

## CONTROLLABILITY RESULTS FOR SEMILINEAR FUNCTIONAL AND NEUTRAL FUNCTIONAL EVOLUTION EQUATIONS WITH INFINITE DELAY

Selma Baghli, Mouffak Benchohra and Khalil Ezzinbi

**Abstract.** In this paper sufficient conditions are given ensuring the controllability of mild solutions defined on a bounded interval for two classes of first order semilinear functional and neutral functional differential equations involving evolution operators when the delay is infinite using the nonlinear alternative of Leray-Schauder type.

[Full text](#)

### References

- [1] M. Adimy, H. Bouzahir and Kh. Ezzinbi, *Existence for a class of partial functional differential equations with infinite delay*, Nonlinear Anal. **46** (2001), 91-112. [MR1845579\(2002e:34126\)](#). [Zbl 1003.34068](#)
- [2] M. Adimy, H. Bouzahir and Kh. Ezzinbi, *Local existence and stability for some partial functional differential equations with infinite delay*, Nonlinear Anal. **48** (2002), 323-348. [MR1869515\(2003e:34145\)](#). [Zbl 0996.35080](#)
- [3] M. Adimy, H. Bouzahir and Kh. Ezzinbi, *Existence and stability for some partial neutral functional differential equations with infinite delay*, J. Math. Anal. Appl. **294** (2004), 438-461. [MR2061336\(2005b:35280\)](#). [Zbl 1050.35119](#)
- [4] N.U. Ahmed, *Semigroup Theory with Applications to Systems and Control*, Pitman Research Notes in Mathematics Series **246** Longman Scientific & Technical, Harlow John Wiley & Sons, Inc., New York, 1991. [MR1100706\(92e:47069\)](#). [Zbl 0727.47026](#)

---

2000 Mathematics Subject Classification: 34G20; 34K40; 93B05.

Keywords: controllability; existence; semilinear functional; neutral functional differential evolution equations; mild solution; fixed-point; evolution system; infinite delay.

\*\*\*\*\*

<http://www.utgjiu.ro/math/sma>

- [5] N.U. Ahmed, *Dynamic Systems and Control with Applications*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2006. [MR2257896\(2007m:93001\)](#). [Zbl 1127.93001](#)
- [6] A. Arara, M. Benchohra, L. Górniewicz and A. Ouahab, *Controllability results for semilinear functional differential inclusions with unbounded delay*, *Math. Bulletin* **3** (2006), 157-183. [Zbl 1150.34594](#)
- [7] S. Baghli and M. Benchohra, *Uniqueness results for partial functional differential equations in Fréchet spaces*, *Fixed Point Theory* **9**(2) (2008), 395-406. [MR2464126](#).
- [8] S. Baghli and M. Benchohra, *Existence results for semilinear neutral functional differential equations involving evolution operators in Fréchet spaces*, *Georgian Math. J.*, to appear.
- [9] S. Baghli and M. Benchohra, *Uniqueness results for evolution equations with infinite delay in Fréchet spaces*, submitted.
- [10] K. Balachandran and J. P. Dauer, *Controllability of nonlinear systems in Banach spaces: A survey. Dedicated to Professor Wolfram Stadler*, *J. Optim. Theory Appl.* **115** (2002), 7-28. [MR1937343\(2004b:93011\)](#). [Zbl 1023.93010](#)
- [11] M. Belmekki, M. Benchohra, K. Ezzinbi and S. K. Ntouyas, *Existence results for some partial functional differential equations with unbounded delay*, *Nonlinear Stud.*, (to appear).
- [12] M. Benchohra, E. P. Gatsori, L. Górniewicz and S. K. Ntouyas, *Controllability results for evolution inclusions with non-local conditions*, *Z. Anal. Anwendungen* **22**(2) (2003), 411-431. [MR2000275\(2004f:93009\)](#). [Zbl 1052.34073](#)
- [13] M. Benchohra, L. Górniewicz, S. K. Ntouyas and A. Ouahab, *Controllability results for nondensely semilinear functional differential equations*, *Z. Anal. Anwendungen* **25** (2006), 311-325. [MR2251956\(2007e:93012\)](#). [Zbl 1101.93007](#)
- [14] M. Benchohra, L. Górniewicz and S. K. Ntouyas, *Controllability of Some Nonlinear Systems in Banach spaces: The fixed point theory approach*, Pawel Wlodkiewicz University College, Plock, 2003. [Zbl 1059.49001](#)
- [15] M. Benchohra, L. Górniewicz and S. K. Ntouyas, *Controllability of neutral functional differential and integrodifferential inclusions in Banach spaces with nonlocal conditions*, *Nonlinear Anal. Forum* **7** (2002), 39-54. [MR1959931\(2004a:93008\)](#). [Zbl 1040.93005](#)
- [16] M. Benchohra, L. Górniewicz and S. K. Ntouyas, *Controllability results for multivalued semilinear differential equations with nonlocal conditions*, *Dynam. Systems Appl.* **11** (2002), 403-414. [MR1941759\(2003k:93005\)](#). [Zbl 1028.34013](#)

