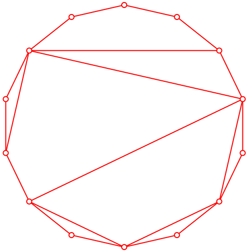
The serpent nest conjecture on accordion complexes

Thibault Manneville (LIX, Polytechnique)

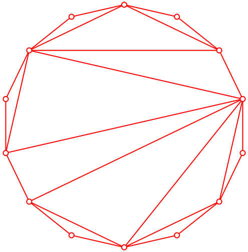
78ème Séminaire lotharingien de combinatoire
March 29th, 2017

Accordion dissections

Dissection = set of pairwise noncrossing diagonals

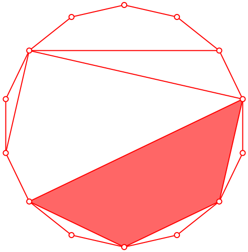


Triangulation = inclusion maximal dissection

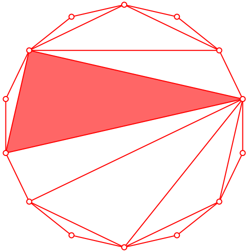


Accordion dissections

Cell = bounded conn. comp. of the complement

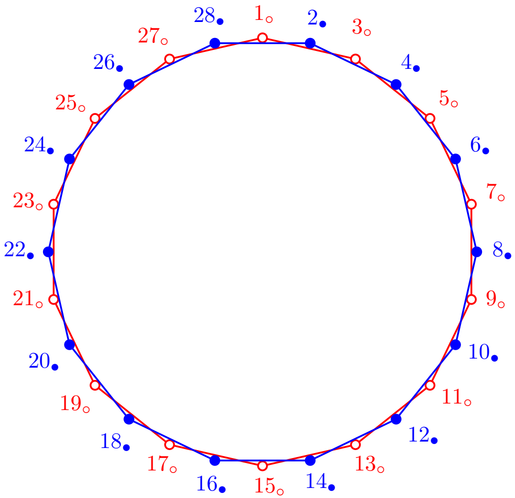


Triangulation = all cells are triangles

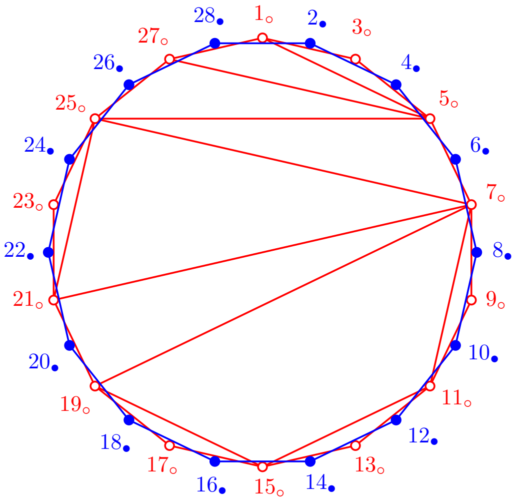


Accordion dissections

Consider interlaced red and blue polygons

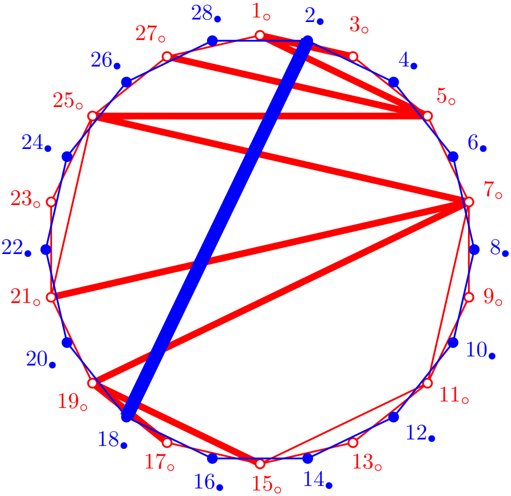


Fix a reference red dissection $D_\circ$



Accordion dissections

D$_\circ$-**accordion diagonal** = blue diagonal crossing a
"blue diagonal" connected set of red diagonals



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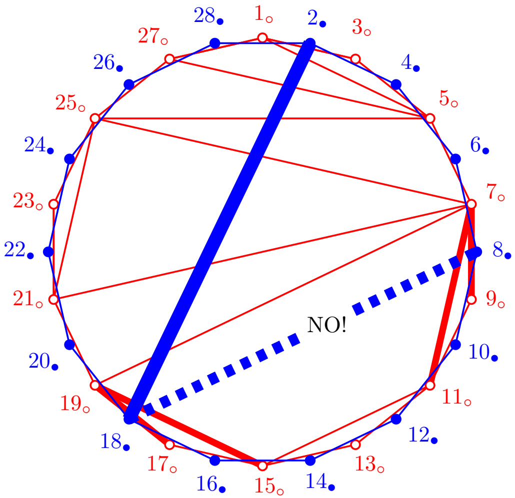
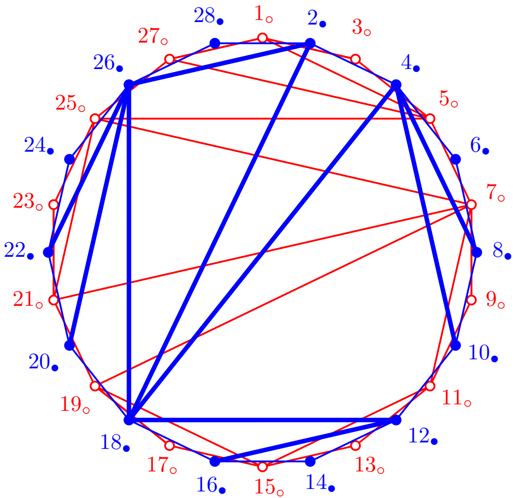


Accordion dissections

Maximal $\mathrm{D}_\circ$-accordion dissection = inclusion max. dissection
"blue dissection" containing blue diagonals



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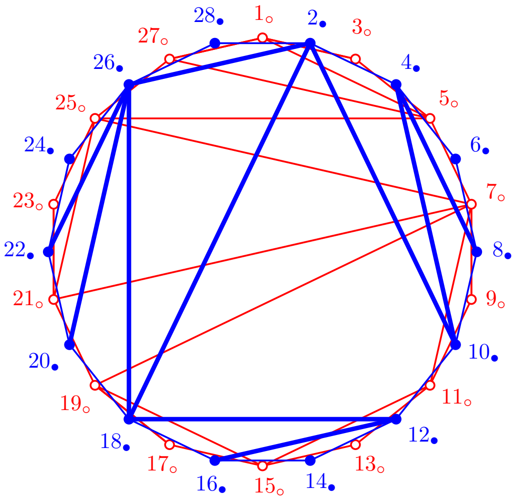

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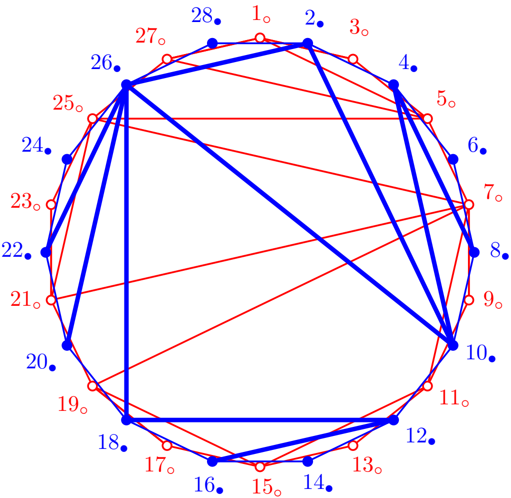
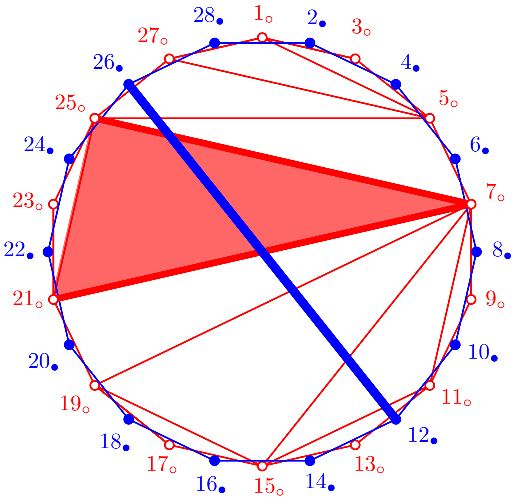


Accordion dissections

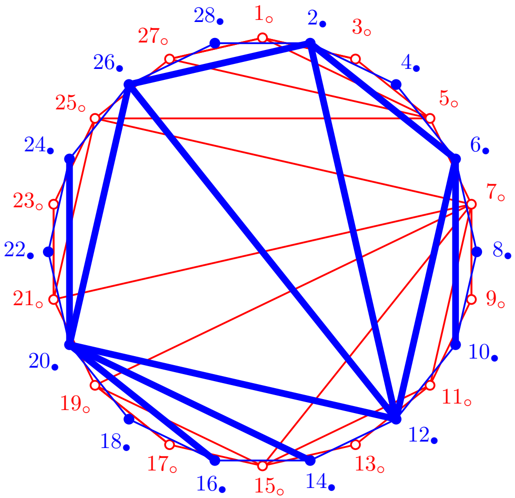
$\mathrm{D}_\circ$ is a triangulation



Accordion dissections

$\mathrm{D}_\circ$ is a triangulation $\implies$ blue dissection = blue triangulations

$C_n := \frac{1}{n+1}\binom{2n}{n}$



History

Baryshnikov, *On Stokes sets* (2001)

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Chapoton, *Stokes posets and serpent nests* (2016)

Are Stokes posets lattices?

