

Homage to John Conway

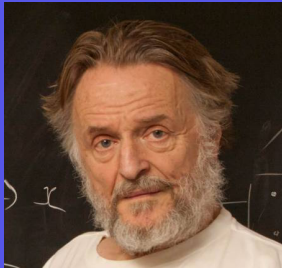


Photo: Princeton University / Denise Applewhite

Theresa Eisenkölbl, Université Lyon 1

John Horton Conway

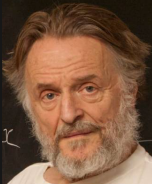


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John Horton Conway

Bio

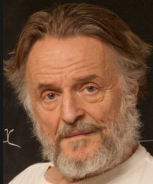


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■ ★26. 12. 1937

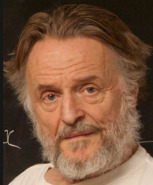


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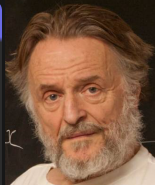


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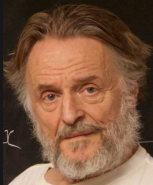


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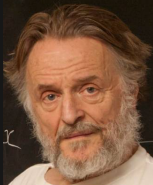


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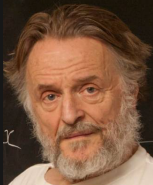


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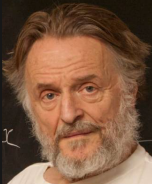


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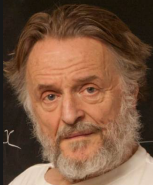


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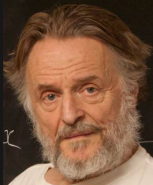


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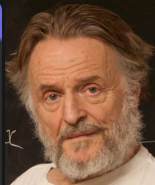


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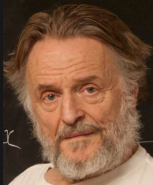


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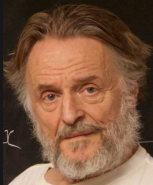


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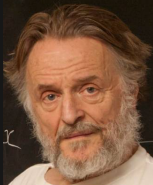


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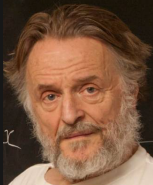


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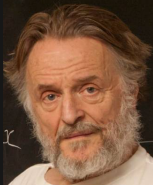


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- *(with Berlekamp and Guy)*
Winning Ways for your Mathematical Plays (1982)

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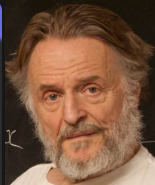


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Winning Ways for your Mathematical Plays (1982)
- *(with Norton, Wilson and Parker)*
Atlas of finite groups (1985)

In an interview, John Conway said that after he made his name with his work on the classification of finite simple groups he felt that he was now free to do whatever he liked.

In an interview, John Conway said that after he made his name with his work on the classification of finite simple groups he felt that he was now free to do whatever he liked.

His advisor, Harold Davenport, said that when he would give John Conway a problem to solve, „he would return with a very good solution to another problem.“

Summer Camps

Summer Camps

Friday 22nd	Saturday 23rd	Sunday 24th
Breakfast	Breakfast	Breakfast
Announcements	Announcements	Announcements
Mark Levi	John Conway	Don Zagier
Mathematics by physical reasoning	Topic decided in consultation with participants	Partitions

Summer Camps

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Photo: MoMISS 2012/Katia Sergeeva

Motivation

Themes of this talk

- Conway group

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- Conway polynomial

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- Conway sequence and constant

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- Sprouts and variants

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- Conway group
- Conway polynomial
- Conway sequence and constant
- Sprouts and variants
- Game of life
- Conway circle

Things Named After John Conway



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Related changes
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List of things named after John Horton Conway

From Wikipedia, the free encyclopedia

This is a list of things named after the English mathematician [John Horton Conway](#) (1937–2020).

