Binary relations lattice

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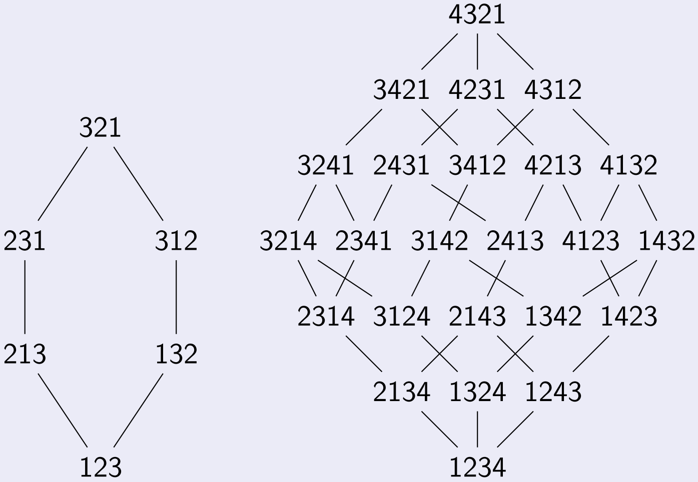
joint work with Joel Gay, Vincent Pilaud et Viviane Pons

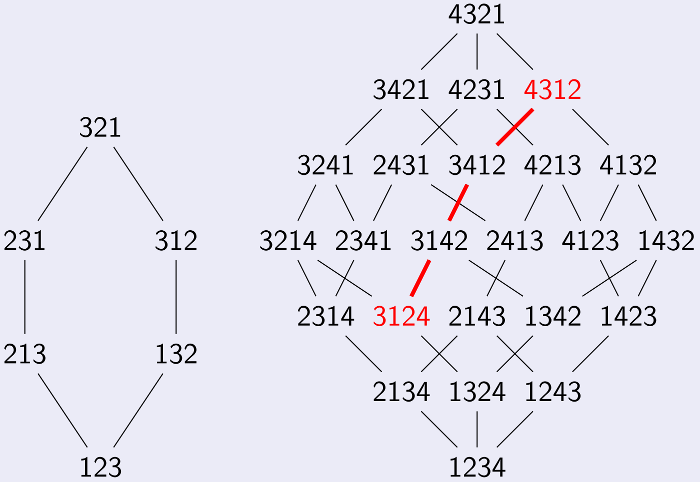
Université Paris-Est Marne-la-Vallée

SLC 76 - April 2016

Right weak order



Right weak order



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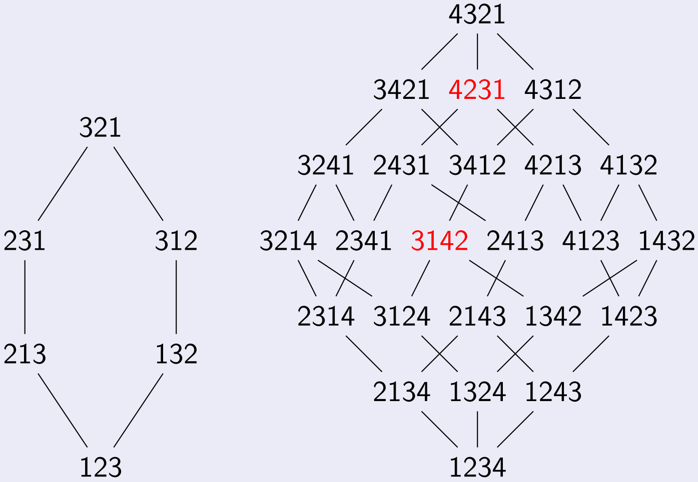