Are Stokes complexes realizable as polytopes?

$\#$(elements of Stokes posets) $=$ $\#$(serpent nests)?

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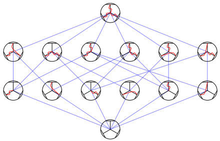

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YES: Garver and McConville, *Oriented Flip Graphs and Noncrossing Tree Partitions* (2016)

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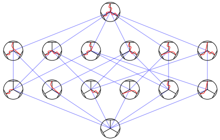

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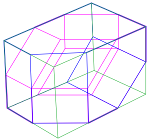

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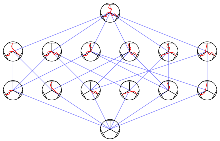

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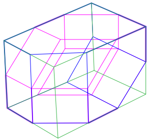

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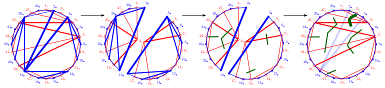
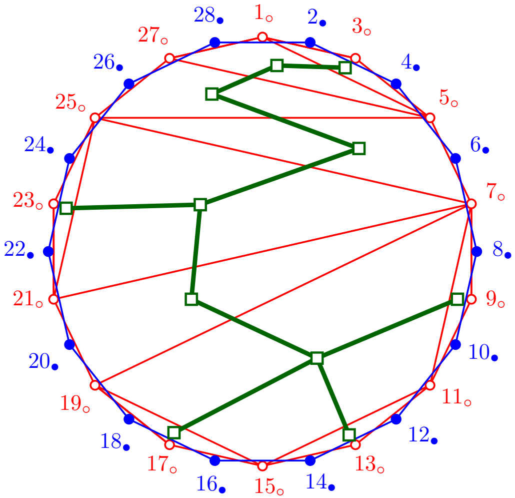
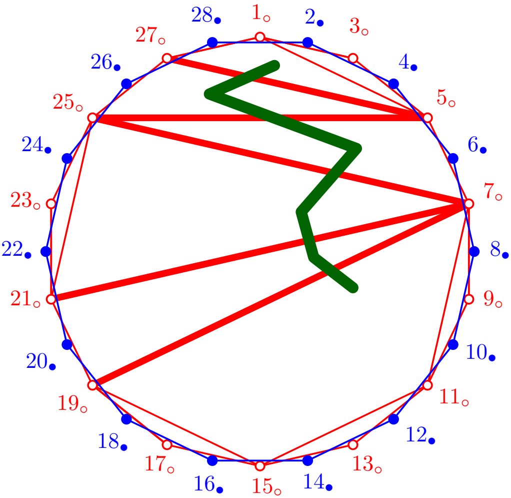


Dual tree $D_\circ^\star$ of $D_\circ$ = vertices: cells of $D_\circ$
edges: internal diagonals of $D_\circ$



Serpent nests

Serpent of $\mathrm{D}_\circ$ = nonempty undirected dual path in $\mathrm{D}_\circ^\star$ crossing a connected set of diagonals



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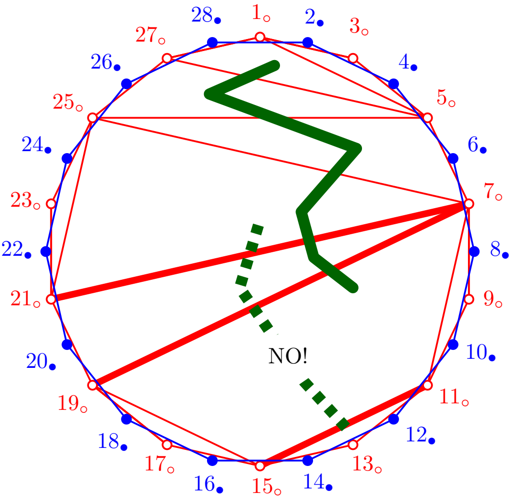

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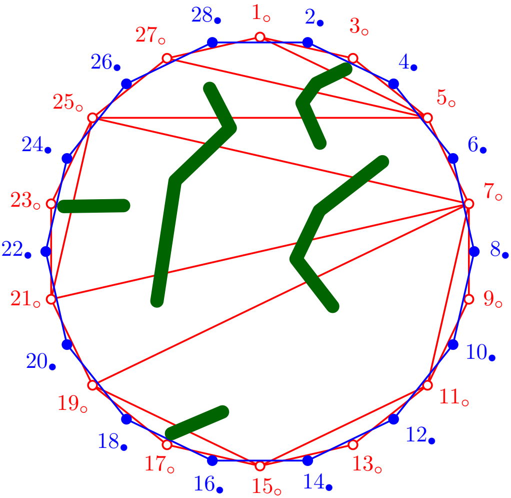
Serpent nest of $\mathrm{D}_\circ$ = set of serpents of $\mathrm{D}_\circ$ with some conditions:



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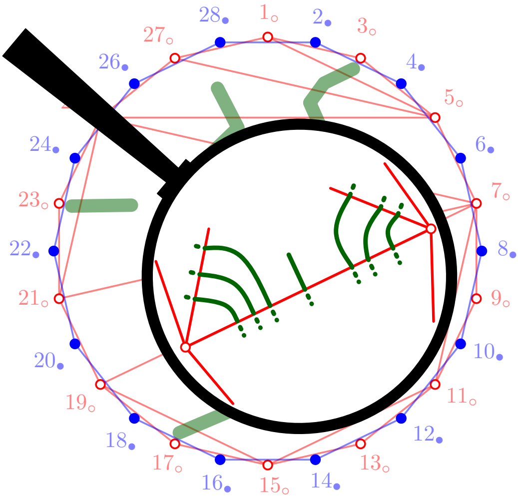
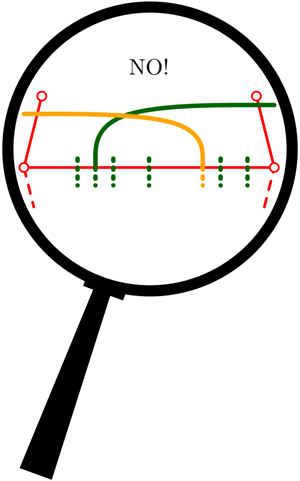
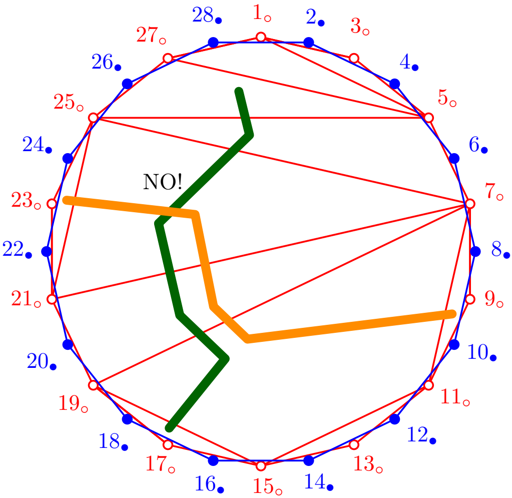


Serpent nests

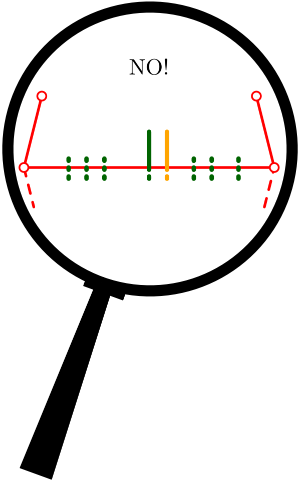
Serpent nest of $\mathrm{D}_\circ$ = set of serpents of $\mathrm{D}_\circ$ with three conditions:
no crossing