- [Conway algebra](#) – an algebraic structure introduced by [Paweł Traczyk](#) and [Józef H. Przytycki](#)^[1]
- [Conway base 13 function](#) – a function used as a counterexample to the converse of the [intermediate value theorem](#)^[2]
- [Conway chained arrow notation](#) – a notation for expressing certain extremely large numbers^[3]
- [Conway circle](#) – a geometrical construction based on extending the sides of a triangle^[4]
- [Conway criterion](#) – a criterion for identifying [prototiles](#) that admit a [periodic tiling](#)^[5]
- [Conway group](#) – any of the groups Co_0 , Co_1 , Co_2 , or Co_3 ^[6]
- [Conway group Co1](#) – one of the [sporadic simple groups](#) discovered by Conway in 1968^[6]
- [Conway group Co2](#) – one of the sporadic simple groups discovered by Conway in 1968^[6]
- [Conway group Co3](#) – one of the sporadic simple groups discovered by Conway in 1968^[6]
- [Conway knot](#) – a particular knot in knot theory
- [Conway notation \(knot theory\)](#) – a notation invented by Conway for describing knots in [knot theory](#)^[7]
- [Conway polyhedron notation](#) – notation invented by Conway used to describe polyhedra^[8]
- [Conway polynomial \(finite fields\)](#) – an irreducible polynomial used in finite field theory^[8]
- [Conway puzzle](#) – a packing problem invented by Conway using rectangular blocks^[9]
- [Conway sphere](#) – a 2-sphere intersecting a given knot in the 3-sphere or 3-ball transversely in four points^[7]
- [Conway triangle notation](#) – notation which allows trigonometric functions of a triangle to be managed algebraically^[8]
- [Conway's 99-graph problem](#) – a problem invented by Conway asking if a certain undirected graph exists^[10]
- [Conway's constant](#) – a constant used in the study of the [Look-and-say sequence](#)^[11]
- [Conway's dead fly problem](#) – does there exist a [Danzer set](#) whose points are separated at a bounded distance from each other?
- [Conway's Game of Life](#) – a cellular automaton defined on the two-dimensional orthogonal grid of square cells^[9]
- [Conway's Soldiers](#) – a one-person mathematical game resembling [peg solitaire](#)^[12]
- [Conway's thrackle conjecture](#) – In graph theory, the conjecture that no [thrackle](#) has more edges than vertices
- [Alexander–Conway polynomial](#) – a knot invariant which assigns a polynomial to each knot type in [knot theory](#)^[7]



The Conway group Co_1 is the quotient of index 2 of the automorphism group of the 24-dimensional Leech lattice.

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The group has 4 157 776 806 543 360 000 elements.

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The group has 4 157 776 806 543 360 000 elements. It is one of three Conway groups among the 26 sporadic groups in the classifications of finite simple groups.

Conway polynomial: Knots and Links

Knot



Conway polynomial: Knots and Links

Knot



Link



Conway polynomial: Knot diagram



Conway polynomial: Definition

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The Conway polynomial ∇ (a variant of the Alexander polynomial) is defined by

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The Conway polynomial ∇ (a variant of the Alexander polynomial) is defined by

$$\nabla \left(\text{unknot} \right) = 1$$

$$\nabla \left(\text{crossing} \right) = \nabla \left(\text{crossing} \right) - z \cdot \nabla \left(\text{crossing} \right)$$

The Conway polynomial is a knot invariant, i.e., it does not change when the knot is continuously deformed in three dimensions.

Conway polynomial: Hopf Link

$$\nabla \left(\begin{array}{c} \nearrow \nearrow \\ \searrow \swarrow \end{array} \right) = \nabla \left(\begin{array}{c} \nearrow \nearrow \\ \swarrow \searrow \end{array} \right) - z \cdot \nabla \left(\begin{array}{c} \searrow \nearrow \\ \swarrow \searrow \end{array} \right)$$

