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THE CONTACT PROBLEM FOR COMPOUND PLATE WITH AN ELASTIC SEMI-INFINITE INCLUSION

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It is considered compound elastic plate, as unlimited elastic medium, consisting of two half planes (y>0 and y<0) with different elastic constants (E_+,μ_+ and E_-,μ_-). We assume that it is in conditions of a plane deformation and it is strengthened on semi-axis (0, $+\infty$) with variable thickness $h_0(x)$, with modulus of elasticity $E_0(x)$ and Poisson's coefficient ν_0. The inclusion is loaded by horizontal forces with intensity $\tau_0(x)$,. When crossing semi-axis (0, $+\infty$) the stress field undergoes a jump, while crossing the rest part of axis ox the jump of strains and displacements fields is not observed.

The equilibrium equations of inclusion element has the form

$$\frac{du^{(1)}(x)}{dx} = \frac{1}{E\ (x)}\int_0^x [\tau^{(1)}(t) - \tau_0(t)]\,dt$$

$$\frac{dv^{(1)}(x)}{dx} = 0, \qquad\qquad x > 0$$

and the equilibrium conditions of the inclusion has the form

$$\int_0^\infty [\tau^{(1)}(t) - \tau_0(t)]dt = 0, \qquad\qquad \int_0^\infty q^{(1)}(t)dt = 0$$

where $u^{(1)}(x)$ and $v^{(1)}(x)$ are the horizontal and vertical displacements of inclusion points; $\tau^{(1)}(x)$ and $q^{(1)}(x)$ - the skippings of tangential and normal contact strains, respectively, subjects to determination $E(x) = \dfrac{h_0(x)E_0(x)}{1-\nu_0^2}$.

The derivations of displacements on the border of plane, dependent on outer load, acting on semi axis, has the form

$$\frac{du^{(2)}(x)}{dx} = -Aq^{(2)}(x) + \frac{B}{\pi}\int_0^\infty \frac{\tau^{(2)}(t)dt}{t-x}$$

$$\frac{dv^{(2)}(x)}{dx} = A\tau^{(2)}(x) + \frac{B}{\pi}\int_0^\infty \frac{q^{(2)}(t)dt}{t-x}$$

where $u^{(2)}(x)$ and $v^{(2)}(x)$ are the boundary values of horizontal and vertical displacements on the semi axis; $\tau^{(2)}(x), q^{(2)}(x)$ - the boundary values of tangential and normal strains, respectively. $A=A(E_+, E_-, \mu_+, \mu_-)$, $B=B(E_+, E_-, \mu_+, \mu_-)$.

Using the contact condition of the inclusion with plate the problem is reduced to the system of integral differential equations with variable coefficient of singular operator. If such coefficient varies with power law we can manage to investigate the obtained equations, to get exact solutions and to establish behavior of unknown contact stresses at the ends of elastic inclusion.