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S-COINCIDENCE AND S-COMMON FIXED POINT THEOREMS FOR TWO PAIRS OF SET-VALUED NONCOMPATIBLE MAPPINGS IN METRIC SPACE

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Abstract. In this work, the new concepts, normal product of two set-valued mappings, s-weakly compatible, s-common fixed point and the (EA_s) property for two pairs of set-valued mappings are introduced, and the s-common fixed point existence theorems for two pairs of set-valued noncompatible mappings under strict contractive condition are proved, without appeal to continuity of any map involved therein and completeness of underlying space. The results presented in this paper generalize, improve, and unify some recent results in this field.

1. Introduction

The problem for common fixed point is an important and interesting, have wide applications to many fields in the mathematic. For these reasons, various variational inclusions have been intensively studied in recent years. In 1994, Pant[18] initiated the study of noncompatible maps satisfying certain contractive conditions, and afterwards Aamri and El Moutawakil[3] defined a property (EA) for single valued maps on a metric space and obtained some common fixed point theorems for such maps under strict contractive conditions. The class of mappings satisfying (EA) property contains compatible as well as noncompatible maps. Kamran extended the property (EA) for a hybrid pair of single valued and set-valued maps in the [12]. Y. Liu et al. [16] obtained coincidence and common fixed point results for two pairs of hybrid maps defining common (EA) property for such pairs. On the other hand, in 1982, Sessa[20] introduced the concept of weakly commuting maps. Jungck[7]

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generalized the notion of weak commutativity by introducing compatible maps and then weakly compatible maps[8]. Jungck and Rhoades[9] further extended weak compatibility to the setting of single valued and multivalued maps. Since then, many interesting coincidence and common fixed point theorems of compatible and weakly compatible maps under various contractive conditions and assuming the continuity of at least one of the mappings, have been obtained by a number of authors. Recently, Ismat Beg and Mujahid Abbas[4] have discussed and studied the fixed point theorems for two hybrid pairs of single valued and multivalued noncompatible maps.

The aim of this paper is introduce to some new concepts, normal product of two set-valued mappings, s-weakly compatible, s-common fixed point and the (EA_s) property for two pairs of set-valued mappings are introduced, and the s-common fixed point existence theorems for two pairs of set-valued noncompatible mappings under strict contractive condition are proved, without appeal to continuity of any map involved therein and completeness of underlying space which extend, unify and improve the earlier comparable results of a number of authors(see, [1]-[14], [16]-[18]). we refer to [1]-[23] and references contained therein.

2. Preliminaries

Let (X, d) be a metric space. We denoted by $CB(X)$ the family of all nonempty closed bounded subsets of X. For $x \in X$ and $A \subseteq X$, $d(x, A) = inf\{d(x, y) : y \in A\}$. Let H be a Hausdorff metric induced by the metric d of X, that is,

$$H(A, B) = max\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\}, \quad for \quad A, B \in CB(X).$$

Let 2^X denote the family of all the nonempty subsets of X, $CB(X)$ denote the family of all nonempty closed bounded subsets of X and $T : X \to 2^X$ be a set-valued mapping and $Tx = T(x)$ for $x \in X$. Let us show some concepts and results.

Definition 2.1. *Let $P, T : X \to 2^X$, then $PTx = \{P(y) : y \in T(x), \forall x \in X\} \subseteq 2^X$ denote a product of P and T. A product PTx is said to be a normal product, if $P, T : X \to CB(X)$ then $PTx \subseteq CB(X)$ for any $x \in X$.*

It is easy to see that $PTx \neq TPx$ for $x \in X$ in general.

Lemma 2.2. *Let 2^X denote the family of all the nonempty subsets of X, and $G, P, T : X \to 2^X$ be three set-valued mappings, then for $x \in X$, the following relations hold:*
(1) $P(G \bigcup T)x = PGx \bigcup PTx$;
(2) $(G \bigcup T)Px = GPx \bigcup TPx$;
(3) $P(G \bigcap T)x = PGx \bigcap PTx$;
(4) $(G \bigcap T)Px = GPx \bigcap TPx$;
(5) if $\overline{T}x = X - Tx$ denote a complement of the mapping Tx for any $x \in X$, then $\overline{P}Tx = \overline{PTx}$.