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$$\nabla \left(\begin{array}{c} \nearrow \\ \nwarrow \\ \nearrow \\ \nwarrow \end{array} \right) = \nabla \left(\begin{array}{c} \nearrow \\ \nwarrow \\ \nearrow \\ \nwarrow \end{array} \right) - z \cdot \nabla \left(\begin{array}{c} \nearrow \\ \nearrow \\ \nwarrow \\ \nwarrow \end{array} \right)$$

$$\nabla \left(\begin{array}{c} \curvearrowright \\ \curvearrowright \end{array} \right) = \nabla \left(\begin{array}{c} \curvearrowright \\ \curvearrowright \end{array} \right) - z \cdot \nabla \left(\begin{array}{c} \curvearrowright \\ \curvearrowright \end{array} \right)$$

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$$\begin{array}{c} \nabla \left(\begin{array}{c} \curvearrowright \\ \curvearrowright \end{array} \right) = \nabla \left(\begin{array}{c} \curvearrowright \\ \curvearrowright \end{array} \right) - z \cdot \nabla \left(\begin{array}{c} \curvearrowright \\ \curvearrowright \end{array} \right) \\ 1 = 1 - z \cdot \nabla \left(\begin{array}{c} \curvearrowright \\ \curvearrowright \end{array} \right) \end{array}$$

Conway polynomial: Hopf Link

$$\nabla \left(\begin{array}{c} \nearrow \\ \nwarrow \\ \nearrow \\ \nwarrow \end{array} \right) = \nabla \left(\begin{array}{c} \nearrow \\ \nwarrow \\ \nwarrow \\ \nearrow \end{array} \right) - z \cdot \nabla \left(\begin{array}{c} \nearrow \\ \nearrow \\ \nwarrow \\ \nwarrow \end{array} \right)$$

$$\begin{array}{c} \nabla \left(\begin{array}{c} \nearrow \\ \nearrow \\ \nwarrow \\ \nwarrow \end{array} \right) = \nabla \left(\begin{array}{c} \nearrow \\ \nwarrow \\ \nearrow \\ \nwarrow \end{array} \right) - z \cdot \nabla \left(\begin{array}{c} \text{circle} \\ \text{circle} \end{array} \right) \\ 1 = 1 - z \cdot \nabla \left(\begin{array}{c} \text{circle} \\ \text{circle} \end{array} \right) \end{array}$$

$$\Rightarrow \nabla \left(\begin{array}{c} \text{circle} \\ \text{circle} \end{array} \right) = 0.$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Hopf Link} \right) = \nabla \left(\text{Crossing} \right) - z \cdot \nabla \left(\text{Hopf Link} \right),$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Hopf Link} \right) = \nabla \left(\text{Crossing} \right) - z \cdot \nabla \left(\text{Hopf Link} \right),$$

$$\nabla \left(\text{Trivial Link} \right) = 1 \text{ and } \nabla \left(\text{Hopf Link} \right) = 0.$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Diagram 1} \right) = \nabla \left(\text{Diagram 2} \right) - z \cdot \nabla \left(\text{Diagram 3} \right),$$

$$\nabla \left(\text{Diagram 4} \right) = 1 \text{ and } \nabla \left(\text{Diagram 5} \right) = 0.$$

$$\nabla \left(\text{Diagram 6} \right) = \nabla \left(\text{Diagram 7} \right) - z \cdot \nabla \left(\text{Diagram 8} \right)$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Diagram 1} \right) = \nabla \left(\text{Diagram 2} \right) - z \cdot \nabla \left(\text{Diagram 3} \right),$$

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$$\begin{aligned} \nabla \left(\text{Diagram 6} \right) &= \nabla \left(\text{Diagram 7} \right) - z \cdot \nabla \left(\text{Diagram 8} \right) \\ &= \nabla \left(\text{Diagram 9} \right) - z \cdot \nabla \left(\text{Diagram 10} \right) \end{aligned}$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Diagram 1} \right) = \nabla \left(\text{Diagram 2} \right) - z \cdot \nabla \left(\text{Diagram 3} \right),$$

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$$\begin{aligned} \nabla \left(\text{Diagram 6} \right) &= \nabla \left(\text{Diagram 7} \right) - z \cdot \nabla \left(\text{Diagram 8} \right) \\ &= \nabla \left(\text{Diagram 9} \right) - z \cdot \nabla \left(\text{Diagram 10} \right) \\ &= 0 - z \cdot 1 = -z. \end{aligned}$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Hopf Link} \right) = -z.$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Hopf Link} \right) = -z.$$

$$\nabla \left(\text{Two Unlinked Circles} \right) = 0.$$

Conway polynomial: Hopf Link

$$\nabla \left(\text{Hopf Link} \right) = -z.$$

$$\nabla \left(\text{Two Unlinked Rings} \right) = 0.$$

Two rings linked together cannot be separated in three dimensions without cutting them.

