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GENERALIZED FUZZY RANDOM SET-VALUED MIXED VARIATIONAL INCLUSIONS INVOLVING RANDOM NONLINEAR $(\mathbf{A}_\omega, \eta_\omega)$-ACCRETIVE MAPPINGS IN BANACH SPACES

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Abstract. The main purpose of this paper is to introduce and study a new class of random generalized fuzzy set-valued mixed variational inclusions involving random nonlinear $(\mathbf{A}_\omega, \eta_\omega)$-accretive mappings in Banach Spaces. By using the random resolvent operator associated with random nonlinear $(\mathbf{A}_\omega, \eta_\omega)$-accretive mappings, an existence theorem of solutions for this kind of random generalized fuzzy set-valued mixed variational inclusions is established and a new iterative algorithm with an random error is suggested and discussed. The results presented in this paper generalize, improve, and unify some recent results in this field.

1. Introduction

Variational inclusions are an important and generalization of classical variational inequalities which have wide applications to many fields including, for example, mechanics, physics, optimization and control, nonlinear programming, economics, and engineering sciences and in face, the problems for random variational inclusions(inequalities) are just so. Motivated and inspired by the recent research works in these fascinating areas , the random variational inclusion(inequalities, equalities, quasi-variational inclusions, quasi-complementarity) problems have been introduced and studied by Ahmad and Bazán [1], Chang[5], Chang and Huang [7], Cho et al. [8], Ganguly and Wadhwa[15], Huang [16], Haung and Cho[17], Huang et al.[18], Khan et al.[24], Lan[25] Noor and Elsanosi[33]. Very recently, the problems of

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random fuzzy generalized variational inclusions involving Random nonlinear mappings have been studied by Zhang and Bi[42] in Hilbert Spaces. On the other hand, Monotonicity techniques were extended and applied in recent years because of their importance in the theory of variational inequalities, complementarity problems, and variational inclusions. In 2003, Huang and Fang[19] introduced a class of generalized monotone mappings, maximal η-monotone mappings, and defined an associated resolvent operator. Using resolvent operator methods, which is a very important method to find solutions of variational inequality and variational inclusion problems, they developed some iterative algorithms to approximate the solution of a class of general variational inclusions involving maximal η-monotone operators. Huang and Fang's method extended the resolvent operator method associated with an η-subdifferential operator due to Ding and Luo[12]. In [13], Fang and Huang introduced another class of generalized monotone operators, H-monotone operators, and defined an associated resolvent operator. They also established the Lipschitz continuity of the resolvent operator and studied a class of variational inclusions in Hilbert spaces using the resolvent operator associated with H-monotone operators. In a paper[14], Fang and Huang et al. further introduced a new class of generalized monotone operators, (H, η)-monotone operators, which provide a unifying framework for classes of maximal monotone operators, maximal η-monotone operators, and H-monotone operators. Recently, Lan et al.[27] introduced a new concept of (A, η)-accretive mappings, which generalizes the existing monotone or accretive operators, and studied some properties of (A, η)-accretive mappings and defined resolvent operators associated with (A, η)-accretive mappings. They also studied a class of variational inclusions using the resolvent operator associated with (A, η)-accretive mappings. For these reasons, various variational inclusions have been intensively studied in recent years. For details, we refer the reader to [1-23, 25-42] and the references therein.

Inspired and motivated by recent research works in this field, the main purpose of this paper is to introduce and study a new class of random generalized fuzzy set-valued mixed variational inclusions involving random nonlinear $(\mathbf{A}, \eta)$-accretive mappings in Banach Spaces. By using the resolvent operator associated with random nonlinear $(\mathbf{A}, \eta)$-accretive mappings, an existence theorem of solutions for this kind of fuzzy set-variational inclusions is established and a new iterative algorithm is suggested and discussed. The results presented in this paper generalize, improve, and unify some recent results in this field.

2. Generalized Fuzzy Random Set-Valued Variational Inclusions

Throughout this paper, we suppose that $(\Omega, \Re, \mu)$ is a complete σ-finite measure space and $\mathbf{X}$ is a separable real Banach Space endowed with dual space X^*, the norm $\|\cdot\|$ and the dual pair $\langle \cdot, \cdot \rangle$, between $\mathbf{X}$ and X^*. We denote by $\Im(\mathbf{X})$ the class of Borel σ-fields in $\mathbf{X}$. Let $2^{\mathbf{X}}$, and $CB(\mathbf{X})$ denote the family of all the nonempty subset of $\mathbf{X}$, and the family of all the nonempty bounded closed sets of $\mathbf{X}$, respectively. The generalized dual mapping J_q:$\mathbf{X} \to 2^X$ is defined by

$$J_q(x) = \{f^* \in X^* : \langle x, f^* \rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \forall x \in \mathbf{X},$$

where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is known that, $J_q(x) = \|x\|^{q-2} J_2(x)\ for\ all\ x \neq 0$, J_q is single-valued if X^* is strictly convex, and if $\mathbf{X} = \mathbf{H}$, the Hilbert space, then J_2 becomes the identity mapping on $\mathbf{H}$. The modulus of smoothness of $\mathbf{X}$ is the function $\pi_{\mathbf{X}} : [0, \infty) \to [0, \infty)$ defined by

$$\pi_{\mathbf{X}}(t) = sup\{\frac{1}{2}(\|x+y\| + \|x-y\|) - 1 : \|x\| \leq 1, \|y\| \leq t\}.$$

A Banach space $\mathbf{X}$ is called uniformly smooth if

$$\lim_{t \to 0} \frac{\pi_X(t)}{t} = 0.$$

$\mathbf{X}$ is called q-uniformly smooth if there exists a constant $c > 0$ such that

$$\pi_{\mathbf{X}}(t) \leq ct^q, (q > 1).$$

Remark 2.1. *That J_q is single-valued if $\boldsymbol{X}$ is uniformly smooth, and Hilbert space and $L_p(2 \leq p < \infty)$ space are 2-uniformly smooth Banach space.In what follows we shall denote the single-valued generalized duality mapping by J_q.*

Definition 2.2. *A mapping $x : \Omega \to \boldsymbol{X}$ is said to be measurable if, for any $B \in \Im(\boldsymbol{X})$, $\{\omega \in \Omega : x(\omega) \in B \in \Re\}$.*

Definition 2.3. *A mapping $\boldsymbol{f} : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$ is called a random mapping if, for any $x \in \boldsymbol{X}, \boldsymbol{f}(\omega, x) = y(\omega)$ is measurable. A random mapping $\boldsymbol{f}$ is said to be continuous (resp., linear, bounded) if for any $\omega \in \Omega$, the mapping $\boldsymbol{f}(\omega, \cdot) : \boldsymbol{X} \to \boldsymbol{X}$ is continuous (resp., linear, bounded).*

Similarly, we can define a random mapping $\mathbf{h} : \Omega \times \mathbf{X} \times \mathbf{X} \to \mathbf{X}$. We shall write $x = x(\omega)$, $y = y(\omega)$, $\mathbf{f}_\omega = \mathbf{f}(\omega, x(\omega))$ and $\mathbf{h}_\omega(x, y) = \mathbf{b}(\omega, x(\omega), y(\omega))$ for all $\omega \in \Omega$ and $x(\omega), y(\omega) \in \mathbf{X}$.

