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LARGE TIME BEHAVIOR OF SOLUTIONS TO NONLINEAR INTEGRO-DIFFERENTIAL SYSTEM ASSOCIATED WITH THE PENETRATION OF A MAGNETIC FIELD INTO A SUBSTANCE

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We study asymptotic behavior of solutions of initial-boundary value problem to the following nonlinear system of integro-differential equations:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial U}{\partial x}\right], \quad \frac{\partial V}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial V}{\partial x}\right], \quad (x,t) \in Q, \tag{1}$$

$$U(0,t) = U(1,t) = V(0,t) = V(1,t) = 0, \quad t \geq 0, \tag{2}$$

$$U(x,0) = U_0(x), \quad V(x,0) = V_0(x), \quad x \in [0,1], \tag{3}$$

where

$$S(x,t) = \int_0^t \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] d\tau, \tag{4}$$

or

$$S(t) = \int_0^t \int_0^1 \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] dx d\tau, \tag{5}$$

$a(S) = (1+S)^p$, $0 < p \leq 1$; $U_0 = U_0(x)$ and $V_0 = V_0(x)$ are given functions; $Q = (0,1) \times (0,\infty)$.

Theorem 1. *If* U_0, $V_0 \in W_2^2(0,1) \cap \overset{\circ}{W}{}_2^1(0,1)$, *then for the solution of the problems (1)-(4) and (1)-(3),(5) the following asymptotic relations hold as* $t \to \infty$*:*

$$\frac{\partial U(x,t)}{\partial x} = O\left(\exp\left(-\frac{t}{2}\right)\right), \quad \frac{\partial V(x,t)}{\partial x} = O\left(\exp\left(-\frac{t}{2}\right)\right),$$

$$\frac{\partial U(x,t)}{\partial t} = O\left(\exp\left(-\frac{t}{2}\right)\right), \quad \frac{\partial V(x,t)}{\partial t} = O\left(\exp\left(-\frac{t}{2}\right)\right)$$

uniformly in x *on* $[0,1]$.

The problem with following boundary conditions is also studied:

$$U(0,t) = V(0,t) = 0, \; U(1,t) = \psi_1, \; V(1,t) = \psi_2, \quad t \geq 0, \tag{6}$$

where $\psi_1 = const \geq 0$, $\psi_2 = const \geq 0$.

Theorem 2. *If* U_0*,* $V_0 \in W_2^2(0,1)$*,* $U_0(0) = V_0(0) = 0$*,* $U_0(1) = \psi_1$*,* $V_0(1) = \psi_2$*,* $\psi_1^2 + \psi_2^2 \neq 0$*, then for the solution of the problems (1),(3),(4),(6) and (1),(3),(5),(6) the following asymptotic relations hold as* $t \to \infty$*:*

$$\frac{\partial U(x,t)}{\partial x} - \psi_1 = O\left(\frac{1}{t^{1+p}}\right), \quad \frac{\partial V(x,t)}{\partial x} - \psi_2 = O\left(\frac{1}{t^{1+p}}\right),$$

$$\frac{\partial U(x,t)}{\partial t} = O\left(\frac{1}{t}\right), \quad \frac{\partial V(x,t)}{\partial t} = O\left(\frac{1}{t}\right)$$

uniformly in x *on* $[0,1]$.