

ON THE REGULARITY OF MILD SOLUTIONS TO COMPLETE HIGHER ORDER DIFFERENTIAL EQUATIONS ON BANACH SPACES.

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Abstract. For the complete higher order differential equation $u^{(n)}(t) = \sum_{k=0}^{n-1} A_k u^{(k)}(t) + f(t)$, $t \in R$ (*) on a Banach space E , we give a new definition of mild solutions of (*). We then characterize the regular admissibility of a translation invariant subspace \mathcal{M} of $BUC(R, E)$ with respect to (*) in terms of solvability of the operator equation $\sum_{j=0}^{n-1} A_j X \mathcal{D}^j - X \mathcal{D}^n = C$. As application, almost periodicity of mild solutions of (*) is proved.

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