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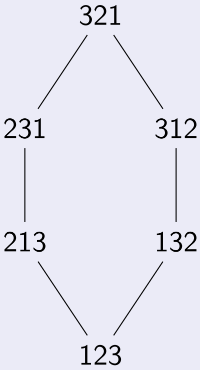
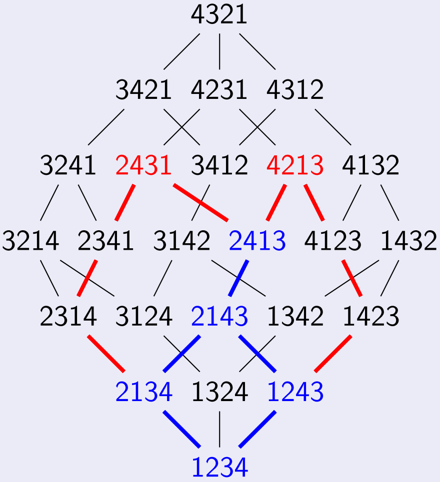
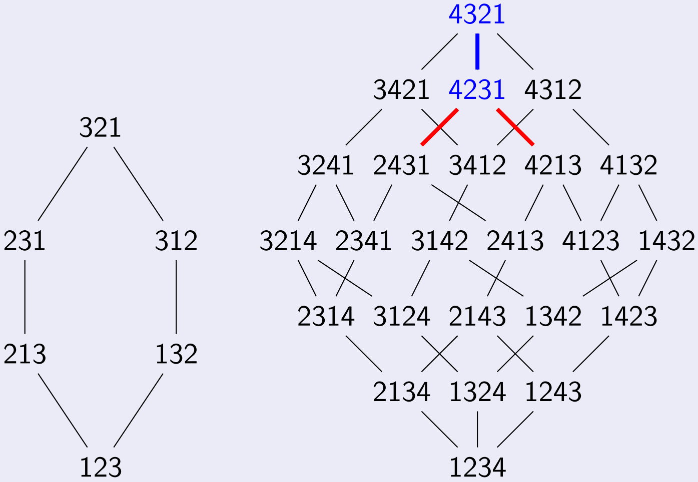




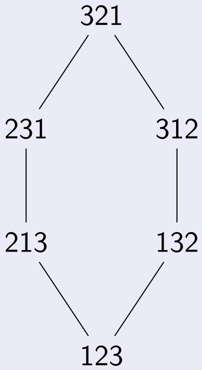
$2413 \wedge 4213 = 2413$

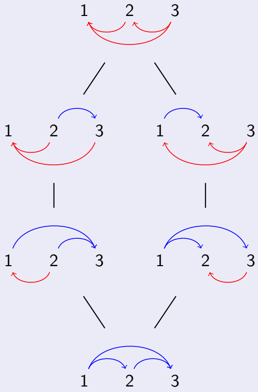
Right weak order



$2413 \wedge 4213 = 2413$ $\qquad$ $2413 \vee 4213 = 4231$

Right weak order on permutation graphs





Binary relations on integer

Let R be a relation of size n.

$$1 \quad 2 \quad \cdots \quad i \quad \cdots \quad j \quad \cdots \quad k \quad \cdots \quad n$$

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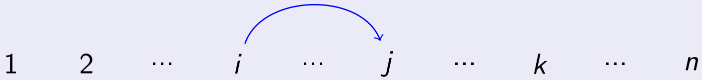

$i \mathrel{\mathrm{R}} j$

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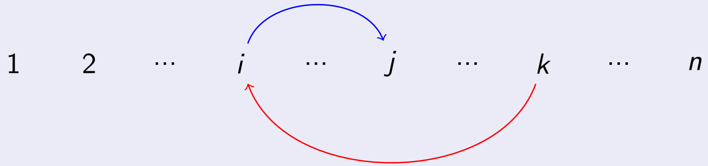

$i \,\mathrm{R}\, j$ $\qquad$ $k \,\mathrm{R}\, i$

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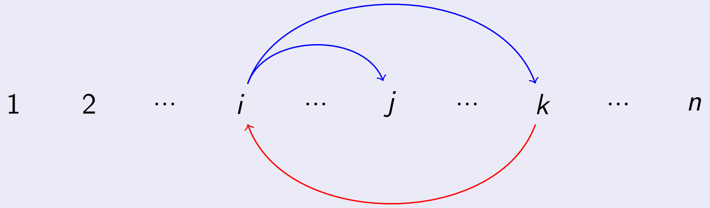

$i \,\mathrm{R}\, j$ $\quad$ $k \,\mathrm{R}\, i$ $\quad$ $i \,\mathrm{R}\, k$

Binary relations on integer

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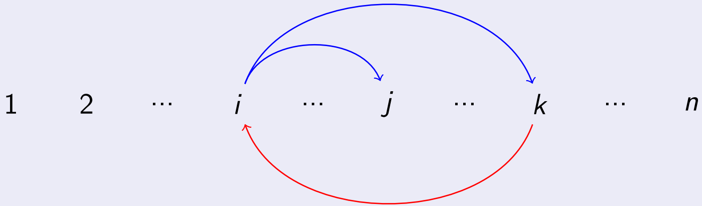

$i \mathrel{\mathrm{R}} j$ $k \mathrel{\mathrm{R}} i$ $i \mathrel{\mathrm{R}} k$

There is $2^{n(n-1)}$ binary relations.

