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A STATISTICAL CASE STUDY OF ACQUIRED KNOWLEDGE

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Abstract. The paper performs a detailed statistical study in a case of acquired knowledge based on historical data from university level courses.

1 Background

Between 2005 and 2015, a number of 420 students registered and finalized four upper-level specialized undergraduate courses, entitled “Financial Mathematics I-IV”, under the author’s coordination. Those courses are at the confluence of mathematics, statistics, finance and economics, with a significant computer science contribution. To be able to register in the Financial Mathematics courses, all 420 students had passed the pre-requisites. Their course performance has been evaluated using the following criteria: a midterm worth 30% of the final grade, a final exam worth 50% and assignments worth 20%. The midterm consisted of 3 questions: 1 (essentially) based on mathematics and statistics knowledge, 1 based on finance and economics, and 1 mixed; the final exam consisted of 6 questions: 1 based on mathematics and statistics, 1 based on finance and economics, and 4 mixed (the latter were considered by both students and instructor the most difficult to answer).

It is the purpose of this paper to perform a thorough *statistical analysis* of the model learning outcomes. Our method is inspired by recent developments in the study of students’ learning and satisfaction outcomes (see [1]-[3]), based on content and process knowledge.

2 Methods

The main variable of interest in our study was the impact of the acquired knowledge on the performance of students. The unit of analysis for all statistical tests was the average performance of the students on: an individual question, midterm, final exam, assignments, and final grade. For the purpose of analysis, the separate scores

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from each question of the midterm and final exam were collapsed into a single pool of scores. The assignments' grades were unweighted cummulative and reported as average scores. The unpaired weighted t-test followed by multiple logit regressions were used to compare midterm vs. final grades, *moderated* by assignments and final exam grades. We used an ANCOVA pairwise comparison test followed by logit regressions to compare the individual scores obtained by students at questions from midterm and final exam, *mediated* by their specific backgrounds. Because of the large numer of contrasts being made, family wise alpha was set at $p < .01$. The mean scores of the examination for both midterm and final exam from previous years showed a significant difference across the questions; for instance, in the first year, we had $F(1, 26) > 1$. This justifies weighing each question accordingly (by the instructor); subsequent years produced Cronbach's alpha between .82 and .95, showing internal consistency of the exam questions.

3 Data Analysis

The failure rate at the midterm was 30.48% (128 students), at the final exam of 14.76% (62 students), and the overall failure rate -at all four courses- was 20.48% (76 students). The assignments grades were, in average, in the range of 80%; however, 5.24% (22 students) failed the assignments, i.e., obtained overall assignments grades less than 50%. The summary for midterm, assignments, final grades and final exam scores are presented in Table 1.

Table 1: Midterm, Assignments, Final Exam and Final Grade Scores

scores	mean±standard deviation
midterm	16.5 ± 4.3
final exam	30.4 ± 6.8
final grade	77.3 ± 3.6
assignments	16.2 ± 8.1

Using an unpaired weighted t-test, at 95% confidence level, we obtained that: The midterm and final exam scores are significantly different [$t = 5.257$, degrees of freedom = 608.7, $p < 0.0001$, confidence band = 4.34%]; the final grades and assignments are not significantly different [$t = 1.814$, degrees of freedom = 404.4, $p = 0.0704$, confidence band = 7.02%]; the final grades and final exam scores are not significantly different [$t = 0.837$, degrees of freedom = 333.4, $p = 0.7984$, confidence band = 7.73%].

Interpretation: Students working on the course assignments improved their peformance from the midterm to the final exam; both assignments and the final exam improved the final grades.

To capture the moderating effect of assignments and final exam grades on the relationship between the midterm and final grades, we performed two multiple logit regressions with final grades as dependent variable, and: (i) midterm grades as indepedent variable; (ii) midterm, assignments, final exam, and the interactions midterm-assignments and midterm-final exam, as four independent variables. In case (i) we obtained the following linear regression equation: final grade $= 0.913 \times$ midterm grade $- 6.578$. The output of regression (ii) is summarized in Table 2. Note that the regression coefficients of both the interactions midterm-assignments and midterm-final exam are significantly different among cases (i) and (ii).