Conway polynomial: Trefoil knot

Exercise

$$\nabla \left(\text{Trefoil Knot} \right) = ?$$

Conway sequence

1

Conway sequence

1
11

Conway sequence

1

11

21

Conway sequence

1
11
21
1211

Conway sequence

1
11
21
1211
111221

Conway sequence

1

11

21

1211

111221

312211

Conway sequence

1
11
21
1211
111221
312211
13112221

Conway sequence

1
11
21
1211
111221
312211
13112221
1113213211

Conway sequence

Conway sequence	Length
1	1
11	2
21	2
1211	4
111221	6
312211	6
13112221	8
1113213211	10
31131211131221	14

Conway sequence: Conway constant

$$\lim_{n \rightarrow \infty} \frac{\ell_{n+1}}{\ell_n} = \lambda$$

Conway sequence: Conway constant

$$\lim_{n \rightarrow \infty} \frac{\ell_{n+1}}{\ell_n} = \lambda$$

with the Conway constant

$$\lambda = 1.303577269034 \dots,$$

the only positive root of

Conway sequence: Conway constant

$$\begin{aligned} & x^{71} - x^{69} - 2x^{68} - x^{67} + 2x^{66} + 2x^{65} + x^{64} - x^{63} - x^{62} - x^{61} - x^{60} - x^{59} \\ & + 2x^{58} + 5x^{57} + 3x^{56} - 2x^{55} - 10x^{54} - 3x^{53} - 2x^{52} + 6x^{51} + 6x^{50} + x^{49} \\ & + 9x^{48} - 3x^{47} - 7x^{46} - 8x^{45} - 8x^{44} + 10x^{43} + 6x^{42} + 8x^{41} - 5x^{40} - 12x^{39} \\ & + 7x^{38} - 7x^{37} + 7x^{36} + x^{35} - 3x^{34} + 10x^{33} + x^{32} - 6x^{31} - 2x^{30} - 10x^{29} \\ & - 3x^{28} + 2x^{27} + 9x^{26} - 3x^{25} + 14x^{24} - 8x^{23} - 7x^{21} + 9x^{20} + 3x^{19} - 4x^{18} \\ & - 10x^{17} - 7x^{16} + 12x^{15} + 7x^{14} + 2x^{13} - 12x^{12} - 4x^{11} - 2x^{10} + 5x^9 + x^7 \\ & - 7x^6 + 7x^5 - 4x^4 + 12x^3 - 6x^2 + 3x - 6. \end{aligned}$$

Conway sequence: Atoms

1

Conway sequence: Atoms

1

11

Conway sequence: Atoms

1

11

21

Conway sequence: Atoms

1
11
21
12 | 11

Conway sequence: Atoms

1

11

21

12 | 11

111221

Conway sequence: Atoms

1

11

21

12 | 11

111221

31 | 22 | 11

Conway sequence: Atoms

1

11

21

12 | 11

111221

31 | 22 | 11

13 | 11 |₂ 2221

Conway sequence: Atoms

1
11
21
12 | 11
111221
31 | 22 | 11
13|11 |₂ 2221
1113|₂21 |₃ 32 | 11

Conway sequence: Atoms

1

11

21

12 | 11

111221

31 | 22 | 11

13 | 11 |₂ 2221

1113 |₂ 21 |₃ 32 | 11

3113 |₃ 12 | 11 |₄ 13 | 1221

Conway sequence: Atoms

1
11
21
12 | 11
111221
31 | 22 | 11
13 | 11 |₂ 2221
1113 |₂ 21 |₃ 32 | 11
3113 |₃ 12 | 11 |₄ 13 | 1221
13 | 2113 |₄ 1112 |₂ 3113 | 11 | 22 | 12