Partial order on relations

Let R be a binary relation

$$\mathrm{R}^{\mathsf{Inc}} = \{i \,\mathrm{R}\, j, i < j\}$$
$$\mathrm{R}^{\mathsf{Dec}} = \{j \,\mathrm{R}\, i, i < j\}$$

Partial order on relations

Let R be a binary relation

$$\mathrm{R}^{\mathsf{Inc}} = \{i \,\mathrm{R}\, j, i < j\}$$
$$\mathrm{R}^{\mathsf{Dec}} = \{j \,\mathrm{R}\, i, i < j\}$$

Let R and S be two binary relations,

$$\mathrm{R} \preccurlyeq \mathrm{S} \Leftrightarrow \qquad \mathrm{R}^{\mathsf{Inc}} \supseteq \mathrm{S}^{\mathsf{Inc}} \text{ and } \mathrm{R}^{\mathsf{Dec}} \subseteq \mathrm{S}^{\mathsf{Dec}}$$

Relations of size 2

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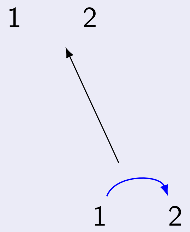
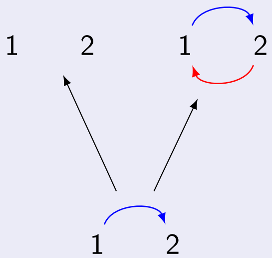


Relations of size 2



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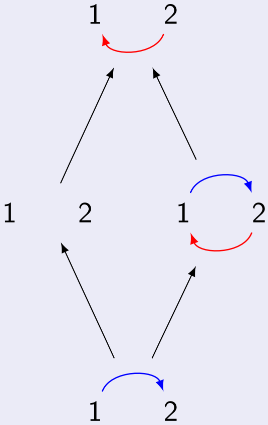
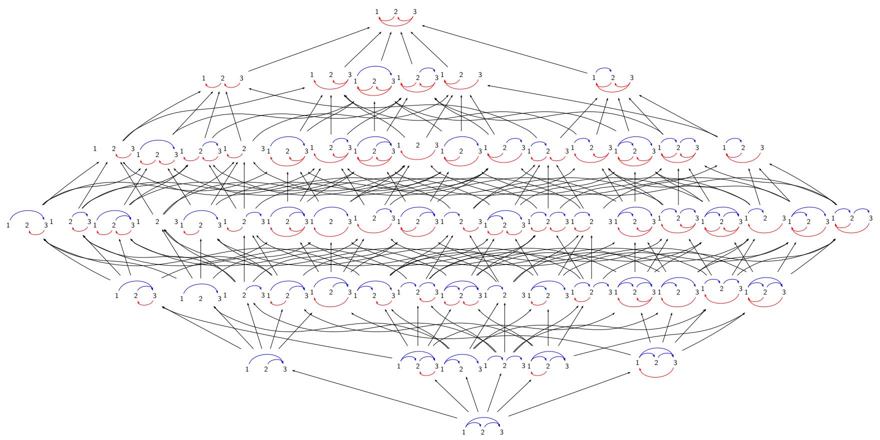
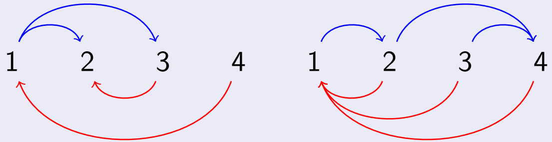




Meet and Join



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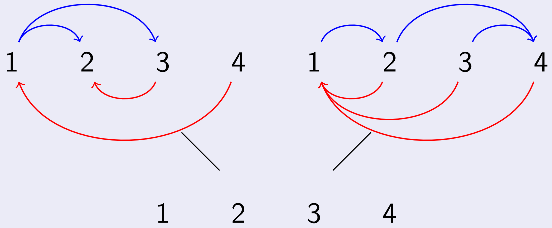

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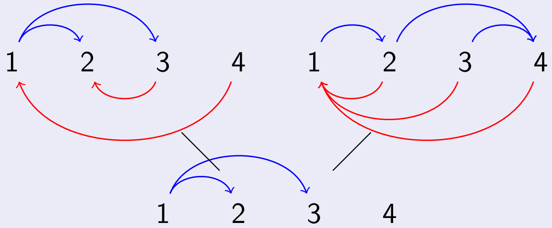