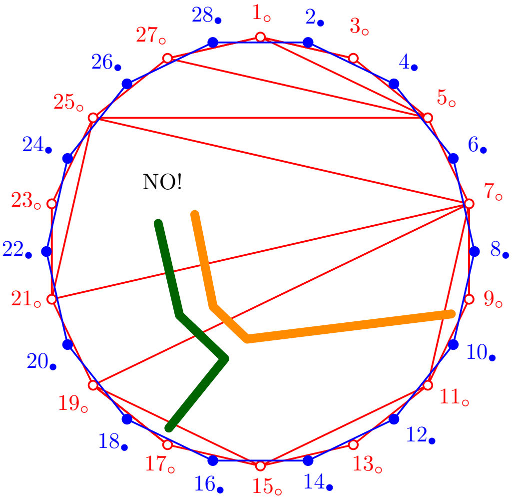




Serpent nests

Serpent nest of $D_\circ$ = set of serpents of $D_\circ$ with three conditions: no crossing, no common arrival





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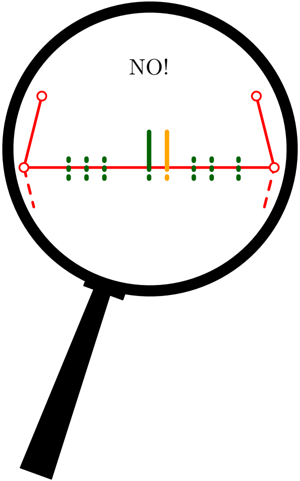

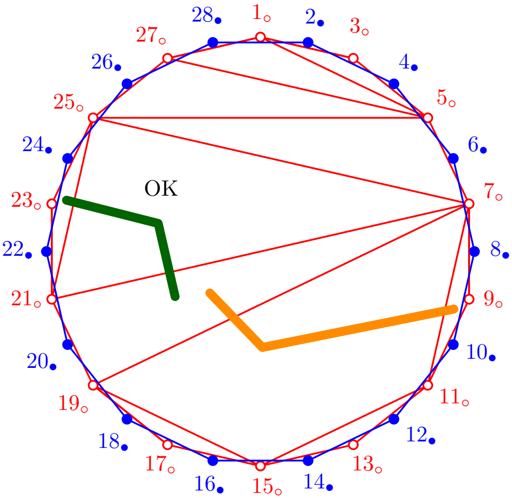

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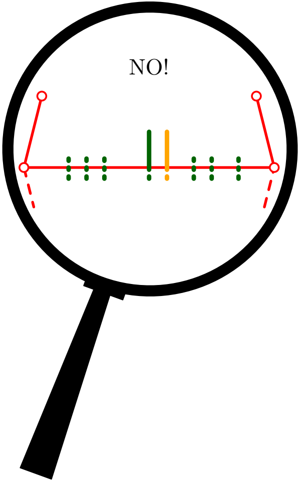

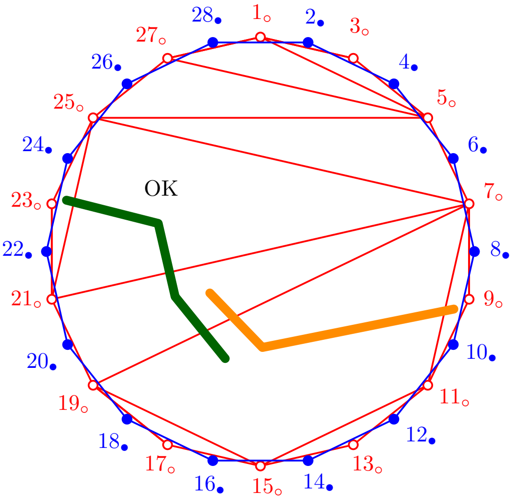

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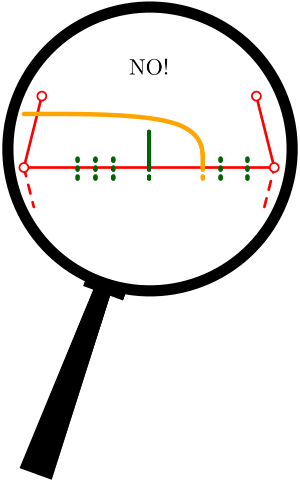
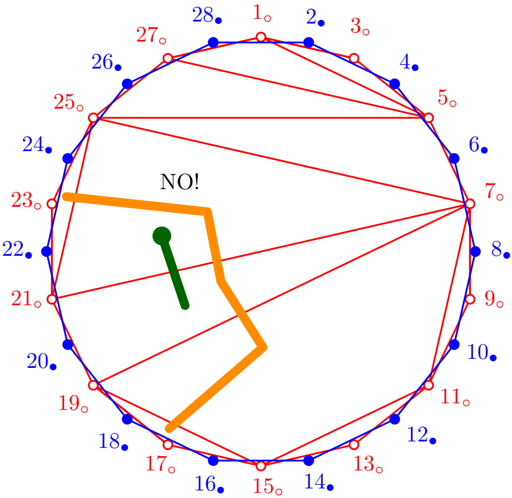
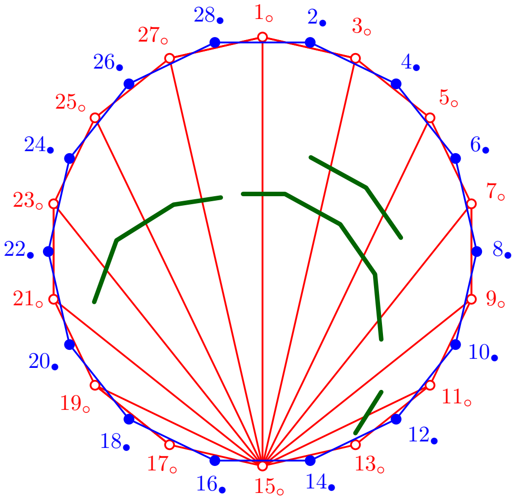




Serpent nests

$\color{red}{D_\circ}$ is a comb triangulation $\Longrightarrow$ serpent nests = noncrossing partitions (C_n)



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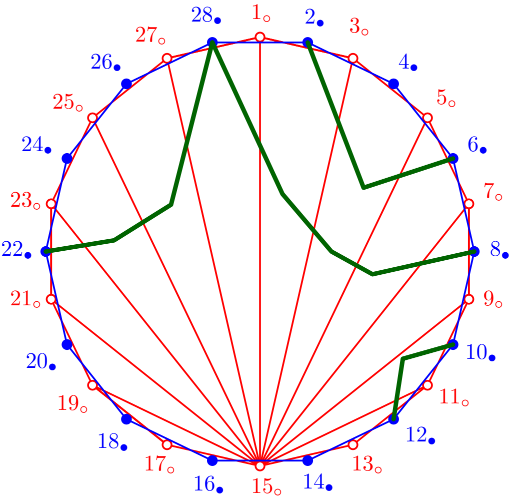
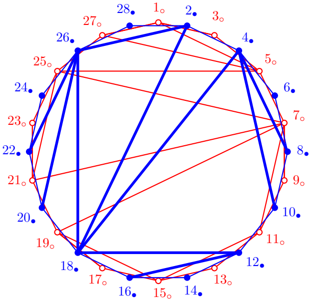
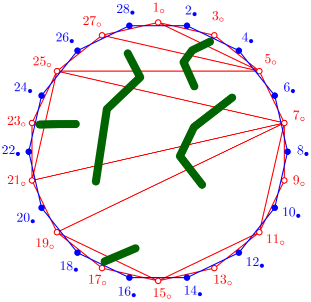


Theorem (M. 2017$^+$)

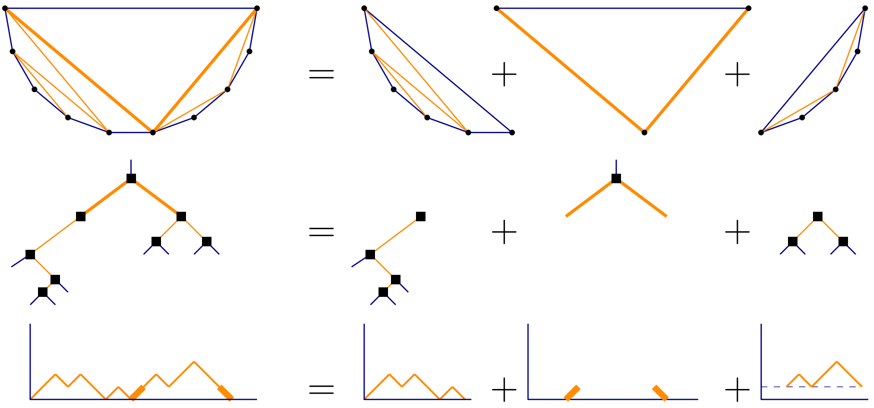
For any dissection $\mathrm{D}_\circ$,
$\#$(*maximal* $\mathrm{D}_\circ$*-accordion dissections*) $= \#$(*serpent nests of* $\mathrm{D}_\circ$)