It is well-known that a measurable mapping is necessarily a random mapping.

Definition 2.4. *A set-valued mapping $Q : \Omega \to 2^X$ is said to be measurable if, for any $B \in \Im(\boldsymbol{X})$, $Q^{-1}(B) = \{\omega \in \Omega : Q(\omega) \bigcap B \neq \emptyset\} \in \Re$.*

Definition 2.5. *A mapping $u : \Omega \to \boldsymbol{X}$ is called a measurable selection of a set-valued measurable mapping $Q : \Omega \to 2^X$ if, for any $\omega \in \Omega$, $u(\omega)$ is measurable and $u(\omega) \in Q(\omega)$.*

Definition 2.6. *A set-valued mapping $\boldsymbol{Q} : \Omega \times \boldsymbol{X} \to 2^X$ is called a random set-valued mapping if, for any $x \in \boldsymbol{X}$, $\boldsymbol{Q}(\cdot, x)$ is measurable(denoted by $\boldsymbol{Q}_{\omega,x}$, or $\boldsymbol{Q}$).*

Let $F(\mathbf{X})$ be a collection of all fuzzy sets over $\mathbf{X}$. A mapping $\widehat{F} : \mathbf{X} \to FF(\mathbf{X})$ is called a fuzzy mapping. For each $x \in \mathbf{X}$, $\widehat{F}(x)$(denote it by $\widehat{F}_x$, in the sequel) is a fuzzy set on $\mathbf{X}$ and $\widehat{F}_x(y)$ is the membership function of y in $\widehat{F}_x$.

Let $\widehat{B} \in F(\mathbf{X}), q \in [0, 1]$. Then the set

$$(\widehat{B})_q = \{x \in \mathbf{X} : \widehat{B}(x) \geq q\}$$

is called a q-cut set of $\widehat{B}$.

Definition 2.7. *A fuzzy mapping $\widehat{Q} : \Omega \to \mathrm{F}(\boldsymbol{X})$ is called a measurable if, for any $a \in (0, 1], (\widehat{Q}(\cdot))_a : \Omega \to 2^X$ is a measurable set-valued mapping.*

Definition 2.8. *A fuzzy mapping* $\widehat{\boldsymbol{Q}}:\Omega\times\boldsymbol{X}\to\mathrm{F}(\boldsymbol{X})$ *is a random fuzzy mapping if, for any* $x\in\boldsymbol{X}$, $\widehat{\boldsymbol{Q}}(\cdot,x):\Omega\times\boldsymbol{X}\to\mathrm{F}(\boldsymbol{X})$ *is a measurable fuzzy mapping(denoted by* $\widehat{\boldsymbol{Q}}_{\omega,x}$*, short down* $\widehat{\boldsymbol{Q}}$*).*

Let $\widehat{\mathbf{S}}_\omega,\widehat{\mathbf{T}}_\omega,\widehat{\mathbf{G}}_\omega$ and $\widehat{\mathbf{P}}_\omega:\Omega\times\mathbf{X}\to F(\mathbf{X})$ be four random fuzzy mappings satisfying the condition $(*)$:

(*) there exists four functions $a,b,c,d:\mathbf{X}\to(0,1]$ such that for all $(\omega,x)\in\Omega\times\mathbf{X}$, we have $(\widehat{\mathbf{S}}_\omega)_{a(x)}$, $(\widehat{\mathbf{T}}_\omega)_{b(x)}$, $(\widehat{\mathbf{G}}_\omega)_{c(x)}$, $(\widehat{\mathbf{P}}_\omega)_{d(x)}\in CB(\mathbf{X})$, where $CB(\mathbf{X})$ denotes the family of all nonempty bounded closed subsets of $\mathbf{X}$.

By using the random fuzzy mappings $\widehat{\mathbf{S}}_\omega,\widehat{\mathbf{T}}_\omega,\widehat{\mathbf{G}}_\omega$ and $\widehat{\mathbf{P}}_\omega$, we can define four random set-valued mappings $\mathbf{S}_\omega,\mathbf{T}_\omega,\mathbf{G}_\omega,\mathbf{P}_\omega:\mathbf{X}\to CB(\mathbf{X})$ by

$$\mathbf{S}_\omega=(\widehat{\mathbf{S}}_\omega)_{a(x)},\quad \mathbf{T}_\omega=(\widehat{\mathbf{T}}_\omega)_{b(x)},\quad \mathbf{G}_\omega=(\widehat{\mathbf{G}}_\omega)_{c(x)},\quad \mathbf{P}_\omega=(\widehat{\mathbf{P}}_\omega)_{d(x)},$$

for each $x\in\mathbf{X}$. In the sequel, $\mathbf{S}_\omega,\mathbf{T}_\omega,\mathbf{G}_\omega$ and $\mathbf{P}_\omega$ are called the set-valued mappings induced by the fuzzy mappings $\widehat{\mathbf{S}}_\omega$, $\widehat{\mathbf{T}}_\omega$, $\widehat{\mathbf{G}}_\omega$ and $\widehat{\mathbf{P}}_\omega$, respectively.