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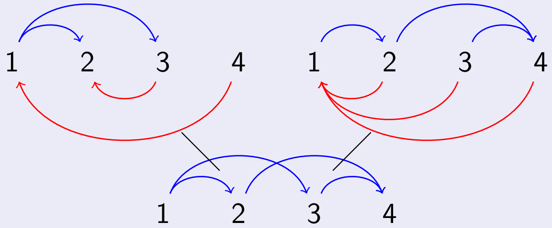

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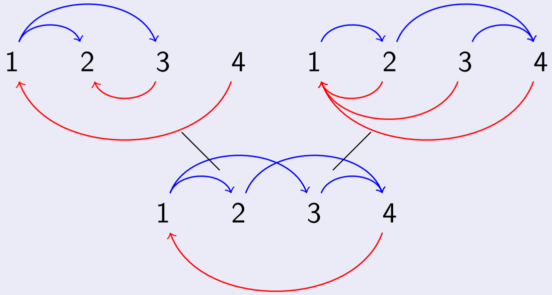

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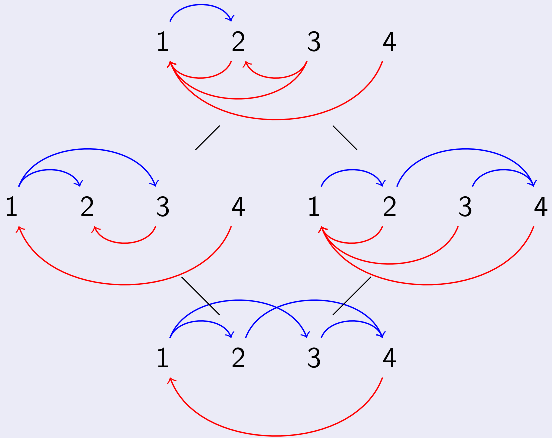
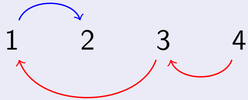
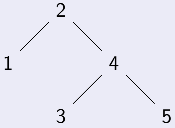


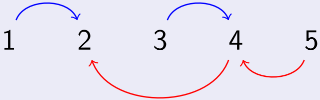
Posets are AWESOME

Objects viewed as posets

4312

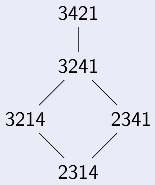


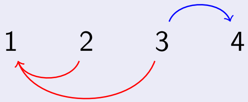


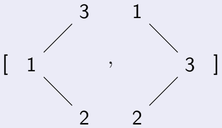


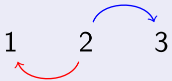
$+--$

Intervals viewed as posets









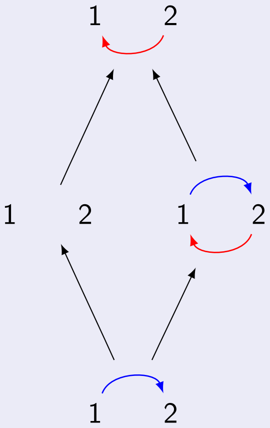
$$[+ - --, + - ++]$$

We want to keep the relations that are:

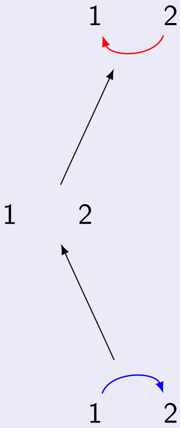
- antisymmetric
- transitives

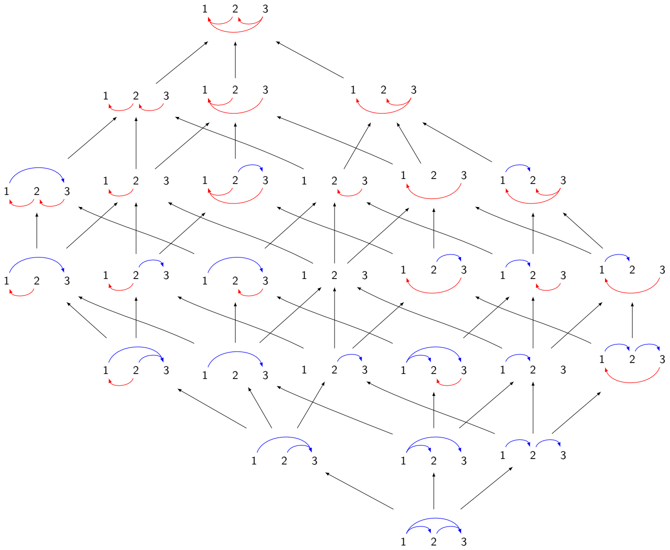
(posets)