$\longleftrightarrow$

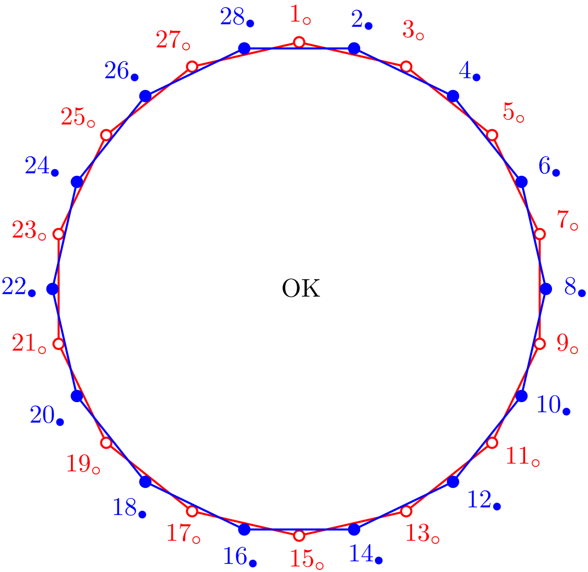


Catalan-like bijection



$$C_{n+1} = \sum_{k=0}^{n} C_k \times 1 \times C_{n-k}$$

Proof: induction on #(diagonals of $D_\circ$)



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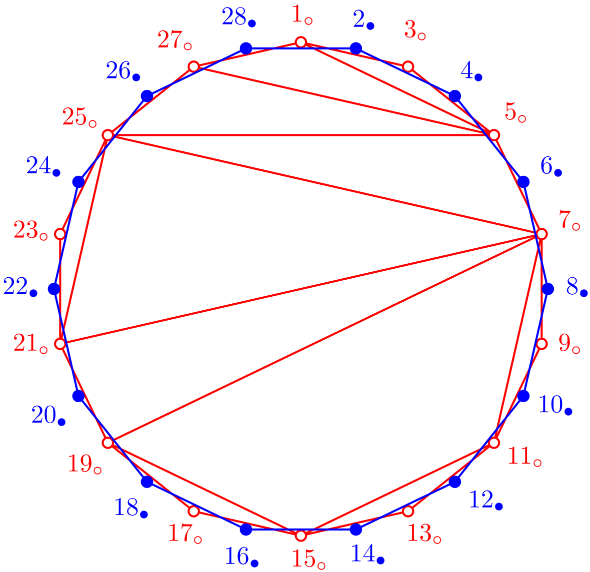

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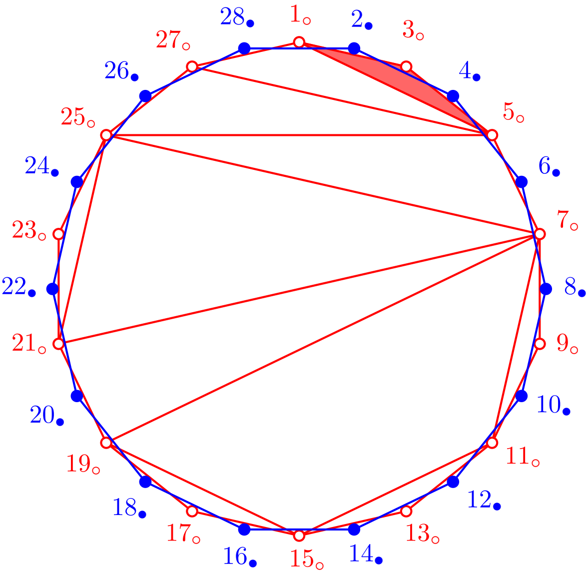
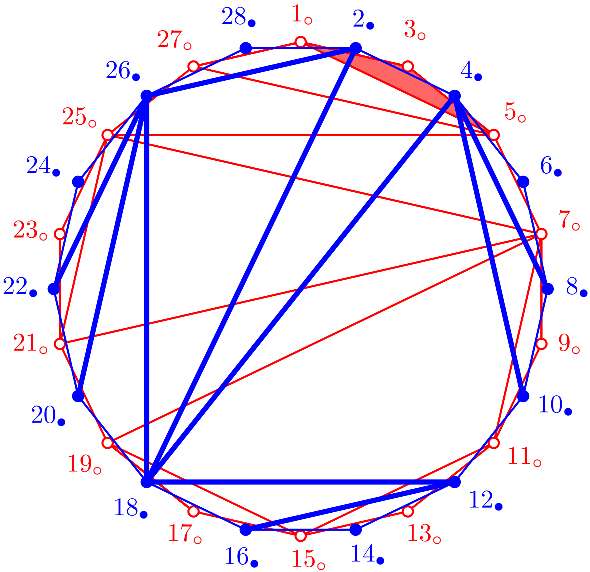
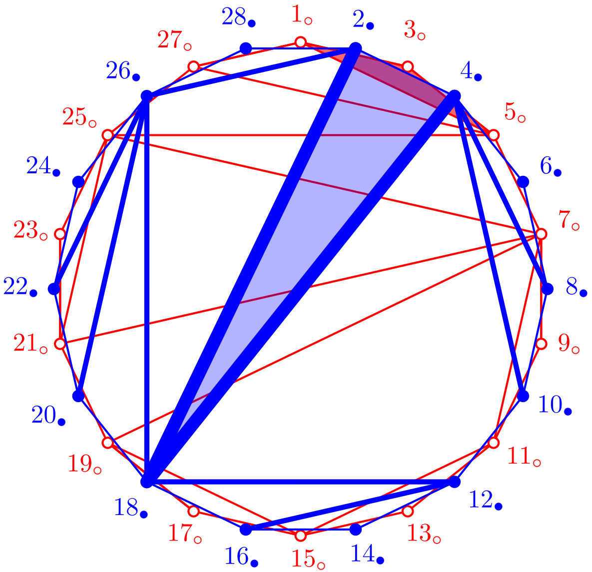


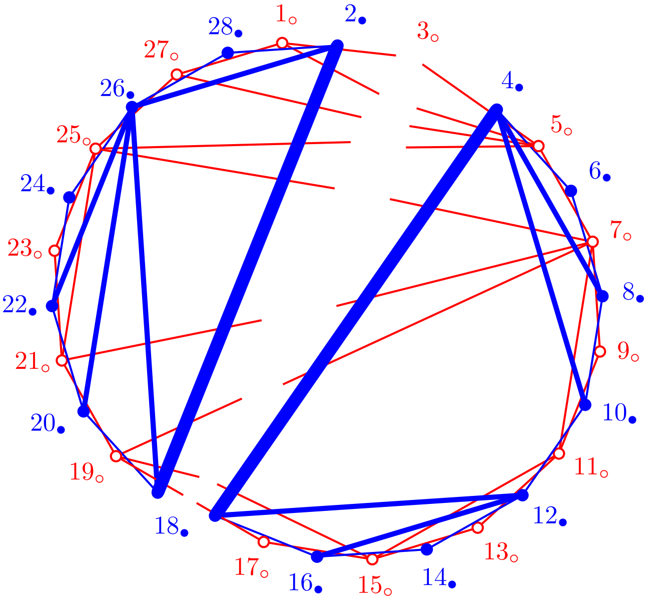
Proof: {maximal $D_\circ$-accordion dissections} → {serpent nests of $D_\circ$}



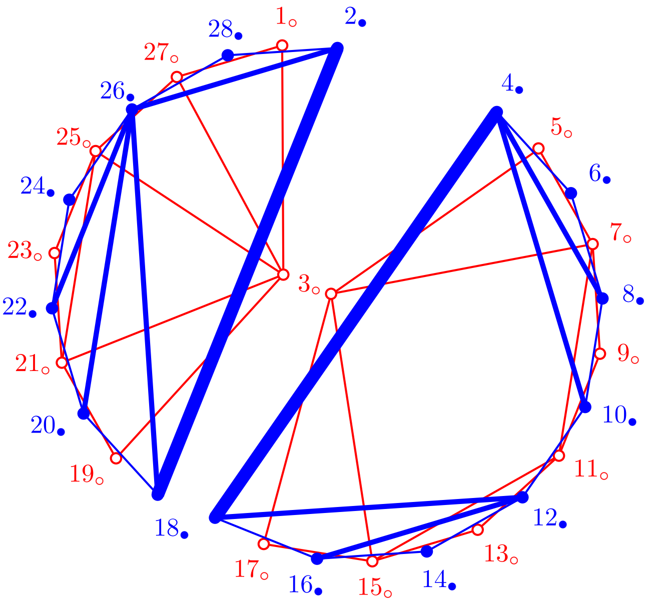
Proof: there exists $x_\bullet \in [6_\bullet, 28_\bullet]$ such that $\{(2_\bullet, x_\bullet), (4_\bullet, x_\bullet)\} \subseteq D_\bullet$