Proof.. This directly follows from the definitions of the product and the complement.

Definition 2.3. *Let $P, T : X \to CB(X)$. A point $x \in X$ is said to be:*
(1) fixed point of P if $x \in P(x)$;
(2)S-coincidence point of a pair (P, T) if $Px \subseteq Tx$;

(3) S-common fixed point of a pair (P,T) if $\{x\} \subseteq Px \cap Tx$.

$\boldsymbol{F}_s(P)$, $\boldsymbol{C}_s(P,T)$ *and* $\boldsymbol{F}_s(P,T)$ *denote set of all fixed points of* P*, set of all coincidence points of the pair* (P,T) *and the set of all common fixed points of the pair* (P,T)*, respectively.*

Definition 2.4. *Let* $P, X :\to CB(X)$ *be two set valued mappings, and a product* PT *be a normal product. Set valued mappings* P,T *are said to be:*

(4) S-compatible if $H(Py_n, Tz_n) \to 0$ *for any* $y_n \in Tx_n$ *and any* $z_n \in Px_n$ *whenever* $\{x_n\}$ *is a sequence in* X *such that* $\lim_{n\to\infty} Px_n = \sigma \subseteq \lim_{n\to\infty} Tx_n = A \in CB(X)$.

(5) S-noncompatible if there exists a sequence $\{x_n\}$ *in* X *such that* $\lim_{n\to\infty} Px_n = \sigma \subseteq \lim_{n\to\infty} Tx_n = A \in CB(X)$*, but* $\lim_{n\to\infty} H(Py_n, Tz_n) \neq 0$ *for any* $y_n \in Tx_n$ *and any* $z_n \in Px_n$*, or nonexistent.*

Definition 2.5. *Let* $P, X :\to CB(X)$ *be two set valued mappings, and a product* PT *be a normal product. The pair* (P,T) *is called:*

(6) S-commuting if $TPx = PTx$ *for all* $x \in X$*;*

(7) S-weakly compatible if they commute at their coincidence points, that is, $PTx = TPx$ *whenever* $x \in \boldsymbol{C}_s(P,T)$*;*

(8) $(IT)_s$*-commuting at* $x \in X$ *if* $PTx \subseteq TPx$.

Definition 2.6. *Let* $P,T : X \to CB(X)$*. The set valued map* P *is said to be* T*-weakly S-commuting at* $x \in X$ *if* $Py \subseteq Ty$ *for any* $y \in Px$.

Definition 2.7. *Mappings* $P,T : X \to CB(X)$ *are said to satisfy property* (EA_s) *if there exists a sequence* $\{x_n\}$ *in* X*, some* $\sigma \subseteq X$*, and* $A \in CB(X)$ *such that* $\lim_{n\to\infty} Px_n = \sigma \subseteq \lim_{n\to\infty} Tx_n = A \in CB(X)$

Now we present an example of set valued mapping pair $\{P,T\}$ which satisfies (EA_s) property and P is T weakly S-commuting at some $x \subseteq \mathbf{C}_s(P,T)$.

Example 2.8. *Let* $X = [0,\infty)$ *with usual metric. Define* $P,T : X \to CB(X)$ *by*

$$Px = \begin{cases} \{0\}, & 0 \leq x < 1 \\ [1,\ 1+x], & 1 \leq x < \infty, \end{cases} \tag{2.1}$$

and

$$Tx = \begin{cases} [0,x], & 0 \leq x < 1 \\ [1,\ 2+x], & 1 \leq x < \infty \end{cases} \tag{2.2}$$

It can be easily verified that the product PT be a normal product, the pair $\{P,T\}$ satisfies (EA_s) property and P is T-weakly S-commuting at $x = 0 \in \mathbf{C}_s(P,T)$. Moreover, $F_s(P,T) \neq \emptyset$.

Lemma 2.9. *([5])Let* $A, B \in CB(X)$*, then for any* $x \in A, d(x,B) \leq H(A,B)$.

3. S-Common Fixed Point

The following result extends Theorem 2.1 of [4], and of course, extends Theorem 1 of [22], Theorem 3 of [11] and improves Theorem 2.3 of [4].