Let $\mathbf{A}_\omega:\Omega\times\mathbf{X}\to\mathbf{X}$, $\eta_\omega:\Omega\times\mathbf{X}\times\mathbf{X}\to\mathbf{X}$ and $\mathbf{F}_\omega:\Omega\times\mathbf{X}\times\mathbf{X}\to\mathbf{X}$ be single-valued random mappings, and $\widehat{\mathbf{S}}_\omega,\widehat{\mathbf{T}}_\omega,\widehat{\mathbf{G}}_\omega$ and $\widehat{\mathbf{P}}_\omega$ be four random fuzzy set-valued mappings. Let $a,b,c,d:\mathbf{X}\to(0,1]$ be four functions, and $\mathbf{M}_\omega:\Omega\times\mathbf{X}\times\mathbf{X}\to 2^X$ be a random set-valued mapping such that for each $\omega\in\Omega$,and $u\in\mathbf{X}$ $\mathbf{M}_\omega(u,\cdot):\mathbf{X}\to 2^X$ is $(\mathbf{A}_\omega,\eta_\omega)$-accretive mapping and $range(\mathbf{P}_\omega)\bigcap dom(\mathbf{M}_\omega(u,\cdot))\neq\emptyset$. We introduce and study the following problem for a new class of random generalized fuzzy set-valued mixed variational inclusions.

For a given element $g:\Omega\to\mathbf{X}$ and any real-valued random variable $k(\omega)>0$, finding measurable mappings $x=x(\omega),u=u(\omega),v=v(\omega),z=z(\omega),y=y(\omega):\Omega\to\mathbf{X}$ such that

$$\widehat{\mathbf{S}}_\omega(u(\omega))\geq a(x),\widehat{\mathbf{T}}_\omega(v(\omega))\geq b(x),\widehat{\mathbf{G}}_\omega(z(\omega))\geq c(x),\widehat{\mathbf{P}}_\omega(y(\omega))\geq d(x)$$

and

$$g(\omega)\in\mathbf{F}_\omega(u,v)+k(\omega)\mathbf{M}_\omega(z,y),\tag{2.1}$$

which is called generalized fuzzy random set-valued mixed variational inclusions involving random nonlinear $(\mathbf{A}_\omega,\eta_\omega)$-accretive mappings in Banach Spaces.

If $\mathbf{S},\mathbf{T},\mathbf{G},\mathbf{P}:\Omega\times\mathbf{X}\to CB(X)$ are four random set-valued mappings, we can define four random fuzzy mappings $\widehat{\mathbf{S}}_\omega,\widehat{\mathbf{T}}_\omega,\widehat{\mathbf{G}}_\omega,\widehat{\mathbf{P}}_\omega:X\to F(\mathbf{X})$ by

$$x\mapsto\chi_{\mathbf{S}_\omega},x\mapsto\chi_{\mathbf{T}_\omega},x\mapsto\chi_{\mathbf{G}_\omega},x\mapsto\chi_{\mathbf{P}_\omega}$$

where $\chi_{\mathbf{S}_\omega}$, $\chi_{\mathbf{T}_\omega}$, $\chi_{\mathbf{G}_\omega}$ and $\chi_{\mathbf{P}_\omega}$ are the characteristic functions of $\widehat{\mathbf{S}}_\omega$, $\widehat{\mathbf{T}}_\omega$, $\widehat{\mathbf{G}}_\omega$, $\widehat{\mathbf{P}}_\omega$, respectively. Taking $a(x)=b(x)=c(x)=1$ for all $x\in\mathbf{X}$, then problem (2.1) equivalent to the following problem:

For a given element $g:\Omega\to\mathbf{X}$ and any real-valued random variable $k(\omega)>0$, finding $x=x(\omega)\in\mathbf{X}$, and measurable mappings $u=u(\omega)\in\mathbf{S}_\omega,v=v(\omega)\in\mathbf{T}_\omega,z=z(\omega)\in\mathbf{G}_\omega$, $y(\omega)\in\mathbf{G}_\omega$ such that

$$g(\omega)\in\mathbf{F}_\omega(u,v)+k(\omega)\mathbf{M}_\omega(z,y),\tag{2.2}$$

which is called generalized random set-valued mixed variational inclusions involving random nonlinear $(\mathbf{A}_\omega,\eta_\omega)$-accretive mappings in Banach Spaces.

For a suitable choice of $\mathbf{A}_\omega, \eta_\omega, \mathbf{F}_\omega, \mathbf{M}_\omega, \widehat{\mathbf{S}}_\omega, \widehat{\mathbf{T}}_\omega, \widehat{\mathbf{G}}_\omega, \widehat{\mathbf{P}}_\omega$, and $\mathbf{S}_\omega, \mathbf{T}_\omega, \mathbf{G}_\omega, \mathbf{P}_\omega$ and the space $\mathbf{X}$, a number of known classes of variational inclusions and variational inequalities can be obtained as special cases of the general set-valued mixed quasi-variational inclusions (2.2).

Remark 2.9. *(i)Some special case of the problem (2.1):*

If $k(\omega) = 1$, $\boldsymbol{X} = X^* = \boldsymbol{E}$ is a Hilbert space and $\boldsymbol{M}_\omega(z, \cdot) = \partial\phi(\cdot)(\forall z \in \boldsymbol{E})$ is the subdifferential of a lower semi-continuous and η-subdifferentiable function $\phi : \boldsymbol{E} \to \boldsymbol{R}\bigcup\{+\infty\}$. Let for any $v \in \boldsymbol{E}$, $\eta_\omega(v, \boldsymbol{g}_\omega) = v - \boldsymbol{g}_\omega(u, x)$,and $\boldsymbol{F}_\omega(u, v) = \boldsymbol{f}_\omega(x) - p_\omega(v)$ and $(\widehat{\boldsymbol{S}}_\omega)_{a(x)} = \boldsymbol{I}$ is a identical mapping in Hilbert Space $\boldsymbol{E}$, and taking $g_\omega = v \in \boldsymbol{H}(\forall\omega \in \Omega)$,then problem(2.1) becomes the following problem:

Find measurable mappings $u, x, y : \Omega \to \boldsymbol{H}$, such that for each $\omega \in \Omega, v \in \boldsymbol{H}$, hold

$$x(\omega) \in \boldsymbol{G}_\omega(u)_{a(u)}, y(\omega) \in \boldsymbol{P}_\omega(u)_{b(u)},$$

and

$$\langle \boldsymbol{f}_\omega(x) - p_\omega(y), v - \boldsymbol{g}_\omega(u, x)\rangle \geq \phi(\boldsymbol{g}_\omega(u, x)) - \phi(v), \tag{2.3}$$

for all $\omega \in \Omega$, and each measurable mappings $u(\omega), v(\omega) \in \boldsymbol{H}$, which is random generalized nonlinear variational inclusions for random fuzzy mappings in Hilbert space. The form of the problem (2.3) was studied by Zhang and Bi[42]. *And a number of known classes of variational inclusions and variational inequalities in [Chang*[5], *Cho and Lan*[9], *Wadhwa*[15], *Huang* [16], *Haung and Cho*[17], *Noor and Elsanosi*[33], *Li*[31]*] can be obtained as special cases of the problem (2.3) when the fuzzy random set-valued mappings all are taken as general random set-valued mappings in the problem.*