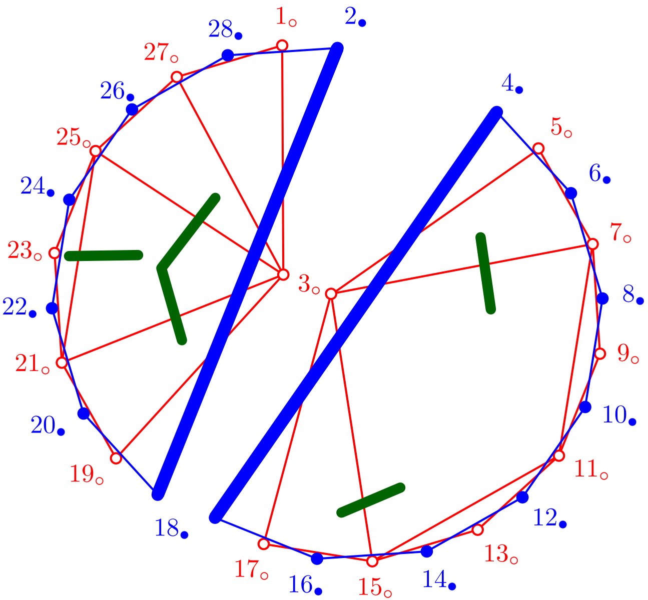
Proof: separate $D_\bullet$ according to $x_\bullet$



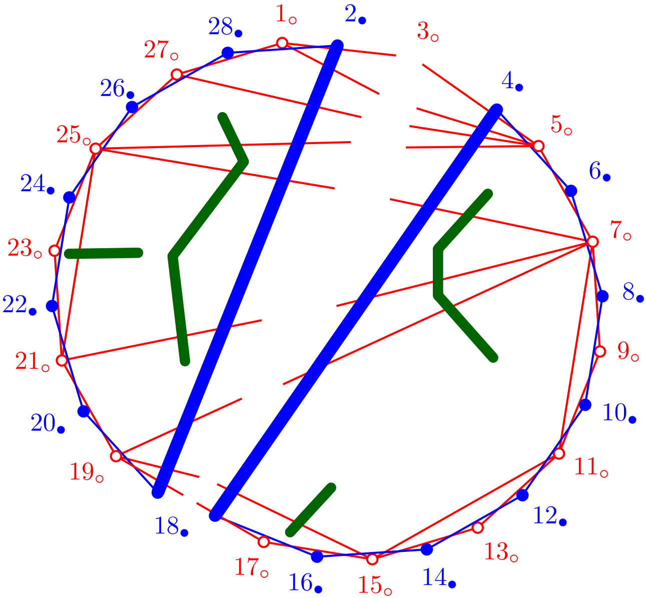
Proof: separate $\textcolor{blue}{D_\bullet}$ according to $x_\bullet$



Proof: apply the bijections obtained inductively on each side



Proof: unfold the serpents



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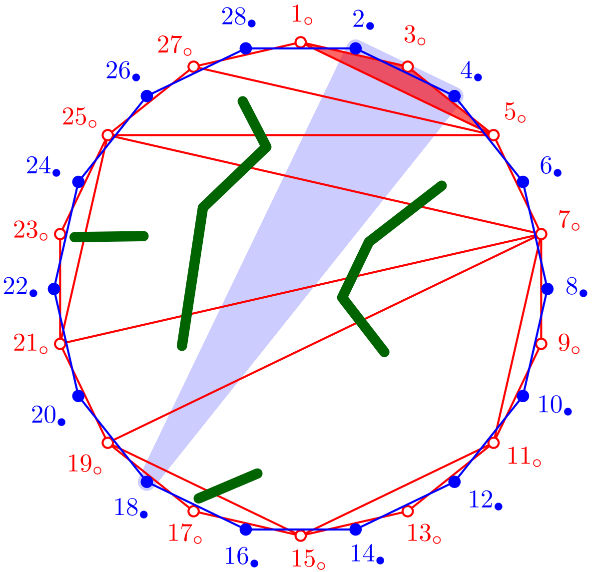
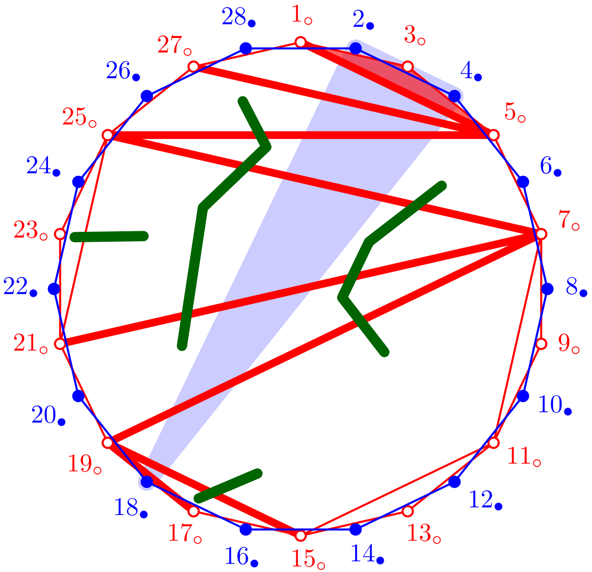
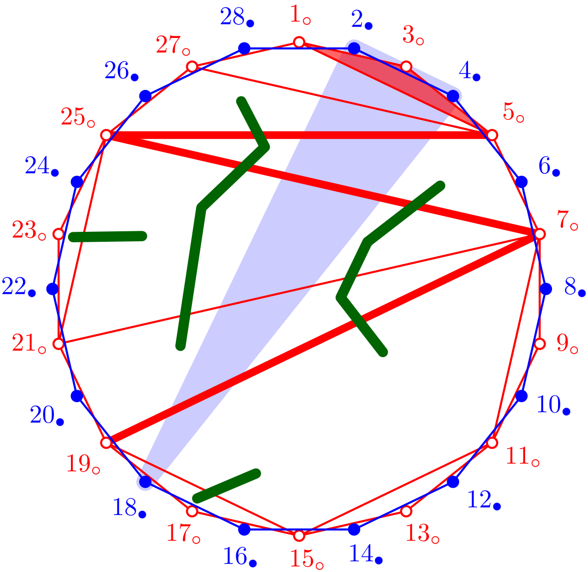


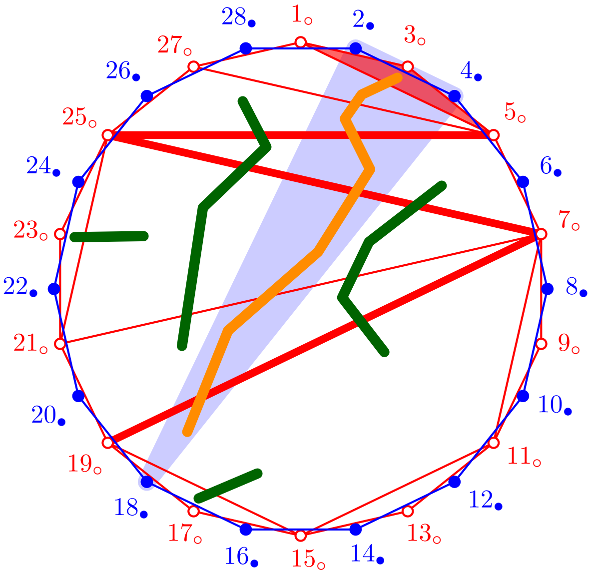
Proof: consider red diagonals crossed by both $(2_\bullet, x_\bullet)$ and $(4_\bullet, x_\bullet)$



Proof: keep only disconnecting diagonals (**zigzag**)



Proof: insert the serpent from $(1_\circ, 3_\circ)$ to the furthest possible one



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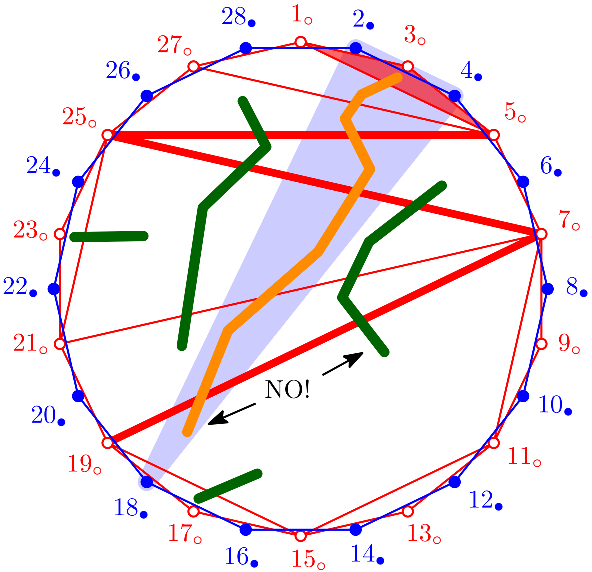