Interpretation: Both assignments and final exam have a moderating effect on the relationship between the midterm and final grades.

Table 2: Output of the multiple logit regression

final grades	coefficient	p-value
midterm	2.418	0.830
assignments	3.107	0.615
final exam	8.450	0.730
midterm & assignments	1.415	0.008
midterm & final exam	3.833	0.001

Among the 420 students, 117 (27.85%) were enroled in a Science program, 256 (60.95%) in a Business program, and 47 (11.20%) in two simultaneous programs from Science and Business, respectively. The summary for midterm and final exam scores, by question, are presented in Table 3 and 4, respectively. Levene's test for equality of error variances in the data in Table 3 gives $F = 14.251$, $p < 0.0001$, hence the conditions for ANCOVA are not met.

Interpretation: Due to different backgrounds and programs, students showed large variation within their midterm scores.

Table 3: Comparison between midterm scores and students' program

midterm scores	question 1	question 2	question 3
business	6.1 ± 1.7	18.4 ± 2.5	8.1 ± 3.5
science	17.2 ± 3.4	10.1 ± 2.3	13.4 ± 2.7
business & science	18.2 ± 1.5	18.7 ± 3.8	20.8 ± 6.8

Table 4: Comparison between final exam scores and students' program

final exam scores	question 1	question 2	questions $3-6$
business	17.8 ± 3.5	30.6 ± 2.7	24.1 ± 4.6
science	32.4 ± 2.5	26.2 ± 3.8	29.8 ± 2.2
business & science	33.6 ± 1.7	33.8 ± 2.8	34.1 ± 0.8

Using an ANCOVA pairwise comparison test for the data in Table 4, at 95% confidence level, we obtained that: There is an overall statistically significant difference between the final exam scores for science and "business & science" program students [$F = 105.612$, standard error = 3.8512, $p < 0.0005$, confidence band = 12.3%]; there is an overall statistically significant difference between the final exam scores for business and "business & science" program students [$F = 281.143$, standard error = 4.6254, $p < 0.0005$, confidence band = 11.5%]; yet, there are no significant differences between the scores of science and business program students [$F = 0.089$, $p = 0.8734$].

Interpretation: Students with science-only or business-only background showed similar improvement at the final exam, yet the performance of the students with both science and business background outperforms the former two categories.

To capture the mediating effect of the students' background on the relationship between midterm and final exam, we performed two logit regressions with the final exam as dependent variable, and: (i) midterm grades as independent variable; (ii) midterm and background (business or science vs. business & science) as two independent variables. In case (i) we obtained the linaer regression equation: final exam grade $= 1.873 \times$ midterm grade $- 3.143$. For Regression (ii), by default, the IBM SPSS statistical software sorts all groups and chooses the most frequent one as default; in our case "business"; the output is summarized in Table 5. Note that the regression coefficient of the midterm grades in case (ii) for "business & science" was smaller than its counterpart in case (i).

Interpretation: "business & science" background has a mediatiating effect on the relationship between midterm and final exam; however, neither "science" nor "business" are mediating variables.

An anonymous student satisfaction questionnaire was administred immediately after the final exam. Students were asked to make a valued judgement in relation to: demographic data (gender and age); the value of integration across disciplines to their learning; perceptions regarding the usefulness of their background in the Financial Mathematics courses; and the value of discussion and receiving feedback to the midterm and assignments. In addition, 3 open-ended questions were added: "The best aspects of this class are..."; "The class can be improved by considering

Table 5: Output of the logit regression

background	score	coefficient	p-value
science	midterm	-4.057	0.085
	final exam	-2.035	0.146
business & science	midterm	1.758	0.007
	final exam	3.851	0.003

the following suggestions:..."; and "Other comments:...". The questionnaire showed an overall rating of 4.8 out of 5, and the MANOVA test employed to analyze the students' satisfaction questionnaire showed a small effect size ($f = 0.10$).

Interpretation: Student evaluations are in agreement with the results obtained by students with different backgrounds.

4 Conclusions

Our statistical analysis shows that, if specific, multidisciplinary, university courses, have the appropriate design, then students with different backgrounds may improve on their performance and obtain good results. Moreover, the difference in scores is explained in part by the design; however, further studies assessing both *how* and *what* to teach are required.

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