(ii)Some special case of the problem (2.2):

If $\boldsymbol{F}_\omega(u, v) = \boldsymbol{f}_\omega(v(\omega)) + u(\omega)$, $\boldsymbol{M}_\omega(\cdot, w) \equiv \boldsymbol{M}_\omega(w(\omega))$, and $\boldsymbol{S}_\omega = \boldsymbol{G}_\omega = \boldsymbol{I}_\omega$ is a identical mapping, and $\boldsymbol{P}_\omega = p_\omega(x)$ is a single-valued mapping in Banach Space, then the problem (2.2) becomes the following problem:

For a given element $g : \Omega \to \boldsymbol{X}$ and any real-valued random variable $k(\omega) > 0$, finding measurable mapping $x : \Omega \to \boldsymbol{X}$ such that

$$g(\omega) \in \boldsymbol{f}_\omega(x) + u + k(\omega)\boldsymbol{M}_\omega(x), \tag{2.4}$$

$(\forall\omega \in \Omega, u = u(\omega) \in \boldsymbol{S}(\omega, x)$, *the problem (2.4) was studied by Cho and Lan*[9] *which is called generalized nonlinear random (A, η) equations with random relaxed cocoercive mappings in Banach Spaces. A number of known classes of random variational inclusions and variational inequalities, random quasi-variational inclusions, random variational-like inclusions, random complementarity and random quasi-complementarity problems were studied previously by many authors(see, Chang*[5]*–Haung and Cho*[17], *Noor and Elsanosi*[33], [9], [42]*), and for examples, [*[16], [25], [3], [9], [11], [31]*] can be obtained as special cases of the problem (2.4).*

(iii) If in the problem (2.4), $\boldsymbol{P}_\omega = p_\omega(x)$, $\boldsymbol{M}_\omega(\cdot, \cdot) \equiv \partial\phi(\omega, \cdot) : \Omega \times \boldsymbol{H} \to \boldsymbol{H}$ is subdifferentiable, and $\phi(\omega, \cdot)$ is the indicator function of a nonempty closed convex set $\boldsymbol{K}$ in $\boldsymbol{H}$ defined in the form:

$$\phi(y) = \begin{cases} 0 & if\ y \in \boldsymbol{K}, \\ \infty & otherwise, \end{cases}$$

then the problem (2.4) becomes the problem of finding measurable mappings $x, u : \Omega \to \boldsymbol{X}$ *such that* $u \in \boldsymbol{T}_\omega(x)$ *and*

$$\langle \boldsymbol{f}_\omega(x) + u(\omega) - g_\omega, y - p_\omega(x)\rangle \geq 0 \qquad (\forall \omega \in \Omega, y \in \boldsymbol{K}), \tag{2.5}$$

Furthermore, these types of variational inclusions can enable us to study many important nonlinear random problems arising in mechanics, physics, random optimization and random control, nonlinear random programming, random economics, regional, structural, and applied sciences with respect random things and produce in a general and unified framework. Let us recall the following results and concepts.

3. Preliminaries

Definition 3.1. *Let* $\boldsymbol{X}$ *be a q-uniformly smooth Banach Space,* $\eta_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ *and* $\boldsymbol{A}_\omega, \boldsymbol{H}_\omega : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$ *be random single-valued mappings. Then a random set-valued mapping* $\boldsymbol{M}_\omega : \Omega \times \boldsymbol{X} \to 2^X$ *is said to be:*

(i) $\hat{H}$*-continuous if, for any* $\omega \in \Omega, \boldsymbol{M}_\omega(\cdot)$ *is continuous in* $\hat{H}(\cdot,\cdot)$*-continuous i.e.: there exists a real-valued random variable* $\alpha_\omega > 0$ *such that*

$$\hat{H}(\boldsymbol{M}_\omega(x_1(\omega)), \boldsymbol{M}_\omega(x_2(\omega))) \leq \alpha_\omega \|x_1(\omega) - x_2(\omega)\| \quad \forall x_1(\omega), x_2(\omega) \in \boldsymbol{X}, \omega \in \Omega$$

where $\hat{H}(\cdot,\cdot)$ *is the Hausdorff metric on* $CB(\boldsymbol{X})$ *defined as follows: for any* $A, B \in CB(\boldsymbol{X})$,

$$\hat{H}(A, B) = max\{\sup_{x\in A}\inf_{y\in B} \boldsymbol{d}(x, y), \sup_{y\in B}\inf_{x\in A} \boldsymbol{d}(x, y)\};$$

(ii) η_ω*-accretive if, for any* $\omega \in \Omega$,

$$\langle u_1(\omega) - u_2(\omega), J_q\eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq 0,$$

for all $x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{M}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{M}_\omega(x_2(\omega))$*;*

(iii) strictly η_ω*-accretive if, for any* $\omega \in \Omega$,

$$\langle u_1(\omega) - u_2(\omega), J_q\eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq 0,$$

for all $x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{M}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{M}_\omega(x_2(\omega))$*; and the equality holds if and only if* $x_1(\omega) = x_2(\omega)$ *for all* $\omega \in \Omega$*;*

(iv) $r_\omega-$*strongly* η*-accretive if there exists a real-valued random variable* $r(\omega) > 0$ *such that*

$$\langle u_1(\omega) - u_2(\omega), J_q\eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq r(\omega)\|x_1(\omega) - x_2(\omega)\|^q$$

, for all $x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{M}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{M}_\omega(x_2(\omega))$*;*

(v) γ_ω*-relaxed* η_ω*-accretive if there exists a real-valued random variable* $\gamma(\omega) > 0$ *such that, for any* $\omega \in \Omega$,

$$\langle u_1(\omega) - u_2(\omega), J_q\eta_\omega(x_1(\omega), x_2(\omega))\rangle \geq -\gamma(\omega)\|x_1(\omega) - x_2(\omega)\|^q$$

, for all $x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{M}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{M}_\omega(x_2(\omega))$*;*

