THE LEVI PROBLEM IN $\mathbb{C}^n$: A SURVEY

Harry J. Slatyer

Abstract. We discuss domains of holomorphy and several notions of pseudoconvexity (drawing parallels with the corresponding concepts from geometric convexity), and present a mostly self-contained solution to the Levi problem. We restrict our attention to domains of $\mathbb{C}^n$.

Full text

References

[1] H. Behnke and K. Stein. Konvergente Folgen nichtschlichter Regularitätsbereiche. *Annali di Matematica Pura ed Applicata*, 28(1):317–326, 1949. MR35338(11,719c). Zbl 0038.05402.

[2] L. Bers. *Introduction to Several Complex Variables: Lectures Delivered at the Courant Institute of Mathematical Sciences, New York University, 1962-1963*. Courant Institute of Mathematical Sciences, New York University, 1964.

[3] H. Boas. Lecture notes on several complex variables. http://www.math.tamu.edu/~boas/courses/650-2007c/notes.pdf, accessed August 2012.

[4] S. Bochner. Analytic and meromorphic continuation by means of Green's formula. *Annals of Mathematics*, 44(4):652–673, 1943. MR9206(5,116f). Zbl 0060.24206.

[5] H. J. Bremermann. Über die Äquivalenz der pseudokonvexen Gebiete und der Holomorphiegebiete im Raum von n komplexen Veränderlichen. *Mathematische Annalen*, 128:63–91, 1954. MR71088(17,82a). Zbl 0056.07801.

[6] H. Cartan and P. Thullen. Zur Theorie der Singularitäten der Funktionen mehrerer komplexen Veränderlichen. *Mathematische Annalen*, 106:617–647, 1932. MR1512777. Zbl 0004.35704.

[7] H. Grauert. On Levi's problem and the imbedding of real-analytic manifolds. *Annals of Mathematics*, 68(2):460–472, 1958. MR0098847(20 #5299). Zbl 0108.07804.

[8] R. Gunning and H. Rossi. *Analytic Functions of Several Complex Variables.* Reprint of the 1965 original. AMS Chelsea Publishing, Providence, 2009. MR180696(31 #4927). Zbl 1204.01045.

[9] F. Hartogs. Einige Folgerungen aus der Cauchyschen Integralformel bei Funktionen mehrerer Veränderlichen. *Sitzungsberichte der Königlich Bayerischen Akademie der Wissenschaften zu München, Mathematisch-Physikalische Klasse*, 36:223–242, 1906. JFM 37.0443.01.

[10] L. Hörmander. L^2 estimates and existence theorems for the $\overline{\partial}$ operator. *Acta Mathematica*, 113:89–152, 1965. MR179443(31 #3691). Zbl 0158.11002.

[11] L. Hörmander. *An Introduction to Complex Analysis in Several Variables.* North-Holland Mathematical Library, 7. North-Holland Publishing Co., Amsterdam, 1990. MR1045639(91a:32001). Zbl 0685.32001.

[12] S. Krantz. *Function Theory of Several Complex Variables.* Reprint of the 1992 edition. AMS Chelsea Publishing, Providence, 2001. MR1846625(2002e:32001). Zbl 1087.32001.

[13] P. Lelong. Les fonctions plurisousharmoniques. *Annales scientifiques de l'École Normale Supérieure*, 62:301–338, 1945. MR18304(8,271f). Zbl 0061.23205.

[14] E. Levi. Studii sui punti singolari essenziali delle funzioni analitiche di due o più variabili complesse. *Annali di Matematica Pura ed Applicata*, 17:61–87, 1910. JFM 41.0487.01.

[15] E. Levi. Sulle ipersuperficie dello spazio a 4 dimensioni che possono essere frontiera del campo di esistenza di una funzione analitica di due variabili complesse. *Annali di Matematica Pura ed Applicata*, 18:69–79, 1911. JFM 42.0449.02.

[16] E. Martinelli. Alcuni teoremi integrali per le funzioni analitiche di piu variabili complesse. *Memorie della Reale Accademia d'Italia, Classe di Scienze Fisiche, Matematiche e Naturali, 7. Serie*, 9:269–283, 1938. Zbl 0022.24002.

[17] A. P. Morse. The behavior of a function on its critical set. *Annals of Mathematics*, 40(2):62–70, 1939. MR1503449. Zbl 0020.01205.

[18] R. Narasimhan. The Levi problem for complex spaces. *Mathematische Annalen*, 142:355–365, 1961. MR148943(26 #6439). Zbl 0106.28603.

[19] F. Norguet. Sur les domaines d'holomorphie des fonctions uniformes de plusieurs variables complexes. (Passage du local au global.). *Bulletin de la Société Mathématique de France*, 82:137–159, 1954. MR71087(17,81c). Zbl 0056.07701.

[20] K. Oka. Sur les fonctions analytiques de plusieurs variables. VI. Domaines pseudoconvexes. *Tôhoku Mathematical Journal*, 49:15–52, 1942. MR14470(7,290a). Zbl 0060.24006.

[21] K. Oka. Sur les fonctions analytiques de plusieurs variables. IX. Domaines finis sans point critique intérieur. *Japanese Journal of Mathematics*, 23:97–155, 1953. MR71089(17,82b). Zbl 0053.24302.

[22] R. M. Range. *Holomorphic Functions and Integral Representations in Several Complex Variables*. Graduate Texts in Mathematics. Springer, New York, 1986. MR847923(87i:32001). Zbl 0591.32002.

[23] B. V. Shabat. *Introduction to Complex Analysis. Part II. Functions of Several Variables*. Translations of Mathematical Monographs, 110. American Mathematical Society, Providence, 1992. MR1192135(93g:32001). Zbl 0799.32001.

[24] Y.-T. Siu. Pseudoconvexity and the problem of Levi. *Bulletin of the American Mathematical Society*, 84(4):481–512, 1978. MR477104(57 #16648). Zbl 0423.32008.

[25] H. J. Slatyer. The Levi Problem: references. http://levi-problem.appspot.com/references.html, accessed April 2016.

[26] V. S. Vladimirov. *Methods of the Theory of Functions of Many Complex Variables*. The M.I.T. Press, Cambridge-London, 1966. MR201669(34 #1551). Zbl 0125.31904.

Harry J. Slatyer

Department of Quantum Science, The Australian National University,

Canberra, ACT 0200, Australia.

e-mail: harry.slatyer@anu.edu.au

License