Antisymmetry



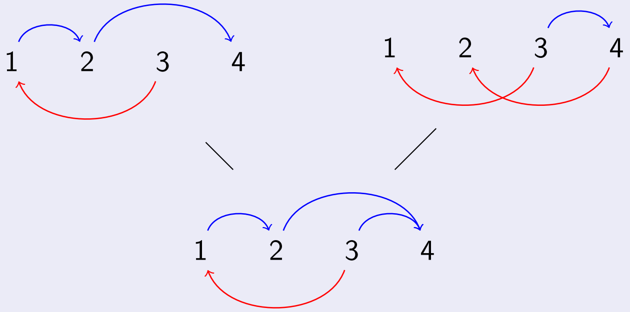
Antisymmetry





Sublattice?

If R and S are antisymmetric, is $\mathrm{R} \wedge \mathrm{S}$ also antisymmetric?



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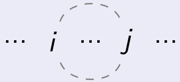

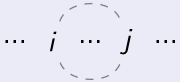

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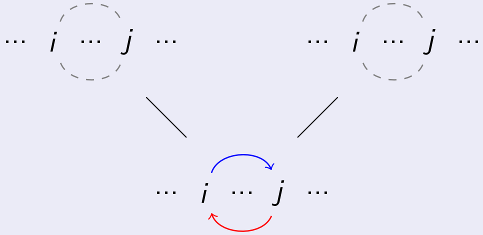

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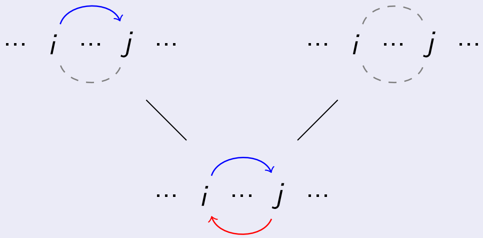

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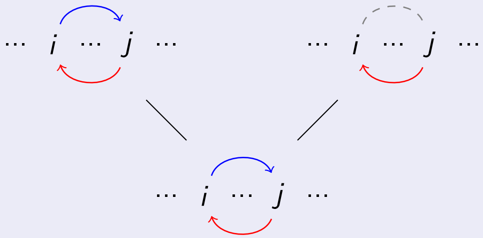
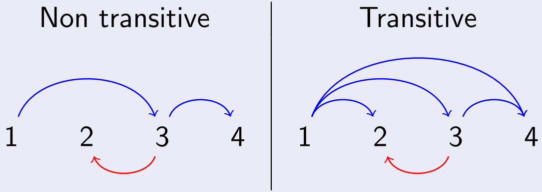
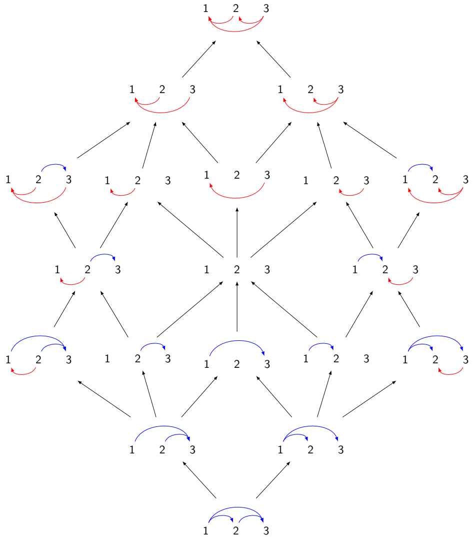
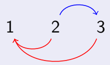


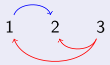
Transitivity

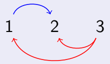



1 2 3

Sublattice?







