

k -triangulations and k -fans of Dyck paths

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This is joint work with Luis Serrano from LaCIM.

You can find the content of this talk (and even more)
in the following papers:

- ▶ A new perspective on k -triangulations ([arXiv:1009.4101](https://arxiv.org/abs/1009.4101))
- ▶ Maximal fillings of moon polyominoes, simplicial complexes, Schubert polynomials (with Luis Serrano, [arXiv:1009.4690](https://arxiv.org/abs/1009.4690))

A bijection between k -triangulations and k -fans of Dyck paths

Theorem

There exists an explicit bijection between k -triangulations of a convex n -gon and k -fans of Dyck paths of semi-length $n - 2k$.

A bijection between k -triangs and k -fans of Dyck paths

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k -triangs $\xrightarrow{\sim}$ k -NE-fillings of a shape λ

$\xrightarrow{\sim}$ reduced pipe dreams for $\sigma_k(\lambda)$

$\xrightarrow{\sim}$ compatible sequences for $\sigma_k(\lambda)$

$\xrightarrow{\sim}$ flagged tableaux for $\lambda^{\text{del } k}$

$\xrightarrow{\sim}$ k -bounded reversed plane partitions for $\lambda^{\text{del } k}$

$\xrightarrow{\sim}$ k -fans of “Dyck paths”

$\xrightarrow{\sim}$ k -SE-fillings of λ

k -triangulations and fillings of shapes

Definition

- ▶ A triangulation of a convex n -gon is what you think it is.
- ▶ A k -triangulation of a convex n -gon is a maximal collection of diagonals in the n -gon not containing a $(k + 1)$ -subset of pairwise crossing diagonals.
- ▶ A k -NE-filling of a shape λ is a maximal $(+,)$ -filling of λ not containing a “NE-chain” of length $k + 1$.

Pipe dreams, compatible sequences, and flagged tableaux

Definition

- ▶ A reduced pipe dream of a permutation $\sigma \in \mathcal{S}_n$ is a $(+, \curvearrowright)$ -filling of the staircase shape $(n-1, \dots, 2, 1)$ which defines a reduced braid for σ .
 - ▶ Introduced by N. Bergeron and S. Billey to combinatorially describe Lascoux-Schützenberger's Schubert polynomials
- ▶ A compatible sequence for σ is an array $(\begin{smallmatrix} a_1, \dots, a_\ell \\ b_1, \dots, b_\ell \end{smallmatrix})$ such that b_1, \dots, b_ℓ is a reduced word for σ plus simple properties.
 - ▶ Introduced by S. Billey, W. Jockush and R. Stanley
- ▶ A k -flagged tableau of shape μ is defined to be a semi-standard tableau for which the i th row is bounded by $k+1$
 - ▶ Introduced by M. Wachs in the context of flagged Schur functions as Schubert polynomials

From compatible sequences to flagged tableaux

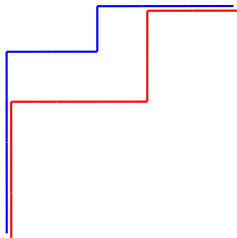
$$\begin{pmatrix} 1 & 1 & 2 & 3 & 3 & 3 & 3 & 5 & 5 & 6 & 6 \\ 5 & 4 & 3 & 6 & 5 & 4 & 3 & 6 & 5 & 7 & 6 \end{pmatrix}$$

3	4	5	6
4	5	6	
5	6		
6	7		

$1 \leq$	1	1	2	3	≤ 3
$2 \leq$	3	3	3		≤ 4
$3 \leq$	5	5			≤ 5
$4 \leq$	6	6			≤ 6

From k -flagged tableaux of shape $\lambda^{\text{del } k}$ to k -SE-fillings of λ

0	0	1	2
1	1	1	
2	2		
2	2		



		+	+	+	+
+	+	+		+	+
+				+	
+	+	+	+	+	
+	+				
+	+				

What can the first steps in the bijection be used for?

Theorem

The **simplicial complex** with facets being k -NE-fillings of shape λ is **vertex-decomposable** and thus **shellable** and **CM** (generalizing the case of the **dual associahedron**).

Theorem

Rotation of the n -gon induces a **cyclic action** on k -triangulations. This action corresponds to **flagged promotion** on k -flagged tableaux of shape $\lambda^{\text{del } a}$.

This transfers a conjectured **cyclic sieving phenomenon** on k -triangulations to k -flagged tableaux.