Theorem 3.1. *Let (X,d) be a metric space, $G,P,Q,T : X \to CB(X)$ be set valued mappings, and the products PT and GQ be two normal products. If the pair $\{G,Q\}$ satisfies (EA_s) property, $G(X) \subseteq P(X) \subseteq CB(X)$ and there exist, $r \in [0,1)$, $p \in Px$ and $Gy \in CB(X)$ for all $x, y \in X, x \neq y$ such that*

$$H(Tx,Qy) < max\{d(p,Gy), rd(p,Tx), rH(Gy,Qy), \frac{1}{2}[d(p,Qy) + H(Gy,Tx)]\}, \tag{3.1}$$

then the pair $\{P,T\}$ and pair $\{G,Q\}$ have S-coincidence points. Moreover, P,G,T and Q have a S-common fixed point if P is T-weakly S-commuting at $x \in \boldsymbol{C}_s(f,T)$ and G is Q-weakly S-commuting at $y \in \boldsymbol{C}_s(G,T)$.

Proof.. Since the pair $\{G,Q\}$ satisfies property (EA_s), there exist a sequence $\{x_n\}$ in X and $\sigma, D \subseteq CB(X)$ such that $\lim_{n\to\infty} Gx_n = \sigma \subseteq D = \lim_{n\to\infty} Qx_n \in CB(X)$. Since, $G(X) \subseteq P(X) \subseteq CB(X)$, for each x_n, there exists $y_n \in X$ such that $Py_n = Gx_n$. Therefore, $\lim_{n\to\infty} Py_n = \lim_{n\to\infty} Gx_n = \sigma \subseteq D = \lim_{n\to\infty} Qx_n \in CB(X)$. Since $\sigma \in P(X) \cap G(X)$, there exists $u, v \in X$ such that $\sigma = Pu = Gv$. We claim that $Pu \subseteq Tu$. If not, then there exists a element $p \in Pu - Tu$, and $H(Pu,Tu) \geq H(p,Tu) > 0$ for $Tu \in CB(X)$. By condition (3.3), we have,

$$\begin{aligned} &H(Tu,Qx_n) \leq max\{d(p,Gx_n), rd(p,Tu), rH(Gx_n,Qx_n), \frac{1}{2}[d(p,Qx_n) + H(Gx_n,Tu)]\} \\ &\leq max\{H(Pu,Gx_n), rH(Pu,Tu), rH(Gx_n,Qx_n), \frac{1}{2}[H(Pu,Qx_n) + H(Gx_n,Tu)]\}, \end{aligned}$$

where $r \in [0,1)$ and $p \in Pu$.

Taking limit $n \to \infty$, we have

$$\begin{aligned} &H(Tu,D) \leq max\{H(Pu,\sigma), rH(Pu,Tu), rH(\sigma,D), \frac{1}{2}[H(Pu,D) + H(D,Tu)]\} \\ &\leq max\{rH(Pu,Tu), \frac{1}{2}H(\sigma,Tu)\}. \end{aligned}$$

It further implies that

$$H(Pu,Tu) = H(\sigma,Tu) \leq H(D,Tu) \leq max\{rH(Pu,Tu), \frac{1}{2}H(Pu,Tu)\},$$

and $H(Pu,Tu) = 0$, which is a contradiction. Thus $Pu \subseteq Tu$.

Now we show that $\lim_{n\to\infty} Ty_n = D$. Otherwise, there exists a positive real number ε, positive integer N, and a subsequence $\{Ty_{n_k}\}$ of $\{Ty_n\}$ such that $H(Ty_{n_k}, D) \geq \varepsilon$, for $n_k \geq N$. From assumption and the Lemma 2.7, it follows that

$$\begin{aligned}
&H(Ty_{n_k}, D) \leq H(Ty_{n_k}, Qx_{n_k}) + H(Qx_{n_k}, D)\\
&\leq max\{d(p_k, Gx_{n_k}), rd(p_k, Ty_{n_k}), rH(Gx_{n_k}, Qx_{n_k}),\\
&\frac{1}{2}[d(p_k, Qx_{n_k}) + H(Gx_{n_k}, Ty_{n_k})]\} + H(Qx_{n_k}, D),\\
&\leq H(Ty_{n_k}, Qx_{n_k}) + H(Qx_{n_k}, D)\\
&\leq max\{H(Py_{n_k}, Gx_{n_k}), rH(Py_{n_k}, Ty_{n_k}), rH(Gx_{n_k}, Qx_{n_k}) + H(Qx_{n_k}, D),\\
&\frac{1}{2}[H(Py_{n_k}, Qx_{n_k}) + H(Gx_{n_k}, Ty_{n_k})]\} + H(Qx_{n_k}, D),
\end{aligned}$$

where $p_k \in Py_{n_k}, Gx_{n_k} \subseteq CB(X)$.