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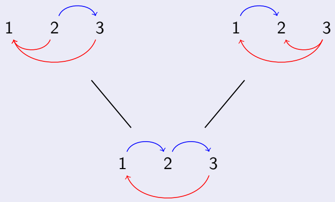

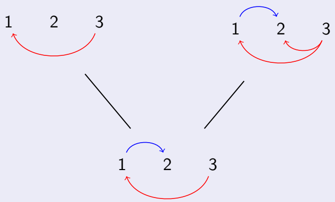

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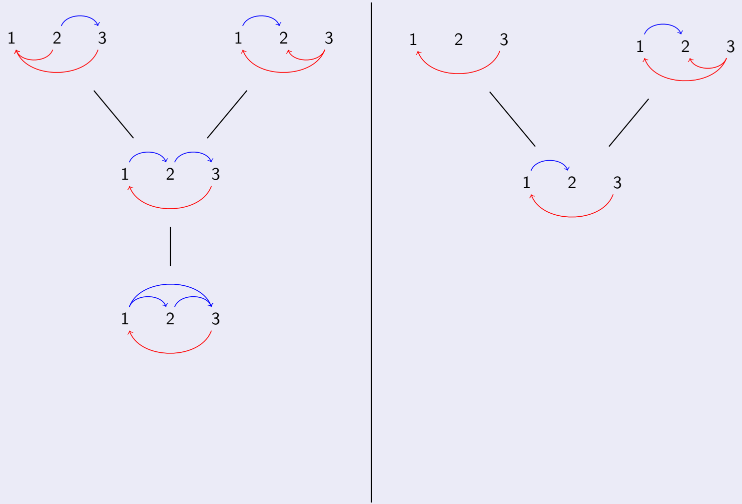

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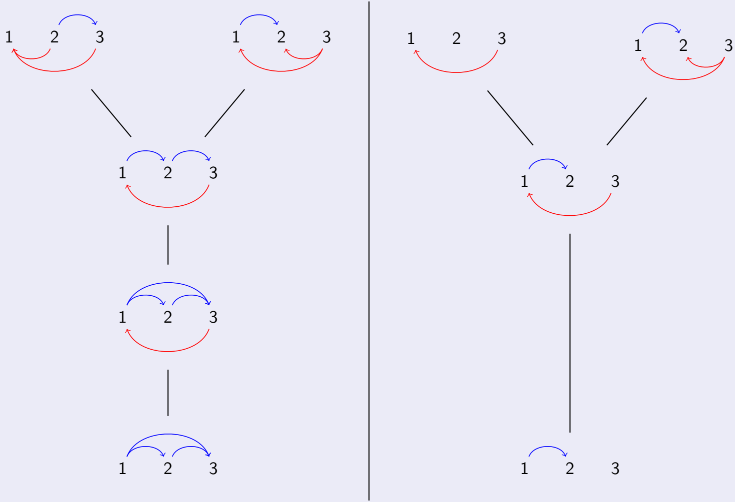
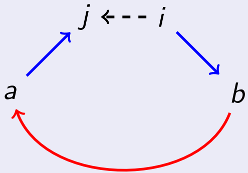


Transitive decreasing deletion

$a \leq j, i \leq b$



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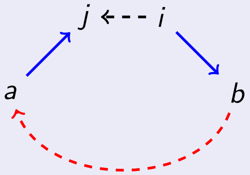

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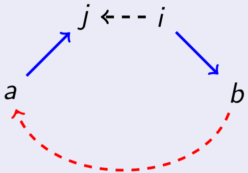

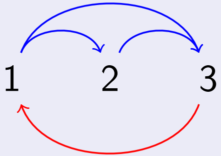

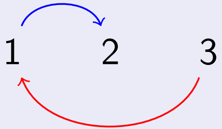

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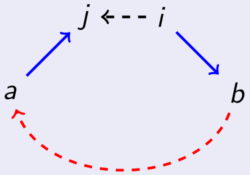

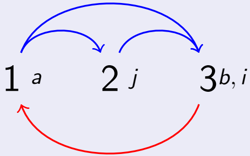

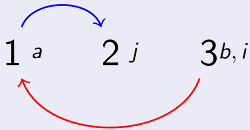

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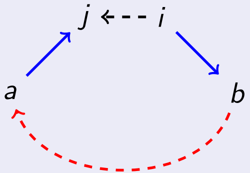
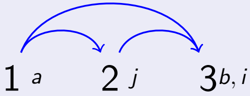
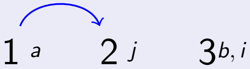
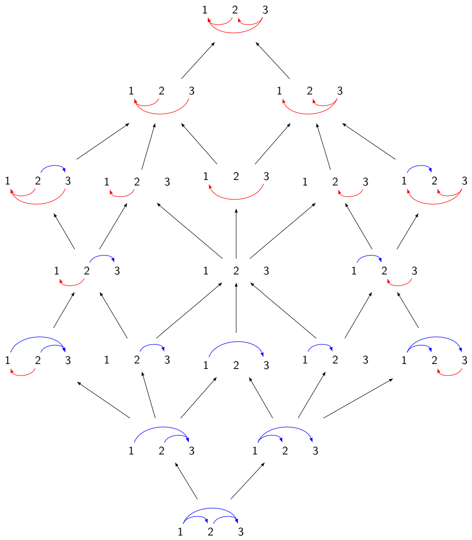