(vi) $\boldsymbol{H}_\omega$*-accretive, if the* $\boldsymbol{M}_\omega$ *is accretive and* $(\boldsymbol{H}_\omega(\cdot) + \rho(\omega)\boldsymbol{M}_\omega(\cdot))(\boldsymbol{X}) = \boldsymbol{X}$ *for all* $\omega \in \Omega$ *and* $\rho(\omega) > 0$*;*

(vii) $(\boldsymbol{H}_\omega, \eta_\omega)$*-accretive, if the* $\boldsymbol{M}_\omega$ *is* η_ω*-accretive and* $(\boldsymbol{H}_\omega(\cdot) + \rho(\omega)\boldsymbol{M}_\omega(\cdot))(\boldsymbol{X}) = \boldsymbol{X}$ *for all* $\omega \in \Omega$ *and* $\rho(\omega) > 0$ *;*

(viii) $(\boldsymbol{A}_\omega, \eta_\omega)$*-accretive, if the* $\boldsymbol{M}_\omega$ *is* γ*-relaxed* η_ω*-accretive and* $(\boldsymbol{A}_\omega(\cdot)+\rho(\omega)\boldsymbol{M}_\omega(\cdot))(\boldsymbol{X}) = \boldsymbol{X}$ *for all* $\omega \in \Omega$ *and* $\rho(\omega) > 0$.

In a similar way, we can define the strictly η_ω-accretivity and strongly η_ω-accretivity of the single-valued mapping $\mathbf{A}_\omega$.

Definition 3.2. *Let* $\boldsymbol{X}$ *be a q-uniformly smooth Banach Space,* $\boldsymbol{A}_\omega : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$ *and* $\boldsymbol{F}_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ *be random single-valued mappings, and* $\boldsymbol{S}_\omega, \boldsymbol{P}_\omega : \Omega \times \boldsymbol{X} \to 2^X$ *be two random set-valued mappings.*

(i) A random set-valued mapping $\boldsymbol{S}_\omega$ *is said to be* m_ω*-relaxed accretive in the second argument, if there exist a real-valued random variable* $m(\omega)$ *such that*

$$\langle u_1(\omega) - u_2(\omega), j_q(x_1(\omega) - x_2(\omega))\rangle \le -m(\omega)\|x_1(\omega) - x_2(\omega)\|^q,$$

$$\forall x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{S}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{S}_\omega(x_2(\omega)), \forall \omega \in \Omega.$$

(ii) A random set-valued mapping $\boldsymbol{S}_\omega$ *is said to be* s_ω*-cocoercive in the second argument, if there exist a real-valued random variable* $s(\omega) > 0$ *such that*

$$\langle u_1(\omega) - u_2(\omega), j_q(x_1(\omega) - x_2(\omega))\rangle \le s(\omega)\|u_1(\omega) - u_2(\omega)\|^q,$$

$$\forall x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{S}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{S}_\omega(x_2(\omega)), \forall \omega \in \Omega.$$

(iii) A random set-valued mapping $\boldsymbol{S}_\omega$ *is said to be* t_ω*-relaxed cocoercive with respct to* $\boldsymbol{A}_\omega$ *in the second argument, if there exist a real-valued random variable* $t(\omega) > 0$ *such that*

$$\langle u_1(\omega) - u_2(\omega), j_q(\boldsymbol{A}_\omega(x_1(\omega)) - \boldsymbol{A}_\omega(x_2(\omega)))\rangle \le -t(\omega)\|u_1(\omega) - u_2(\omega)\|^q,$$

$$\forall x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{S}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{S}_\omega(x_2(\omega)), \forall \omega \in \Omega.$$

(iv) A random set-valued mapping $\boldsymbol{S}_\omega$ *is said to be* $(\varsigma_\omega, \kappa_\omega)$*-relaxed cocoercive in the second argument, if there exist two positive real-valued random variables* $\varsigma(\omega)$ *and* $\kappa(\omega)$ *such that*

$$\langle u_1(\omega)-u_2(\omega), j_q(\boldsymbol{A}_\omega(x_1(\omega))-\boldsymbol{A}_\omega(x_2(\omega)))\rangle \le -\varsigma(\omega)\|u_1(\omega)-u_2(\omega)\|^q+\kappa(\omega)\|x_1(\omega)-x_2(\omega)\|^q,$$

$$\forall x_1(\omega), x_2(\omega) \in \boldsymbol{X}, u_1(\omega) \in \boldsymbol{S}_\omega(x_1(\omega)), u_2(\omega) \in \boldsymbol{S}_\omega(x_2(\omega)), \forall \omega \in \Omega.$$

(v) A random single-valued mapping $\boldsymbol{F}_\omega$ *is said to be* (μ_ω, ν_ω)*-Lipschitz continuous, if there exist two random variables* $\mu_\omega, \nu_\omega : \Omega \to (0, +\infty)$ *such that*

$$\|\boldsymbol{F}_\omega(x_1(\omega), y_1(\omega)) - \boldsymbol{F}_\omega(x_2(\omega), y_2(\omega))\| \le \mu_\omega\|x_1(\omega) - x_2(\omega)\| + \nu_\omega\|y_1(\omega) - y_2(\omega)\|,$$

$$\forall x_i(\omega), y_i(\omega) \in \boldsymbol{X}, i = 1, 2;$$

(vi) A random single-valued mapping $\boldsymbol{F}_\omega$ *is said to be* $(\varphi_\omega, \psi_\omega)$*-*$\boldsymbol{S}_\omega$*-relaxed cocoercive with respect to* $\boldsymbol{A}_\omega\boldsymbol{P}_\omega$ *in the second argument of* $\boldsymbol{F}(\cdot,\cdot,\cdot)$*, if there exist two random variables* $\varphi, \psi : \Omega \to (0, +\infty)$ *such that*

$$\langle \boldsymbol{F}_\omega(u_1, \cdot) - \boldsymbol{F}_\omega(u_2, \cdot), J_q(\boldsymbol{A}_\omega(y_1) - \boldsymbol{A}_\omega(y_2))\rangle \ge -\varphi_\omega\|\boldsymbol{F}_\omega(u_1, \cdot) - \boldsymbol{F}_\omega(u_2, \cdot)\|^q + \psi_\omega\|x_1 - x_2\|^q,$$

$$\forall x_i(\omega) \in \boldsymbol{X}, u_i(\omega) \in \boldsymbol{S}_\omega(x_i(\omega)), y_i(\omega) \in \boldsymbol{P}_\omega(x_i(\omega))(i = 1, 2), \omega \in \Omega;$$

