SEMIMARTINGALE LOCAL TIME AND THE AMERICAN PUT OPTION

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In this paper we investigate the American put option problem in a general diffusion model and its related optimal stopping problem. The main result of the investigation is the representation of a value function of the American put option as a sum of the corresponding European type value function and the early exercise premium. In the finance literature this representation is known under the name of the early exercise premium. It was derived by many authors in terms of the well known Black–Scholes model (i.e. when the dynamics of share values is described by geometric Brownian motion). Among them were I. Karatzas and N. El. Karoui (A new approach to the Skorohod problem, and its applications. *Stochastics and Stochastics Rep.* **34**(1991), No. 1–2, 57–82) and S. Jacka (Optimal stopping and the American put. *Math. Finance* **1**(1991), No. 2, 1–14). The former authors proceeded from the results of the theory of balayage of general type random processes, while Jacka used the fact that a value function of the American put option is a solution of the problem with a free boundary (Stefan's problem). A thorough investigation of the American put option problem in the Black–Scholes model was carried out by R. Myneni (The pricing of the American option. *Ann. Appl. Probab.* **2**(1992), No. 1, 1–23). It should be however noted that the proofs of many authors somewhat lack the clearness.