Apply limit $k \to \infty$,

$$\lim_{n\to\infty} H(Ty_{n_k}, \sigma) \leq \lim_{n\to\infty} H(Ty_{n_k}, D) \leq max\{r \lim_{n\to\infty} H(\sigma, Ty_{n_k}), \frac{1}{2} \lim_{n\to\infty} H(\sigma, Ty_{n_k})\},$$

which is a contradiction. Hence $\lim_{n\to\infty} Ty_n = D$.

We can show that $Gv \subseteq Qv$. In the face, if not, then for any $g \in Gv - Qv$ and $Qv \in CB(X)$, $H(Gv, Qv) \geq d(g, Qv) > 0$. By condition (3.3), we have,

$$H(Ty_n, Qv) \leq max\{H(Py_n, Gv), rH(Py_n, Ty_n), rH(Gv, Qv), \frac{1}{2}[H(Py_n, Qv) + H(Gv, Ty_n)]\},$$

where $r \in [0, 1)$ and $p_n \in Py_n$.

Taking limit $n \to \infty$, we have

$$\begin{aligned}
&H(D, Qv) \leq max\{H(\sigma, Gv), rH(\sigma, D), rH(Gv, Qv), \frac{1}{2}[H(\sigma, Qv) + H(Gv, D)]\}\\
&\leq max\{rH(Gv, Qv), \frac{1}{2}H(Gv, Qv)\}.
\end{aligned}$$

It further implies that

$$H(Gv, Qv) = H(\sigma, Qv) \leq H(D, Qv) \leq max\{rH(Gv, Qv), \frac{1}{2}H(Gv, Qv)\},$$

and $H(Gv, Qv) = 0$, which is a contradiction. Thus $Gv \subseteq Qv$.

Now, we show that $\{u, v\} \subseteq Pu \cap Tu \cap Gv \cap Qv$. P, G, T and Q have a S-common fixed point. By assumption, $P^2u \subseteq TPu$ and $G^2v \subseteq QGv$ because that P is T-weakly S-commuting at $u \in \mathbf{C}_s(f, T)$ and G is Q-weakly S-commuting at $v \in \mathbf{C}_s(G, T)$. Also, using the Lemma 2.7, we obtain, $H(Pu, Gg) \leq H(Tu, Qg)$ for any $g \in Gv$. We claim that $u \in Pu$.

If not, then condition (3.3) implies that

$$\begin{aligned}
&H(Tu,Qp) < max\{d(p,Gp), rd(p,Tu), rH(Gp,Qp), \frac{1}{2}[d(g,Qp)+H(Gp,Tu)]\},\\
&\le max\{H(Pu,Gp), rH(Pu,Tu), rH(Gp,Qp), \frac{1}{2}[H(Gp,Qp)+H(Gp,Tu)]\}\\
&\le max\{H(Tu,Qp), rH(Pu,Tu), rH(Qp,Qp), \frac{1}{2}[H(Qp,Qp)+H(Qp,Tu)]\}\\
&= H(Tu,Qp)
\end{aligned}$$

for $p \in Pu = Gv$ and any $g \in Gp \subseteq Qp$, which is a contradiction and the claim follows. And we claim that $v \in Gv$ as same as the way. It further implies $\{u, v\} \subseteq Pu \cap Tu \cap Gv \cap Qv$.