Definition 3.3. *The random mapping* $\eta_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ *is said to be* τ_ω*-Lipschitz continuous if there exists a real-valued random variable* $\tau(\omega) > 0$ *such that*

$$\|\eta_\omega(x(\omega), y(\omega))\| \le \tau_\omega\|x(\omega) - y(\omega)\|, \quad \forall x(\omega), y(\omega) \in \boldsymbol{X}, \quad \text{and} \quad \forall \omega \in \Omega.$$

Definition 3.4. *Let $\boldsymbol{X}$ be a Banach Space , $\eta_\omega : \Omega\times \boldsymbol{X}\times \boldsymbol{X}\to \boldsymbol{X}$ be a random single-valued mapping, $\boldsymbol{A}_\omega : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$ be a strictly η_ω-accretive random single-valued mapping and $\boldsymbol{M}_\omega : \Omega \times \boldsymbol{X} \to 2^X$ be a $(\boldsymbol{A}_\omega, \eta_\omega)$-accretive random mapping, and $\rho_\omega : \Omega \to (0, +\infty)$ be a random variable. The random resolvent operator $\boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega,\boldsymbol{M}_\omega} : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$ is defined by*

$$\boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega,\boldsymbol{M}_\omega}(y) = (\boldsymbol{A}_\omega + \rho_\omega \boldsymbol{M}_\omega)^{-1}(y),$$

for all $\omega \in \Omega$, $y = y(\omega) \in \boldsymbol{X}$, and $\{\omega \in \Omega : 0 < \rho_\omega \in B\} \in \Re$.

Lemma 3.5. *([10])Let $\boldsymbol{X}$ be a Banach Space, $\boldsymbol{M}_\omega : \Omega \times \boldsymbol{X} \to CB(\boldsymbol{X})$ be a $\hat{H}$-continuous random set-valued mapping. then for any measurable mapping $x : \Omega \to \boldsymbol{X}$, the random set-valued mapping $\boldsymbol{M}_\omega(x(\omega)) : \Omega \to CB(\boldsymbol{X})$ is measurable.*

Lemma 3.6. *([10]) Let $\boldsymbol{M}_\omega, \boldsymbol{Q}_\omega : \Omega\times \boldsymbol{X}\to CB(\boldsymbol{X})$ be two measurable set-valued mappings, $\varepsilon > 0$ be a constant and $x : \Omega \to \boldsymbol{X}$ be a measurable selection of $\boldsymbol{M}_\omega$. Then there exists a measurable selection $y : \Omega \to \boldsymbol{X}$ of $\boldsymbol{Q}_\omega$ such that for any $\omega \in \Omega$,*

$$\|x(\omega) - y(\omega)\| \leq (1 + \varepsilon)\hat{H}(\boldsymbol{M}_\omega(\cdot), \boldsymbol{Q}_\omega(\cdot)).$$

Lemma 3.7. *([27])Let $\boldsymbol{X}$ be a q-uniformly smooth and real separable Banach Space, $\eta_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ be τ_ω-Lipschtiz continuous mapping, $\boldsymbol{A}_\omega : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$ be an r_ω-strongly η-accretive mapping, and $\boldsymbol{M}_\omega(\cdot, y) : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to 2^X(\forall y \in \boldsymbol{X})$ be an $(\boldsymbol{A}_\omega, \eta_\omega)$-accretive mapping. Then the generalized random resolvent operator $\boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega,\boldsymbol{M}_\omega} : \boldsymbol{X} \to \boldsymbol{X}$ is $\tau_\omega^{q-1}/(r_\omega - m_\omega\rho_\omega)$-Lipschitz continuous, that is,*

$$\|\boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega,\boldsymbol{M}_\omega}(x) - \boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega,\boldsymbol{M}_\omega}(y)\| \leq \frac{\tau_\omega^{q-1}}{r_\omega - m_\omega\rho_\omega}\|x - y\| \qquad \text{for all } x, y \in \boldsymbol{X}, \omega \in \Omega.$$

where $\rho_\omega, r_\omega, m_\omega : \Omega \to (0, +\infty)$ are real-valued measurable, and $0 < \rho_\omega < \frac{r_\omega}{m_\omega}$.

Lemma 3.8. [38]*Let $\boldsymbol{X}$ be a real uniformly smooth Banach space. Then $\boldsymbol{X}$ is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that for all $x, y \in \boldsymbol{X}$,*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, J_q(x)\rangle + c_q\|y\|^q$$

4. Iterative algorithm with an random error of solutions

We first transfer the problem (2.1) into a fixed point problem.

Lemma 4.1. *Measurable $x, u, v, z, y : \Omega \to \boldsymbol{X}$ is a system solution of random generalized set-valued mixed variational inclusions the problem (2.1) if and only if for each $\omega \in \Omega$, holds the following relation*

$$y(\omega) = \boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\boldsymbol{M}_\omega}[\boldsymbol{A}_\omega(y) + \rho_\omega g_\omega - \rho_\omega \boldsymbol{F}_\omega(u, v)], \tag{4.1}$$

where $u \in \boldsymbol{S}_\omega, v \in \boldsymbol{T}_\omega, z \in \boldsymbol{G}_\omega, y \in \boldsymbol{P}_\omega$, $\rho_\omega, k_\omega : \Omega \to (0, +\infty)$ are two real-valued random variables, and $\boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\boldsymbol{M}_\omega} = (\boldsymbol{A}_\omega + \rho_\omega k_\omega \boldsymbol{M}_\omega)^{-1}$ is a resolvent operator in Banach Space $\boldsymbol{X}$.

Proof. The proof directly follows from the definition of $\mathbf{R}^{\mathbf{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\mathbf{M}_\omega}$ and so it is omitted.