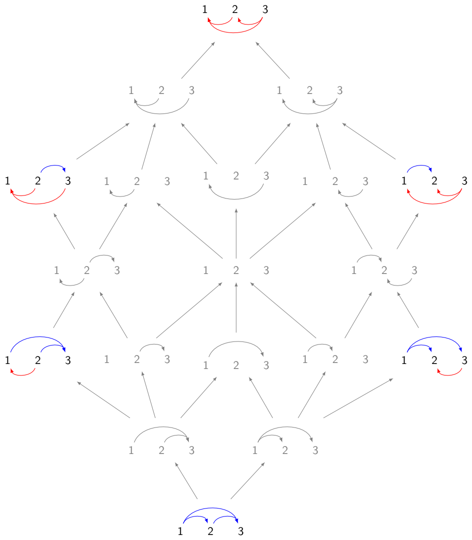




Poset lattice



Weak Order Element Poset (WOEP) $\simeq$ Right weak order



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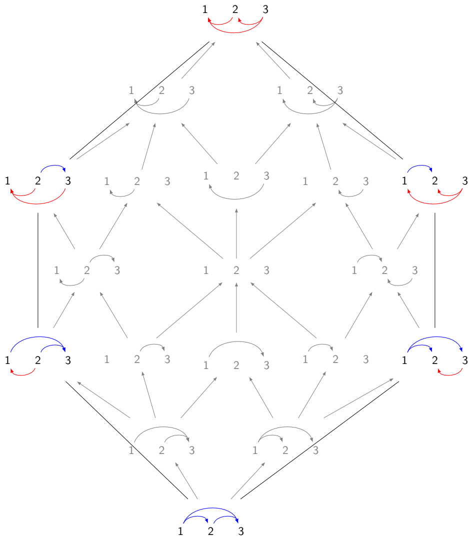
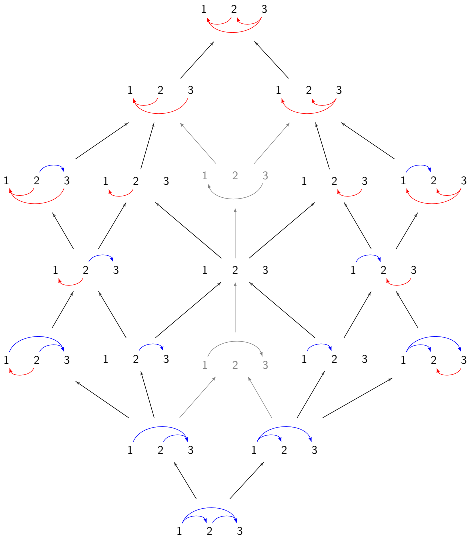
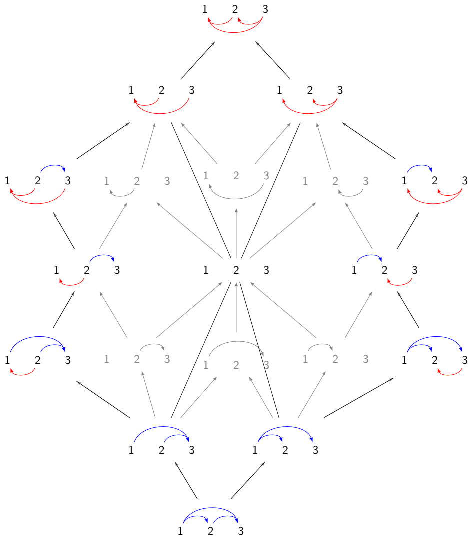


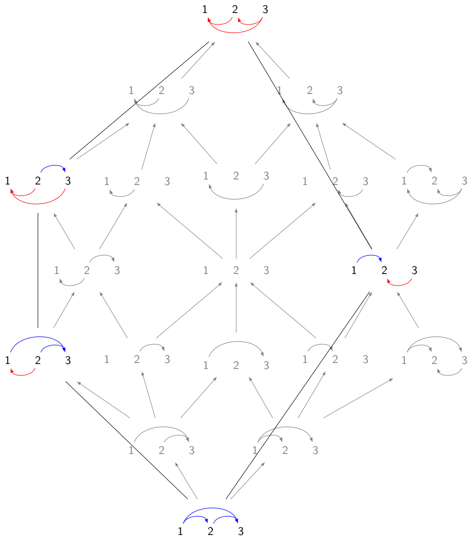
Weak Order Interval Poset (WOIP)