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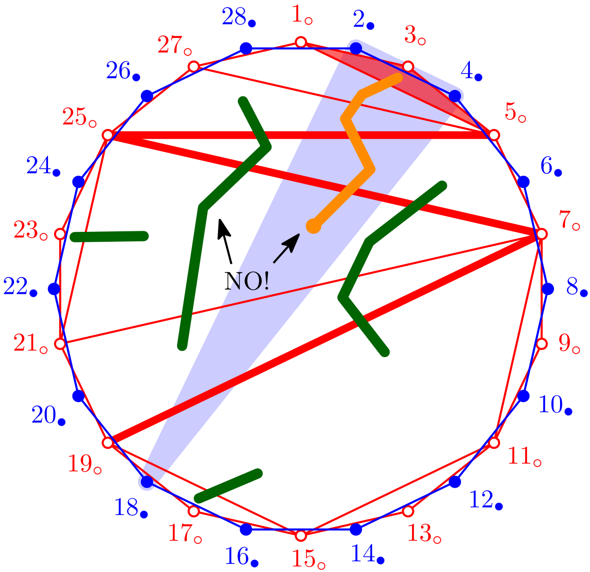

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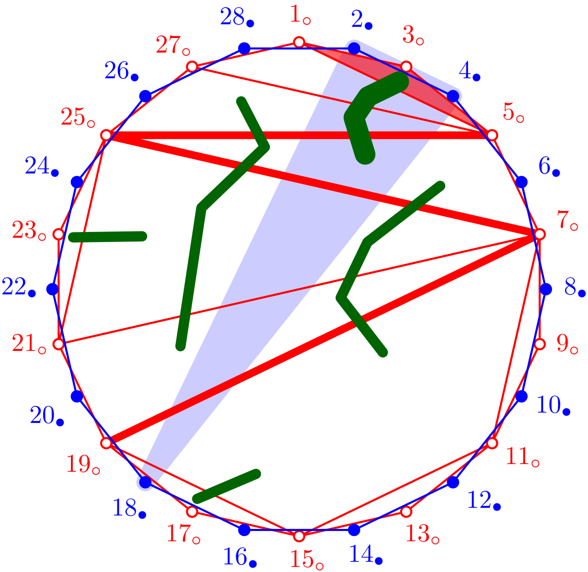
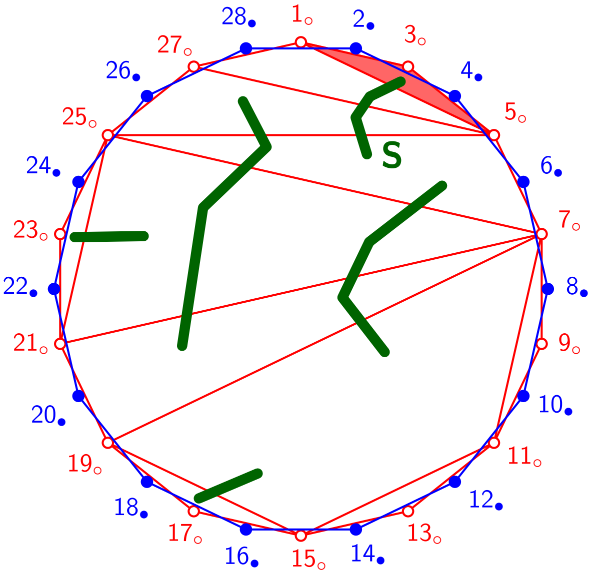
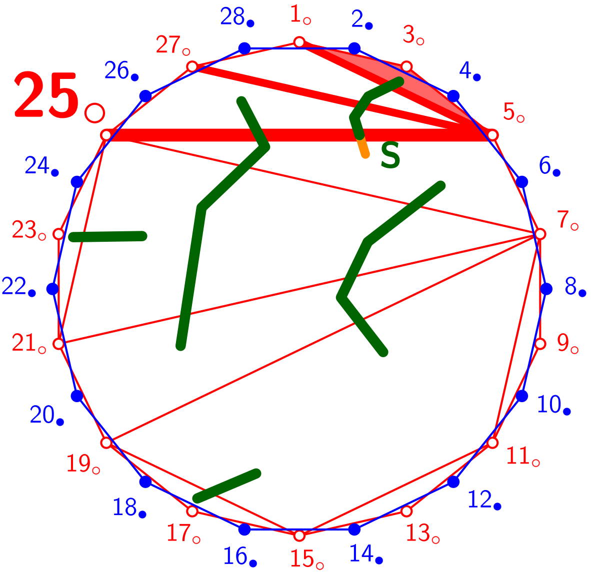


Proof: $\{$serpent nests of $\mathrm{D}_\circ\} \to \{$maximal $\mathrm{D}_\circ$-accordion dissections$\}$



Proof: validly extend S around successive pivots



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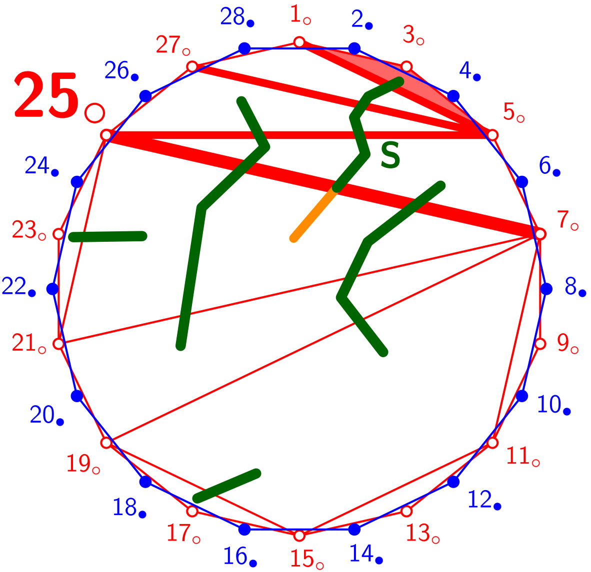

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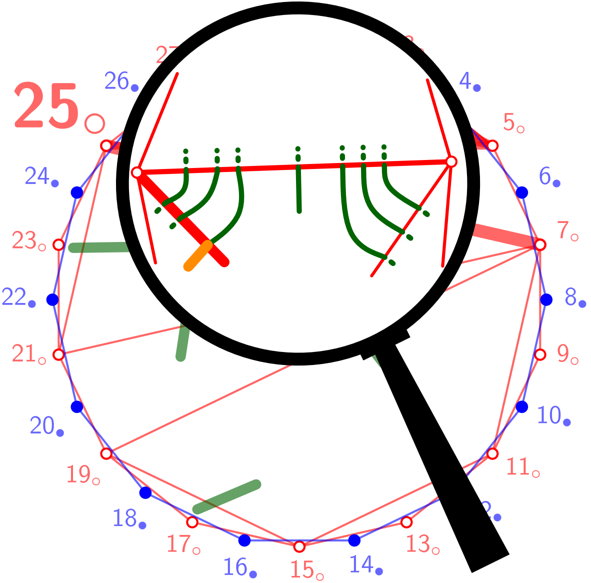