Based on Lemma 4.1 and Nadler [32], we can develop a new iterative algorithm for solving the random generalized fuzzy set-valued mixed variational inclusions (2.1) with random nonlinear $(\mathbf{A}_\omega, \eta_\omega)$-accretive mappings as follows:

Algorithm 4.2. *Let $\widehat{\boldsymbol{S}}_\omega, \widehat{\boldsymbol{T}}_\omega, \widehat{\boldsymbol{G}}_\omega, \widehat{\boldsymbol{P}}_\omega : \Omega \times \boldsymbol{X} \to \mathrm{F}(\boldsymbol{X})$ be random fuzzy mappings satisfying condition $(*)$ and $\boldsymbol{S}_\omega, \boldsymbol{T}_\omega, \boldsymbol{G}_\omega, \boldsymbol{P}_\omega : \Omega \times \boldsymbol{X} \to CB(\boldsymbol{X})$ be the set-valued mappings induced by the fuzzy mappings $\widehat{\boldsymbol{S}}_\omega, \widehat{\boldsymbol{T}}_\omega, \widehat{\boldsymbol{G}}_\omega, \widehat{\boldsymbol{P}}_\omega$, respectively. Let $\boldsymbol{A}_\omega : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$, $\eta_\omega, \boldsymbol{F}_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ be single-valued mappings and let $\boldsymbol{M}_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to 2^X$ be a random set-valued mapping such that for each fixed $\omega \in \Omega$, and for any a measurable mapping $z : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$, $\boldsymbol{M}_\omega(z, \cdot) : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to 2^X$ be a $(\boldsymbol{A}_\omega, \eta_\omega)$-accretive mapping and range$(\boldsymbol{P}_\omega) \bigcap dom\boldsymbol{M}_\omega(\cdot, t) \neq \emptyset$. For any given $x_0 : \Omega \to \boldsymbol{X}$, the set-mappings $\boldsymbol{S}_\omega(x_0), \boldsymbol{T}_\omega(x_0), \boldsymbol{G}_\omega(x_0), \boldsymbol{P}_\omega(x_0) : \Omega \times \boldsymbol{X} \to CB(\boldsymbol{X})$ all are measurable by lemma (3.6). We know that, for any $x_0 \in \boldsymbol{X}$, the set-mappings $\boldsymbol{S}_\omega(x_0), \boldsymbol{T}_\omega(x_0), \boldsymbol{G}_\omega(x_0), \boldsymbol{P}_\omega(x_0)$ are measurable and there exists measurable selections $u_0 \in \boldsymbol{S}_\omega(x_0), v_0 \in \boldsymbol{T}_\omega(x_0), z_0 \in \boldsymbol{G}_\omega(x_0(\cdot)), y_0 \in \boldsymbol{P}(\cdot, x_0)$(see* [22]*). Set*

$$x_1(\omega) = (1 - \varpi)x_0 + \varpi[x_0 - y_0 + \boldsymbol{R}^{\boldsymbol{A}_\omega, \eta_\omega}_{\rho_\omega k_\omega, \boldsymbol{M}_\omega}(\boldsymbol{A}_\omega(y_0) + \rho_\omega g_\omega - \rho_\omega \boldsymbol{F}_\omega(u_0, v_0))] + e_0,$$

where $k_\omega, \rho_\omega, \boldsymbol{A}_\omega, \boldsymbol{M}_\omega, \boldsymbol{F}_\omega$ are the same as in Lemma (4.1), $1 > \varpi > 0$ is a constant, and $e_0 = e_0(\omega) : \Omega \to X$ is a measurable function which is an random error to take into account a possible inexact computation of the proximal point. Then, it is easy to know that $x_1 : \Omega \to \boldsymbol{X}$ is a measurable mapping. Since $u_0 \in \boldsymbol{S}_\omega(x_0), v_0 \in \boldsymbol{T}_\omega(x_0), z_0 \in \boldsymbol{G}_\omega(x_0), y_0 \in \boldsymbol{P}_\omega(x_0)$, by Lemma (3.7) ,there exists measurable selections $u_1 \in \boldsymbol{S}_\omega(x_1)$, $v_1 \in \boldsymbol{T}_\omega(x_1), z_1 \in \boldsymbol{G}_\omega(x_1), y_1 \in \boldsymbol{P}_\omega(x_1$ such that , for all $\omega \in \Omega$,

$$\|u_0 - u_1\| \leq (1 + \frac{1}{1})\hat{H}(\boldsymbol{S}_\omega(x_0), \boldsymbol{S}_\omega(x_1)),$$
$$\|v_0 - v_1\| \leq (1 + \frac{1}{1})\hat{H}(\boldsymbol{T}_\omega(x_0), \boldsymbol{T}_\omega(x_1)),$$
$$\|z_0 - z_1\| \leq (1 + \frac{1}{1})\hat{H}(\boldsymbol{G}_\omega(x_0), \boldsymbol{G}_\omega(x_1)),$$
$$\|y_0 - y_1\| \leq (1 + \frac{1}{1})\hat{H}(\boldsymbol{P}_\omega(x_0), \boldsymbol{P}_\omega(x_1)),$$

By induction, we can define a measurable sequences x_n, u_n, v_n, z_n, and $y_n : \Omega \to \boldsymbol{X}$ inductively satisfying

$$\begin{cases} x_{n+1} = (1 - \varpi)x_n + \varpi[x_n - y_n + \boldsymbol{R}^{\boldsymbol{A}_\omega, \eta_\omega}_{\rho_\omega k_\omega, \boldsymbol{M}_\omega}(\boldsymbol{A}_\omega(y_n) + \rho_\omega g_\omega - \rho_\omega \boldsymbol{F}_\omega(u_n, v_n))] + e_n, \\ u_n \in \boldsymbol{S}_\omega(x_n), \quad \|u_n - u_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(\boldsymbol{S}_\omega(x_n), \boldsymbol{S}_\omega(x_{n+1})), \\ v_n \in \boldsymbol{T}_\omega(x_n), \quad \|v_n - v_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(\boldsymbol{T}_\omega(x_n), \boldsymbol{T}_\omega(x_{n+1})), \\ z_n \in \boldsymbol{G}_\omega(x_n), \quad \|z_n - z_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(\boldsymbol{G}_\omega(x_n), \boldsymbol{G}_\omega(x_{n+1})), \\ y_n \in \boldsymbol{P}_\omega(x_n) \quad \|y_n - y_{n+1}\| \leq (1 + \frac{1}{n+1})\hat{H}(\boldsymbol{P}_\omega(x_n), \boldsymbol{P}_\omega(x_{n+1})), \end{cases} \tag{4.2}$$

where $n = 0, 1, 2, \cdots$, $0 < \varpi < 1$ is a constant, $e_n = e_n(\omega) : \Omega \to \boldsymbol{X}(n \geq 0)$ is an random error to take into account a possible inexact computation of the proximal point.