\*\*\*\*\*

Surveys in Mathematics and its Applications **4** (2009), 15 – 39

<http://www.utgjiu.ro/math/sma>

- [17] M. Benchohra and S. K. Ntouyas, *Controllability results for multivalued semilinear neutral functional equations*, Math. Sci. Res. J. **6** (2002), 65–77. [MR1901589\(2003d:93010\)](#). [Zbl 1036.34092](#)
- [18] A. Bensoussan, G. Da Prato, M.C. Delfour and S.K. Mitter, *Representation and Control of Infinite Dimension Systems*, Vol. **2**, Systems and Control: Foundations and Applications, Birkhauser, Boston, 1993. [MR1246331\(94m:49001\)](#). [Zbl 0790.93016](#)
- [19] E.N. Chukwu and S.M. Lenhart, *Controllability questions for nonlinear systems in abstract spaces*. J. Optim. Theory Appl. **68**(3) (1991), 437–462. [MR1097312\(92d:93022\)](#). [Zbl 0697.49040](#)
- [20] C. Corduneanu and V. Lakshmikantham, *Equations with unbounded delay*, Nonlinear Anal., **4** (1980), 831–877. [MR586852\(81i:34061\)](#). [Zbl 0449.34048](#)
- [21] R. Curtain and H.J. Zwart, *An Introduction to Infinite Dimensional Linear Systems Theory*, Springer-Verlag, New-York, 1995. [MR1351248\(96i:93001\)](#). [Zbl 0839.93001](#)
- [22] K.J. Engel and R. Nagel, *One-Parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000. [MR1721989\(2000i:47075\)](#). [Zbl 0952.47036](#)
- [23] K. Ezzinbi, *Existence and stability for some partial functional differential equations with infinite delay*, Electron. J. Differential Equations, **2003**(116) (2003), 1–13. [MR2022064\(2005f:34210\)](#). [Zbl 1070.34103](#)
- [24] A. Friedman, *Partial Differential Equations*, Holt, Rinehat and Winston, New York, 1969. [MR0445088\(56 #3433\)](#). [Zbl 0224.35002](#)
- [25] X. Fu, *Controllability of neutral functional differential systems in abstract space*, Appl. Math. Comput. **141** (2003), 281–296. [MR1972908\(2004a:93010\)](#). [Zbl 01981452](#)
- [26] X. Fu, *Controllability of abstract neutral functional differential systems with unbounded delay*, Appl. Math. Comput. **151** (2004), 299–314. [MR2044202\(2004k:34158\)](#). [Zbl 1044.93008](#)
- [27] X. Fu and K. Ezzinbi, *Existence of solutions for neutral functional differential evolution equations with nonlocal conditions*, Nonlinear Anal. **54** (2003), 215–227. [MR1979731\(2004d:34169\)](#). [Zbl 1034.34096](#)
- [28] E. P. Gatsori, *Controllability results for nondensely defined evolution differential inclusions with nonlocal conditions*, J. Math. Anal. Appl., **297** (2004), 194–211. [MR2080376\(2004d:34169\)](#). [Zbl 1059.34037](#)

