Non-local in time problems for some equations of mathematical physics

D. Gordeziani, M. Avalishvili, G. Avalishvili

Nonclassical boundary and initial boundary value problems often arise when constructing mathematical models of various phenomena of physics, biology and ecology. Non-local boundary and initial boundary value problems formulated for equations of mathematical physics are nonclassical problems, where instead of the boundary or initial conditions a certain dependence of the values of the unknown function on the boundary on its' values in internal points of the considered domain is given. The nonclassical problems with non-local boundary conditions are used for mathematical modeling of pollution processes in rivers, seas, which are caused by sewage. Non-local boundary conditions simulate decreasing of pollution under influence of natural factors of filtration and settling that cause self-purification of the environment. In nonclassical problems with non-local initial conditions instead of classical initial conditions combinations of the initial values of the unknown function and its' values at later times are given. Non-local in time problems are obtained when modeling the processes of radionuclides propagation in Stokes fluid, diffusion and flow in porous media.

The first investigation of nonclassical initial boundary value problem for the homogeneous heat equation with integral non-local boundary condition was carried out by J. Cannon [1]. Further, similar integral space non-local problems were studied by L. Kamynin, N. Yonkin, A. Samarskii, A. Friedman, A. Bouziani. Different nonclassical boundary value problem with discrete non-local boundary condition first was considered by A. Bitsadze and A. Samarskii [2]. The authors formulated non-local problem for general elliptic equation and in the case of rectangular space domain and Laplace operator applying methods of the theory of integral equations they proved the existence and uniqueness of solution. The problem considered by A. Bitsadze and A. Samarskii and its' generalizations were studied by D. Gordeziani [3] applying different approach. D. Gordeziani suggested rather general iteration procedure, which not only proved the existence of solution, but also provided an opportunity to construct algorithm for numerical solution, since on each step of the procedure classical boundary value problems were considered. Later on, space non-local problems for elliptic, parabolic and hyperbolic equations subject to discrete and integral non-local boundary conditions were considered and investigated by V. Il'in, E. Moiseev, B. Paneyakh, A. Skubachevskii and the others.

Nonclassical initial boundary value problem for parabolic equation with discrete non-local in time initial condition first was considered and investigated by D. Gordeziani [4]. Further, non-local in time problems for evolution equations of mathematical physics were investigated by V.V. Shelukhin, C.V. Pao, R. Ewing, R. Lazarov, Y. Lin and the others.

In the present paper we investigate non-local in time problems for the first, the second order evolution and Schrödinger type equations in abstract Hilbert spaces with general linear non-local initial conditions. We prove the existence and uniqueness of solutions of the non-local in time problems under suitable conditions on non-local operators and construct algorithms of approximation of the solutions of the non-local problems by solutions of the corresponding classical problems. We consider applications of the obtained general results to specific problems and show that the existence and uniqueness of solutions of non-local in time problems for hyperbolic and Schrödinger equations depend on algebraic properties of the expressions involving time moments and geometric characteristics of the space domain. In addition, we investigate nonclassical problem for nonlinear Navier-Stokes equations subject to general nonlinear non-local initial condition, prove the existence of solution and construct an algorithm of approximation by solutions of the classical initial boundary value problems.

References

1. J.R. Cannon, The solution of the heat equation subject to specification of energy, Quart. Appl. Math. **21** (1963), 2, 155-160.

2. A.V. Bitsadze, A.A. Samarskii, On some generalizations of linear elliptic problems, DAN SSSR **185** (1969), 4, 739-740 (in Russian).

3. D.G. Gordeziani, On one method of solution of Bitsadze-Samarskii boundary value problem, Abstr. of Rep. of the Sem. of Inst. Appl. Math. TSU **2** (1970), 38-40 (Russian).

4. D.G. Gordeziani, On some initial conditions for parabolic equations, Rep. of Enlarged Sess. of the Sem. of I. Vekua Inst. Appl. Math. **4** (1989), 57-60.