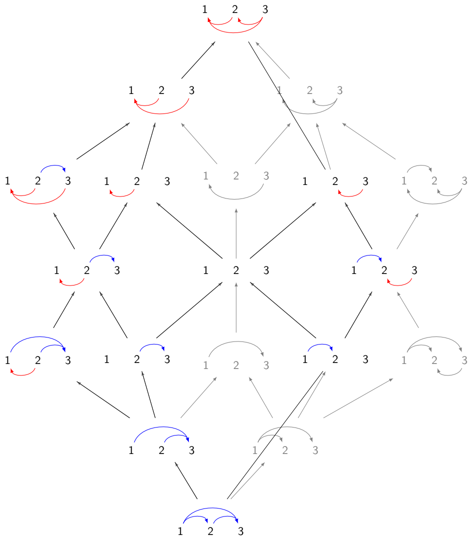
Weak Order Face Poset (WOFP)



Tamari Order Element Poset (TOEP) $\simeq$ Tamari order



Tamari Order Interval Poset (TOIP)



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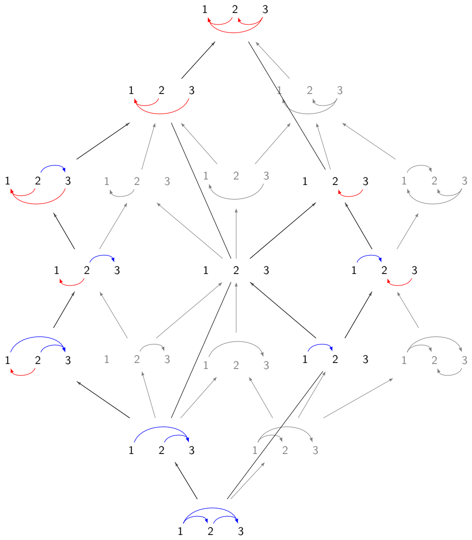
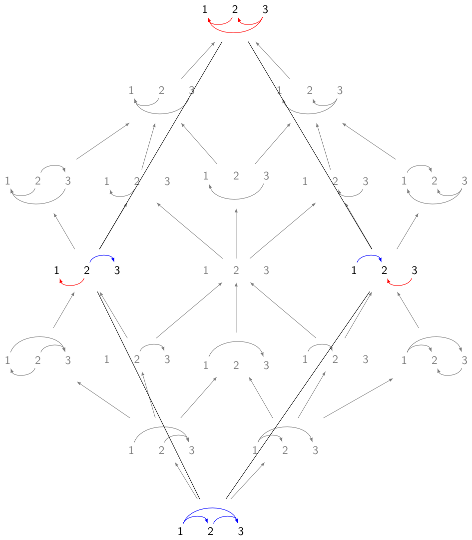
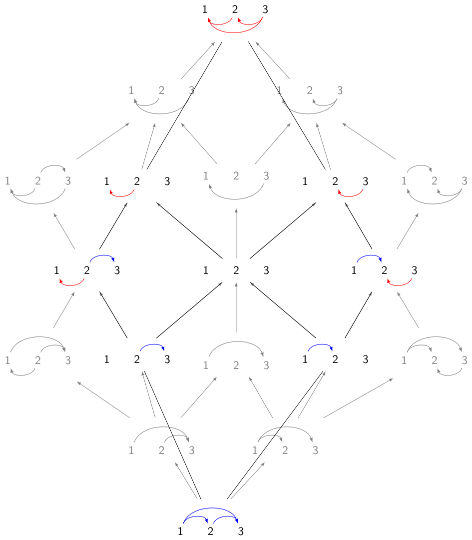


Boolean Order Element Poset (BOEP) $\simeq$ Boolean order



Boolean Order Interval Poset (BOIP = BOFP)



Results

- A lattice on posets.
- A set of rules on posets that automatically yields sublattices.

Work in progress

- "Correction" algorithms for the subposets that are not sublattices.
- Study associated Hopf algebras.
- Study associated polytopes.
- Study the same problems in the Coxeter group framework.