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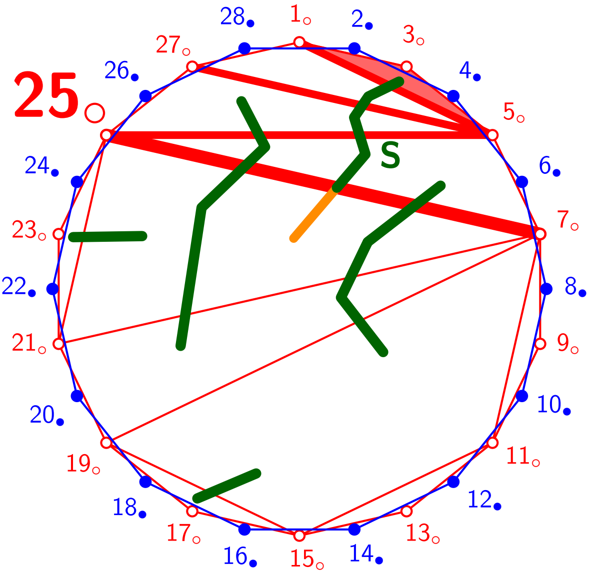
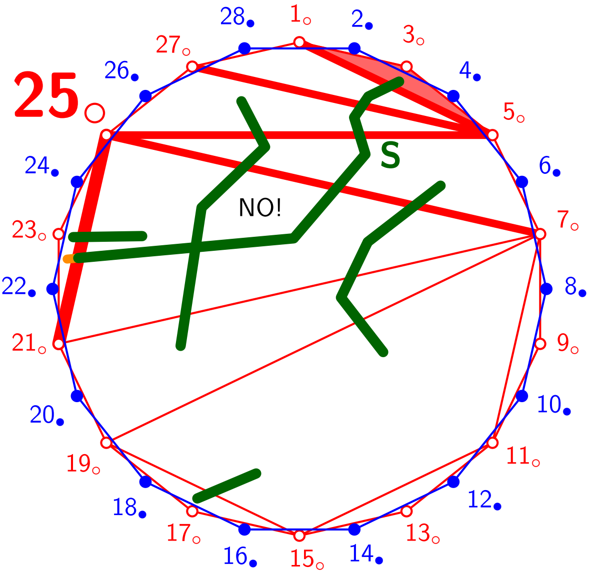
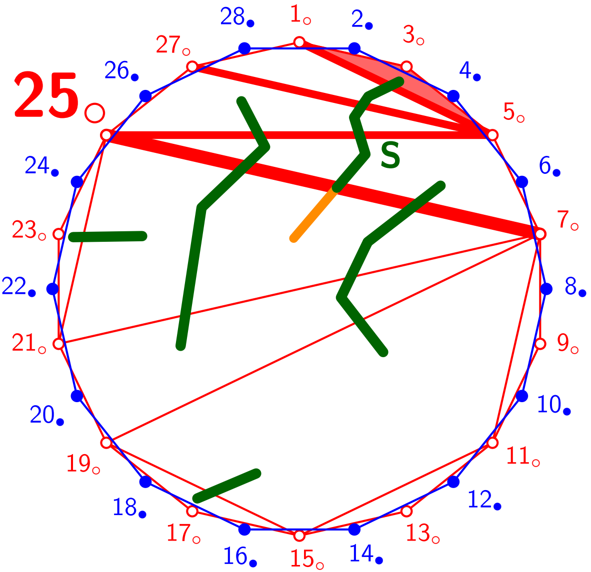


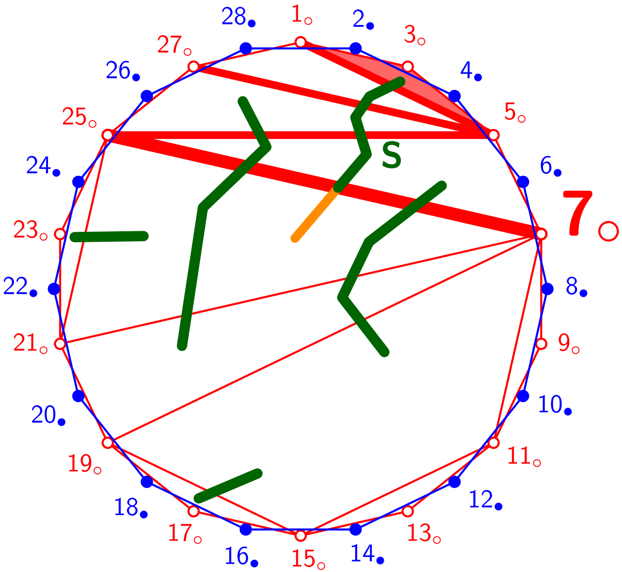
Proof: validly extend **S** around successive pivots



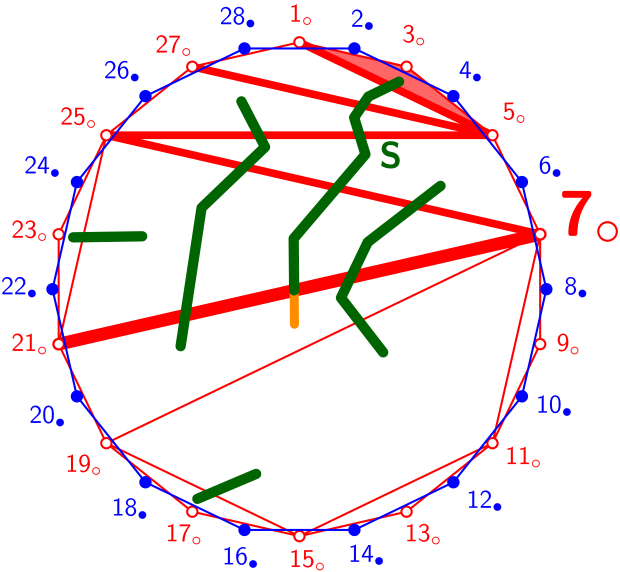
Proof: validly extend S around successive pivots



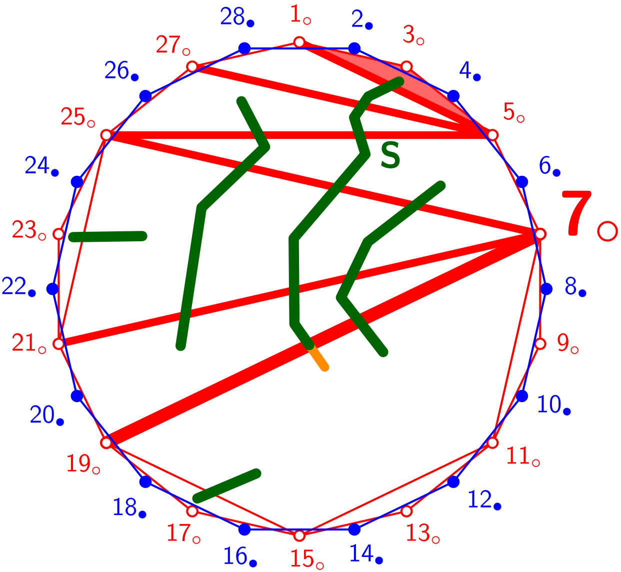
Proof: validly extend S around successive pivots



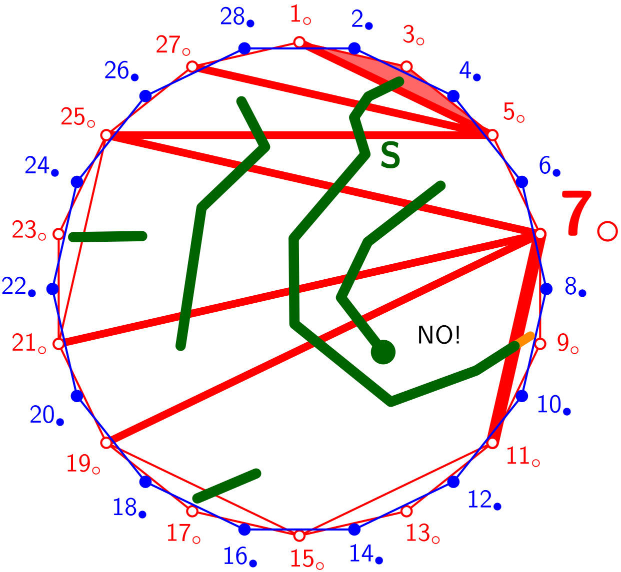
Proof: validly extend S around successive pivots



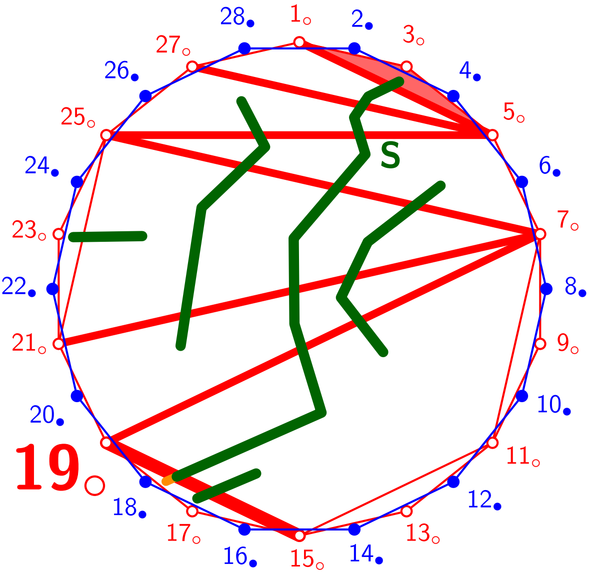
Proof: validly extend S around successive pivots



