ldots} \frac{\left((\frac{\partial}{\partial t_1})^{n_1}(2\frac{\partial}{\partial t_2})^{n_2} \ldots Z_{\Omega_2}\right)\left((\frac{\partial}{\partial x_1})^{n_1}(2\frac{\partial}{\partial x_2})^{n_2} \ldots Z_{\Omega_1}\right)}{1^{n_1}n_1!2^{n_2}n_2! \ldots}$$

Règle de Leibniz généralisée, opérateurs différentiels adjoints, etc.

CAS SPÉCIAUX

Version combinatoire des opérateurs de Hammond

$$\Omega(X,D) = \Theta(D)$$

Opérateurs de multiplication combinatoires

$$\Omega(X,D) = G(X)$$

Opérateurs de différences finies combinatoires

$$\Omega(X,D) = \Phi(X,\Delta), \quad \Delta = E_{+}(D)$$

Opérateurs de pointages combinatoires

$$\Omega(X,D) = \Lambda(XD)$$

Opérateurs différentiels « classiques », etc

FORMULES GÉNÉRALES

$$\Omega(X,D)F(X) := \Omega(X,T) \times_T F(X+T)|_{T:=1}$$

$$\frac{X^a D^k}{A} \frac{X^n}{H} = \frac{X^a T^k}{A} \times_T \frac{(X+T)^n}{H}|_{T:=1}$$

$$\frac{(X+T)^n}{H} = \sum_{k=0}^{n} \sum_{\omega \,\underline{\in}\, S_{n-k,k} \backslash S_n / H} \frac{X^{n-k} T^k}{\omega H \omega^{-1} \cap S_{n-k,k}}$$

$$\frac{X^a T^k}{A} \times_T \frac{X^b T^k}{B} = \sum_{\tau \,\underline{\in}\, (\pi_2 A) \backslash S_k / (\pi_2 B)} \frac{X^{a+b} T^k}{A \times_{S_k} B^\tau}, \quad \text{where } B^\tau = (id_{[b]}, \tau) B (id_{[b]}, \tau^{-1}).$$

$$\left[\frac{X^a T^k}{A}\right]_{T:=1} = \frac{X^a}{\pi_1 A}$$

LABELLE, G., LAMATHE. C.
"A Theory of general combinatorial differential operators",
19th Conference on
Formal Power Series and Algebraic Combinatorics,
FPSAC07, Nankai University, Tianjin, Chine;
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http://www.fpsac.cn/conpap.htm

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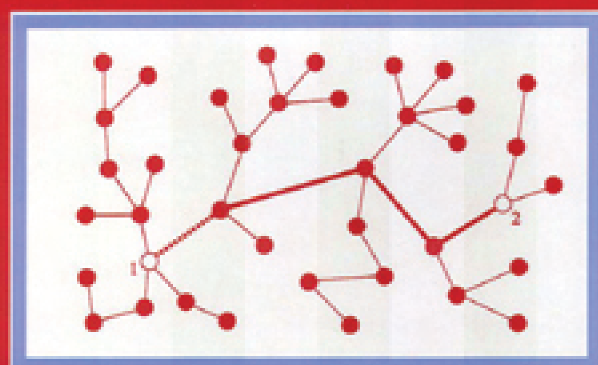

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