J. Nonlinear Sci. Appl. 1 (2008), no. 2, 121–122

The Journal of Nonlinear Sciences and Applications
http://www.tjnsa.com

A COUNTEREXAMPLE TO "COMMON FIXED POINT THEOREM IN PROBABILISTIC QUASI-METRIC SPACE"

DOREL MIHEŢ

Abstract. We give a counterexample to the paper "Common fixed point theorem in probabilistic quasi-metric space" published in the first issue of this journal.

For details on the concepts used in the paper, the reader is referred to the book [1].

A *probabilistic quasi-metric space* is a triple $(X, \mathcal{F}, \tau)$ where X is a nonempty set, τ is a continuous triangular function and $\mathcal{F}$ is a mapping from $X \times X$ to D_+ satisfying the following properties:

(PQM1) $F_{p,q} = F_{q,p} = \varepsilon_0$ if and only if $p = q$;

(PQM2) $F_{p,r} \geq \tau(F_{p,q}, F_{q,r})$ for all $p, q, r \in X$.

Let $(X, \mathcal{F}, \tau)$ be a probabilistic quasi-metric space. Two self mappings f, g of X are said to be *R-weakly commuting* if there exists $R > 0$ such that

$$F_{fgx,gfx}(t) \geq F_{fx,gx}(t/R)$$

for all $x \in X$ and $t > 0$.

The following version of the well known common fixed point theorem of Jungck [2] appears in [3], Theorem 2.3:

Let $(X, \mathcal{F}, \tau)$ be a left complete PQM space with $\tau \geq \tau_W$ and let f, g be two R-weakly commuting self mappings of X satisfying the following conditions:

i)$f(X) \subset g(X)$

ii) f or g is continuous

iii)$F_{fx,fy} \geq C(F_{gx,gy})$ for all $x, y \in X$, where $C : D_+ \to D_+$ is a continuous function such that $C(F) > F$ for each $f \in D_+$ with $F \neq \varepsilon_0$.

Date: Received: 28 September 2008.

2000 *Mathematics Subject Classification.* Primary 54E70; Secondary 54H25.

Key words and phrases. Probabilistic metric spaces; quasi-metric spaces; fixed point theorem; R-weakly commuting maps; triangle function.

Then f and g have a unique common fixed point in X.

Here we give an example to show that the above result is not correct.

1. Main results

Example 1.1. Let $X = R$ and $d(x, y) = |x - y|$.

Then the space $(X, \mathcal{F}, \tau_M)$ where

$$F_{x,y}(t) = \begin{cases} 0, & \text{if} \quad t \leq d(x, y) \\ 1, & \text{if} \quad t > d(x, y) \end{cases}$$

for all $t > 0$ and $\tau_M(F, G)(x) := \sup_{s+t=x} Min\{F(s), G(t)\}$ is a complete probabilistic metric space, for $F_{p,q}(t) > 1 - t$ iff $|x - y| \leq t$.

Let $C(F) = \sqrt{F}$ and $f, g : X \to X,\ fx = x, gx = x + 1$.

Then, the mappings f, g are continuous and $f(X) = g(X) = X$.

Also, since $f \circ g = g \circ f = g$, f and g are R-weakly commuting.

Next, the mapping C is continuous and $C(F) > F\ \forall F \neq \varepsilon_0$ (recall that $F > G$ means $F \geq G$ and $F \neq G$).

On the other hand, as $F_{fx,fy} = F_{gx,gy}$ for all $x, y \in X$ and $F_{x,y}$ takes only the values 0 and 1, it is easy to verify the equality:

$$F_{fx,fy} = C(F_{gx,gy})\ \forall x, y \in X.$$

Therefore, all the conditions of Theorem 2.3 in [3] are satisfied.

However, f and g have not any common fixed point in X.

The result can be corrected if the competeness of X is replaced by the stronger condition of G-completeness (see [4]).

References

[1] M. Grabiec, Y. J. Cho, V. Radu, On nonsymmetric topological and probabilistic structures, Nova Publishers 2006. (document)

[2] Gerald Jungck, Commuting Mappings and Fixed Points, The American Mathematical Monthly, Vol. 83, No. 4 (1976), 261-263. (document)

[3] A.R. Shabani, S. Ghasempour, Common fixed point theorem in probabilistic quasi-metric space, J. Nonlinear Sci. Appl. 1 (2008), no. 1, 31-35. (document), 1.1

[4] R. Vasuki, P. Veeramani, Fixed point theorems and Cauchy sequences in fuzzy metric spaces, Fuzzy Sets and Systems, 135 (3) (2003), 409-413. 1

West University of Timişoara, Faculty of Mathematics and Computer Science; Bv. V. Parvan 4, 300223 Timişoara, Romania

E-mail address: mihet@math.uvt.ro