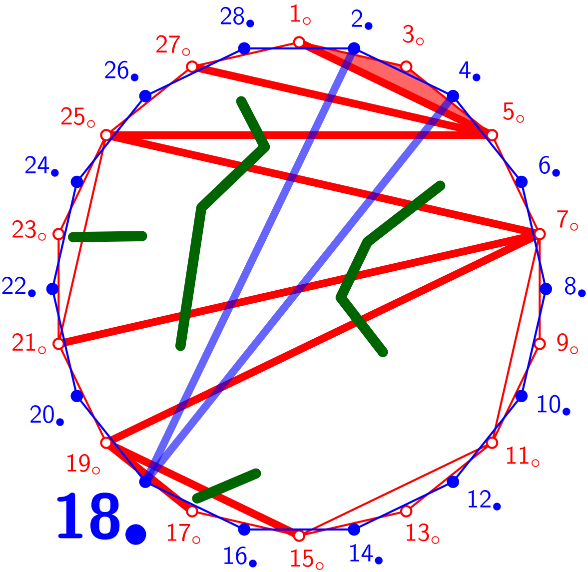
Proof: validly extend S around successive pivots



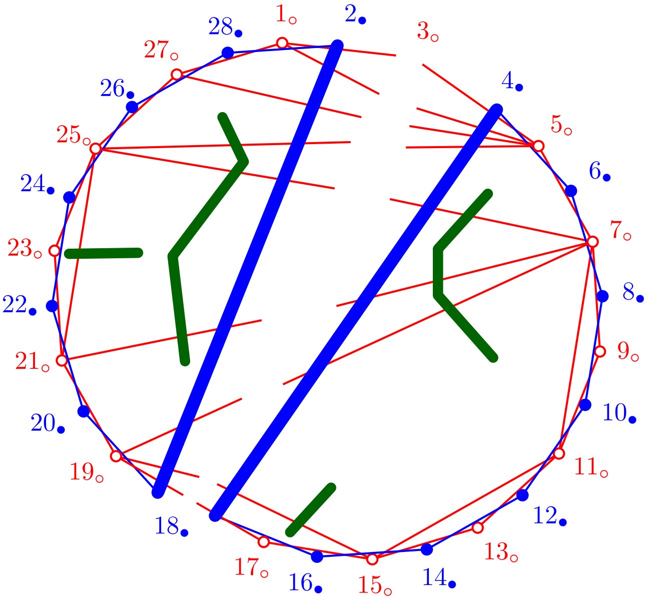
Proof: validly extend S around successive pivots



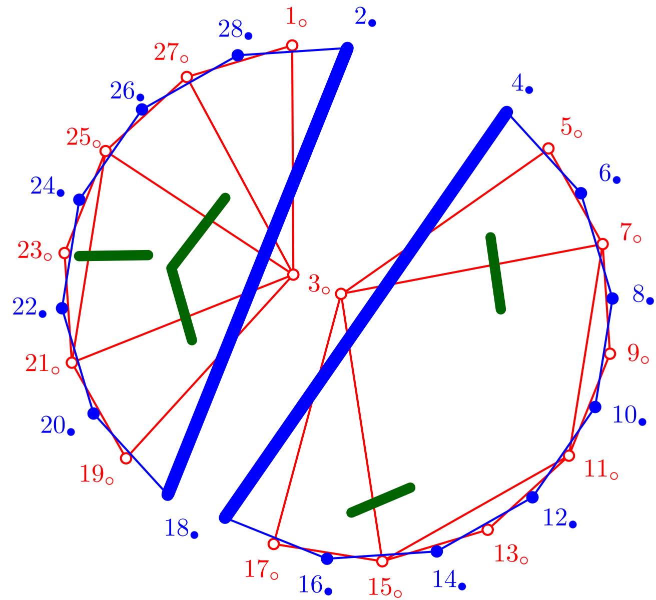
Proof: validly extend S around successive pivots



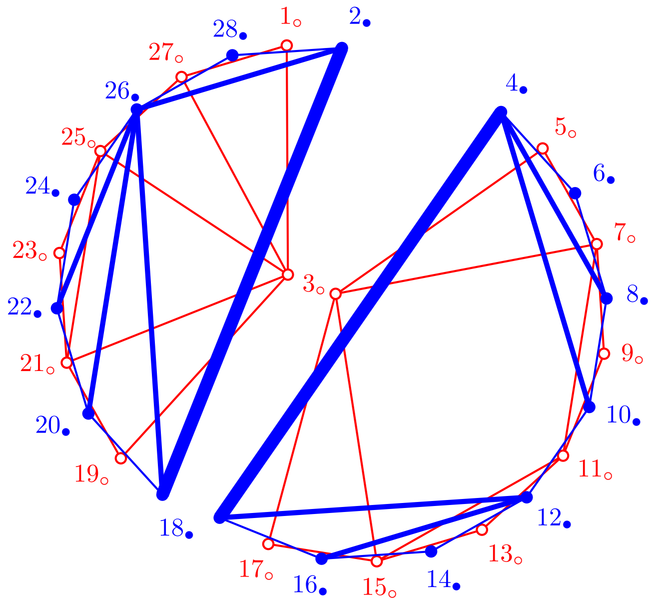
Proof: separate according to $x_\bullet$



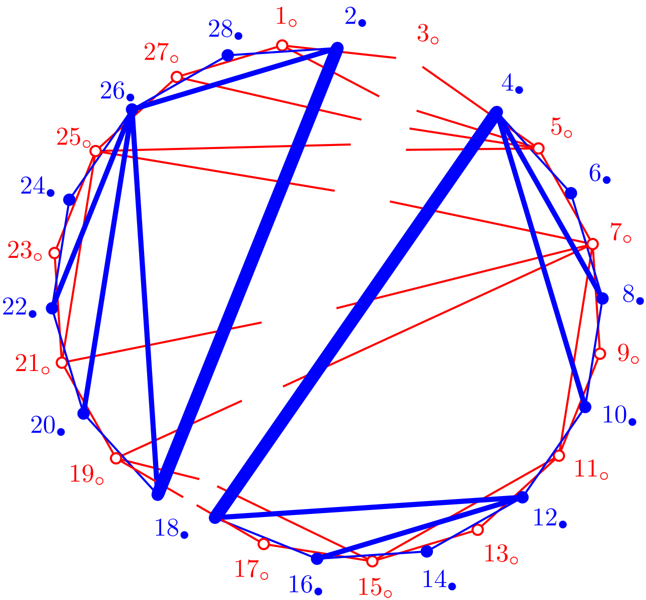
Proof: apply reverse bijections inductively obtained



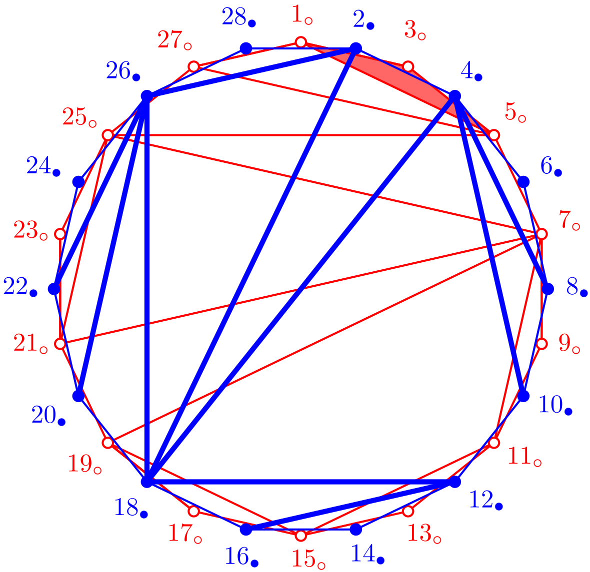
Proof: apply reverse bijections inductively obtained



Proof: glue back together

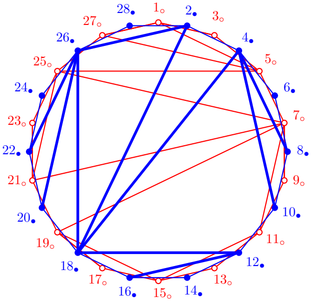


Proof: glue back together

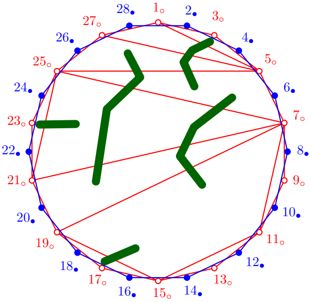


Theorem (M. 2017^{+})

For any dissection $\mathrm{D}_\circ$,
$\#$(*maximal* $\mathrm{D}_\circ$-*accordion dissections*) $= \#$(*serpent nests of* $\mathrm{D}_\circ$)



$\longleftrightarrow$



THANK YOU FOR YOUR KIND LISTENING!

Quesssssstions?