Lastly, we claim that $u = v$. If not, then condition (3.3) implies that

$$\begin{aligned}
&H(Tu,Qv) < max\{d(p,Gv), rd(p,Tu), rH(Gv,Qv), \frac{1}{2}[d(p,Qv)+H(Gv,Tu)]\}\\
&\le max\{H(Pu,Qv), rH(Pu,Tu), rH(Gv,Qv), \frac{1}{2}[H(Gv,Qv)+H(Gv,Tu)]\}\\
&\le max\{H(Tu,Qv), rH(Pu,Tu), rH(Gv,Qv), \frac{1}{2}[H(Gv,Qv)+H(Qv,Tu)]\}\\
&= H(Tu,Qv)
\end{aligned}$$

where $r \in [0, 1)$, $p \in Pu = Gv \subseteq Qv$, which is again a contradiction and the claim follows. As was stated above the $\{u\} \subseteq Pu \cap Gu \cap Tu \cap Qu$, that is, P, G, T and Q have a S-common fixed point u. This completes the proof.

Corollary 3.2. *Let (X, d) be a metric space, $P, T, G, Q : X \to CB(X)$ be set-valued mappings. The pair $\{G, Q\}$ is S-noncompatible, $G(X) \subseteq P(X) \subseteq CB(X)$ and there exist, $r \in [0, 1)$, $p \in Px$ and $Gy \in CB(X)$ for all $x, y \in X, x \neq y$ such that*

$$H(Tx,Qy) < max\{d(p,Gy), rd(p,Tx), rH(Gy,Qy), \frac{1}{2}[d(p,Qy)+H(Gy,Tx)]\}, \qquad (3.2)$$

then the pair $\{P, T\}$ and pair $\{G, Q\}$ have S-coincidence points. Moreover, P, G, T and Q have a S-common fixed point if P is T-weakly S-commuting at $x \in \boldsymbol{C}_s(f, T)$ and G is Q-weakly S-commuting at $y \in \boldsymbol{C}_s(G, T)$.

Let $P = f$ and $G = g$ be two single-valued mappings, then corollary 3.2 extends corollary 2.2 of [4], to set valued mappings.

Remark 3.3. *Let (X, d) be a metric space, $P, T, G, Q : X \to CB(X)$ be set-valued mappings. The pair $\{G, Q\}$ satisfies (EA_s) property, $G(X) \subseteq P(X) \subseteq CB(X)$, If taking $1 \ge r \ge \frac{1}{2}$ in Theorem 3.1, and for $p \in Px, Gy \in CB(X)(\forall x, y \in X, x \neq y)$ such that*

$$H(Tx,Qy) < max\{d(p,Gy), d(p,Tx), H(Gy,Qy), \frac{1}{2}[d(p,Qy)+H(Gy,Tx)]\}, \qquad (3.3)$$

then pairs $\{P, T\}$ and $\{G, Q\}$ have S-coincidence points. Moreover, P, G, T and Q have a S-common fixed point if P is T-weakly S-commuting at $x \in \boldsymbol{C}_s(f, T)$ and G is Q-weakly S-commuting at $y \in \boldsymbol{C}_s(G, T)$.

Let $\varphi : (0, +\infty) \to (o, +\infty)$ be a continuous and nondecreasing function such that $0 < \varphi(t) < t$ for each $t \in (0, +\infty)$. The following corollary improves Theorem 2.5 of [12], Theorem 2.10 of [16], and Theorem 2.1 of [4].

Corollary 3.4. *Let (X, d) be a metric space, $G, P, Q, T : X \to CB(X)$ be set valued mappings. If the pair $\{G, Q\}$ satisfies (EA_s) property, $G(X) \subseteq P(X) \subseteq CB(X)$ and there exist, $r \in [0, 1)$, $p \in Px$ and $Gy \in CB(X)$ for all $x, y \in X, x \neq y$ such that*

$$H(Tx, Qy) < \varphi(max\{d(p, Gy), rd(p, Tx), rH(Gy, Qy), \frac{1}{2}[d(p, Qy) + H(Gy, Tx)]\}), \quad (3.4)$$

then the pair $\{P, T\}$ and pair $\{G, Q\}$ have S-coincidence points. Moreover, P, G, T and Q have a S-common fixed point if P is T-weakly S-commuting at $x \in \boldsymbol{C}_s(f, T)$ and G is Q-weakly S-commuting at $y \in \boldsymbol{C}_s(G, T)$.

Proof. The proof directly follows from the definition of $\varphi : (0, +\infty) \to (o, +\infty)$ and the method proved Theorem 3.2, and so it is omitted.

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