\*\*\*\*\*

- [29] A. Granas and J. Dugundji, *Fixed Point Theory*, Springer-Verlag, New York, 2003. [MR1987179](#)(2004d:58012). [Zbl 1025.47002](#)
- [30] J. Hale and J. Kato, *Phase space for retarded equations with infinite delay*, Funkcial. Ekvac. **21** (1978), 11-41. [MR0492721](#)(58 #11793). [Zbl 0383.34055](#)
- [31] J. K. Hale and S. M. Verduyn Lunel, *Introduction to Functional Differential Equations*, Applied Mathematical Sciences **99**, Springer-Verlag, New York, 1993. [MR1243878](#)(94m:34169). [Zbl 0787.34002](#)
- [32] H. R. Henriquez, *Existence of periodic solutions of neutral functional differential equations with unbounded delay*, Proyecciones **19**(3) (2000), 305-329. [MR1808725](#)(2001k:35283).
- [33] E. Hernandez, *Regularity of solutions of partial neutral functional differential equations with unbounded delay*, Proyecciones **21**(1) (2002), 65-95. [MR1909696](#)(2003d:34169).
- [34] E. Hernandez, *A Massera type criterion for a partial neutral functional differential equation*, Electron. J. Differential Equations **2002**(40) (2002), 1-17. [MR1907716](#)(2003f:34150). [Zbl 0999.35100](#)
- [35] Y. Hino, S. Murakami, and T. Naito, *Functional Differential Equations with Unbounded Delay*, Lecture Notes in Mathematics, **1473**, Springer-Verlag, Berlin, 1991. [MR1122588](#)(92g:34088). [Zbl 0732.34051](#)
- [36] F. Kappel, W. Schappacher, *Some considerations to the fundamental theory of infinite delay equation*, J. Differential Equations **37** (1980), 141-183. [MR587220](#)(81j:34117). [Zbl 0466.34036](#)
- [37] V. Kolmanovskii, and A. Myshkis, *Introduction to the Theory and Applications of Functional-Differential Equations*. Mathematics and its Applications, 463. Kluwer Academic Publishers, Dordrecht, 1999. [MR1680144](#)(2000c:34164). [Zbl 0917.34001](#)
- [38] Y.C. Kwun, J.Y. Park and J.W. Ryu, *Approximate controllability and controllability for delay Volterra system*, Bull. Korean Math. Soc. **28** (2) (1991), 131-145. [MR1127732](#)(92k:93023). [Zbl 0770.93009](#)
- [39] L. Lasiecka and R. Triggiani, *Exact controllability of semilinear abstract systems with application to waves and plates boundary control problems*, Appl. Math. Optimiz. **23** (1991), 109-154. [MR1086465](#)(92a:93021). [Zbl 0729.93023](#)
- [40] G. Li, Sh. Song and Ch. Wu, *Controllability of evolution inclusions with nonlocal conditions*, J. Systems Sci. Complexity, **18**(1) (2005), 35-42. [MR2122874](#)(2005k:93016). [Zbl 1119.93018](#)

\*\*\*\*\*

- [41] G. Li and X. Xue, *Controllability of evolution inclusions with nonlocal conditions*, Appl. Math. Comput., **141**, (2003), 375-384. [MR1972917\(2004d:93014\)](#). [Zbl 1029.93003](#)
- [42] X. Li and J. Yong, *Optimal Control Theory for Infinite Dimensional Systems*, Birkhauser, Berlin, 1995. [MR1312364\(96k:49004\)](#). [Zbl 0817.49001](#)
- [43] K. Naito, *On controllability for a nonlinear Volterra equation*, Nonlinear Anal. **18** (1) (1992), 99-108. [MR1138645\(93a:93019\)](#). [Zbl 0768.93011](#)
- [44] S. Nakagiri and R. Yamamoto, *Controllability and observability for linear retarded systems in Banach space*, Inter. J. Control **49**(5) (1989), 1489-1504. [MR998053\(90h:93014\)](#). [Zbl 0676.93029](#)
- [45] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, New York, 1983. [MR710486\(85g:47061\)](#). [Zbl 0516.47023](#)
- [46] M.D. Quinn and N. Carmichael, *An approach to nonlinear control problems using the fixed point methods, degree theory and pseudo-inverses*, Numer. Funct. Anal. Optim. **7** (1984-1985), 197-219. [MR767382\(86e:58019\)](#). [Zbl 0563.93013](#)
- [47] K. Schumacher, *Existence and continuous dependence for differential equations with unbounded delay*, Arch. Rational Mech. Anal. **64** (1978), 315-335. [MR0477379\(57 #16908\)](#). [Zbl 0383.34052](#)
- [48] R. Triggiani, *On the stabilizability problem in Banach space*, J. Math. Anal. Appl. **52**(3) (1975), 383-403. [MR0445388\(56 #3730\)](#). [Zbl 0326.93023](#)
- [49] J. Wu, *Theory and Applications of Partial Functional Differential Equations*, Applied Mathematical Sciences **119**, Springer-Verlag, New York, 1996. [MR1415838\(98a:35135\)](#). [Zbl 0870.35116](#)
- [50] K. Yosida, *Functional Analysis*, 6<sup>th</sup> edn. Springer-Verlag, Berlin, 1980. [MR617913\(82i:46002\)](#). [Zbl 0435.46002](#)
- [51] J. Zabczyk, *Mathematical Control Theory*, Birkhauser, Berlin, 1992. [MR1193920\(93h:49001\)](#). [Zbl 1071.93500](#)

\*\*\*\*\*

Selma Baghli  
Laboratoire de Mathématiques,  
Université de Sidi Bel-Abbès,  
BP 89, 22000 Sidi Bel-Abbès, Algérie.  
e-mail: selma\_baghli@yahoo.fr

Mouffak Benchohra  
Laboratoire de Mathématiques,  
Université de Sidi Bel-Abbès,  
BP 89, 22000 Sidi Bel-Abbès, Algérie.  
e-mail: benchohra@univ-sba.dz

Khalil Ezzinbi  
Laboratoire de Mathématiques,  
Faculté des Sciences de Semlalia,  
BP 2390, Marrakech, Morocco.  
e-mail: ezzinbi@ucam.ac.ma

\*\*\*\*\*