Conway sequence: Atoms

1
11
21
12 | 11
111221
31 | 22 | 11
13 | 11 |₂ 2221
1113 |₂ 21 |₃ 32 | 11
3113 |₃ 12 | 11 |₄ 13 | 1221
13 | 2113 |₄ 1112 |₂ 3113 | 11 | 22 | 12
1113 |₂ 1221133112 |₃ 13 | 2113 |₂ 21 |₂ 22 |₂ 1112

Conway sequence: Atoms

11132 13211
311312 | 11131221
1321131112 |₂ 3113 | 112212
11131221133112 |₃ 13211321221112
3113212221232112 |₄ 1113122113121122
132113121132111213122112 |₅ 31131122211311122122
111312211311122113123112111311222112 |₆ 1321132132211331221122

Conway sequence: Atoms

11132 13211
311312 | 11131221
1321131112 |₂ 3113112212
11131221133112 |₃ 13211321221112
3113212221232112 |₄ 1113122113121122
132113121132111213122112 |₅ 31131122211311122122
111312211311122113123112111311222112 |₆ 1321132132211331221122

Conway sequence: Atoms

11132 and 13211 are 2 of 92 atoms. It just remains to show how each atom decays into other atoms. The Conway constant λ is the largest eigenvalue of the transition matrix.

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λ is the growth constant for any starting string of positive integers with one exception:

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Sprouts: Rules

Example with $n = 3$.

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- Two players alternate.

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Example with $n = 3$.

- Two players alternate.
- Start with $n \in \mathbb{N}$ vertices.

Example with $n = 3$.

- Two players alternate.
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- A move consists of connecting two vertices and placing a new vertex on the new edge.

Example with $n = 3$.

- Two players alternate.
- Start with $n \in \mathbb{N}$ vertices.
- A move consists of connecting two vertices and placing a new vertex on the new edge.
- The new edge may return to the starting vertex.

Example with $n = 3$.

- Two players alternate.
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- The maximal degree of each vertex is three.
- A player who does not have a legal move loses.

Sprouts: Does it end?

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Each move uses up two free spots and provides one new spot.

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Let A and B be the players and let A be the starting player. The table lists the winner for the given number n of starting vertices.

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Open problem.

Like Sprouts, but instead of vertices, we use crosses that mark four free ends for edges. On a new edge, we place a notch that provides two new free ends, one in each direction from the edge.

Brussels Sprouts: Does it end?

Each move removes two free ends and introduces two free ends, so there are always $4n$ free ends. A face contains at least one free end, so there can be at most $4n$ faces. However, it is not possible to play indefinitely without drawing more than $4n$ faces.

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<i>Winner</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>

Brussels Sprouts: Who wins?

Game ends when each face contains exactly one free end after m moves. There are $2m$ edges and $n + m$ vertices in the end. Each move removes two free ends and introduces two free ends, so there are always $4n$ free ends and therefore $4n$ faces in the end.

Brussels Sprouts: Who wins?

Euler's formula

$$(n + m) - 2m + 4n = 2.$$

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Players win according to the parity of n independently of their chosen moves.

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Boring.

Planted Brussels Sprouts

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Caleb Ji, James Propp: *Brussels Sprouts, Noncrossing Trees, and Parking Functions*,
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Nonetheless, games that are trivial from the point of view of strategy may still pose interesting questions for enumerative combinatorics. (“If you can’t beat ’em, count how many ways they can beat you.”)

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We show that the endstates of the game are in natural bijection with noncrossing trees and that the game histories are in natural bijection with both parking functions and factorizations of a cycle of S_n .

Game of Life: Rules

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Is this really a game? „Zero-player game“

Balanced rules allow complex behavior: Glider

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Example of changed rules.

Several video interviews with John Conway are available on Numberphile.

Homework: Conway Circle