From Algorithm 4.2, we can get algorithm for solving problems (2.2) as follows:

Algorithm 4.3. *For any given for any $x_0(\cdot) \in \boldsymbol{X}$, the set-valued mappings $\boldsymbol{S}_\omega(x_0)$, $\boldsymbol{T}_\omega(x_0)$, $\boldsymbol{G}_\omega(x_0)$, $\boldsymbol{P}_\omega(x_0)$ are measurable and there exists measurable selections $u_0 \in \boldsymbol{S}_\omega(x_0)$, $v_0 \in \boldsymbol{T}_\omega(x_0)$, $z_0 \in \boldsymbol{Q}_\omega(x_0), y_0 \in \boldsymbol{P}_\omega(x_0)$, we can get the measurable iterative sequences $\{x_n\}, \{u_n\}$,*

$\{v_n\}$, $\{z_n\}$ and $\{y_n\} : \Omega \to \boldsymbol{X}$ as follows:

$$\begin{cases} x_{n+1} = (1-\varpi)x_n + \varpi[x_n - y_n + \boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\boldsymbol{M}_\omega}(\boldsymbol{A}_\omega(y_n) + \rho_\omega g_\omega - \rho_\omega \boldsymbol{F}_\omega(u_n,v_n))] + e_n, \\ u_n \in \boldsymbol{S}_\omega(x_n), \quad \|u_n - u_{n+1}\| \le (1+\frac{1}{n+1})\hat{H}(\boldsymbol{S}_\omega(x_n), \boldsymbol{S}_\omega(x_{n+1})), \\ v_n \in \boldsymbol{T}_\omega(x_n), \quad \|v_n - v_{n+1}\| \le (1+\frac{1}{n+1})\hat{H}(\boldsymbol{T}_\omega(x_n), \boldsymbol{T}_\omega(x_{n+1})), \\ z_n \in \boldsymbol{G}_\omega(x_n), \quad \|z_n - z_{n+1}\| \le (1+\frac{1}{n+1})\hat{H}(\boldsymbol{G}_\omega(x_n), \boldsymbol{S}_\omega(x_{n+1})), \\ y_n \in \boldsymbol{P}_\omega(x_n) \quad \|y_n - y_{n+1}\| \le (1+\frac{1}{n+1})\hat{H}(\boldsymbol{P}_\omega(x_n), \boldsymbol{P}_\omega(x_{n+1})), \end{cases} \tag{4.3}$$

where $n = 0,1,2,\cdots$, $0 < \varpi < 1$ is a constant, $e_n(\omega) : \Omega \to \boldsymbol{X}(n \ge 0)$ is an random error to take into account a possible inexact computation of the proximal point.

Remark 4.4. *If we choose suitable $\eta, \boldsymbol{A}, \boldsymbol{F}, \boldsymbol{S}, \boldsymbol{T}, \boldsymbol{G}, \boldsymbol{P}$ and M, then Algorithm 4.3 can be degenerated to a number of algorithms involving many known algorithms which due to classes of variational inequalities, and variational inclusions (see, for examples,* [4], [11], [16], [25], [31], [35]*).*

Now we prove the existence of solutions of problem (2.1) and the convergence of iterative sequences generated by Algorithm 4.2.

5. Existence and convergence

In this section, we will prove the existence of solution for problem (2.1) and the convergence of the iterative sequences generated by Algorithm 3.2.

Theorem 5.1. *Let $\boldsymbol{X}$ be a q-uniformly smooth and real separable Banach Space, $\eta_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ be τ_ω-Lipschtiz continuous random mapping, $\boldsymbol{A}_\omega : \Omega \times \boldsymbol{X} \to \boldsymbol{X}$ be r_ω-strongly $\eta_\omega(\cdot,\cdot)$-accretive random mapping and α_ω-Lipschitz continuous. Let $\widehat{\boldsymbol{S}}_\omega$, $\widehat{\boldsymbol{T}}_\omega$, $\widehat{\boldsymbol{G}}_\omega$, $\widehat{\boldsymbol{G}}_\omega : \Omega \times \boldsymbol{X} \to \mathrm{F}(\boldsymbol{X})$ be fuzzy random mappings satisfying condition $(*)$ and $\boldsymbol{S}_\omega, \boldsymbol{T}_\omega, \boldsymbol{G}_\omega$, $\boldsymbol{P}_\omega : \Omega \times \boldsymbol{X} \to CB(\boldsymbol{X})$ be random set-valued mappings induced by the fuzzy random mappings $\widehat{\boldsymbol{S}}_\omega$, $\widehat{\boldsymbol{T}}_\omega$, $\widehat{\boldsymbol{G}}_\omega$, and $\widehat{\boldsymbol{P}}_\omega$, respectively. suppose that $\boldsymbol{S}_\omega, \boldsymbol{T}_\omega, \boldsymbol{G}_\omega, \boldsymbol{P}_\omega$ be $\hat{H}$-Lipschitz continuous with random variables $\gamma_\omega, \xi_\omega, \zeta_\omega, \chi_\omega$, respectively. Let $\boldsymbol{P}_\omega$ be $(\varsigma_\omega, \kappa_\omega)$-relaxed cocoercive in the second argument of $\boldsymbol{P}_\omega(\cdot)$. Let $\boldsymbol{F}_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ be Lipschitz continuous random mapping with random variables (μ_ω, ν_ω), and $\boldsymbol{F}_\omega$ be $(\varphi_\omega, \psi_\omega)$-$\boldsymbol{S}_\omega$-relaxed cocoercive with respect to $\boldsymbol{A}_\omega \boldsymbol{P}_\omega$ in the second argument of $\boldsymbol{F}_\omega(\cdot,\cdot)$, and let $g_\omega : \Omega \to \boldsymbol{X}$ be a real random variable. Suppose $\boldsymbol{M}_\omega : \Omega \times \boldsymbol{X} \times \boldsymbol{X} \to 2^X$ such that for each measurable $y \in \boldsymbol{X}$, $\boldsymbol{M}_\omega(\cdot, y) : \boldsymbol{X} \to 2^X$ be $(\boldsymbol{A}_\omega, \eta_\omega)$-accretive random mapping and range$(\boldsymbol{P}_\omega) \bigcap dom \boldsymbol{M}_\omega(\cdot, y) \ne \emptyset$. If for any $x, y, z \in \boldsymbol{X}$ there exists a random real-valued variable $\delta_\omega > 0$ such that:*

$$\|\boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\boldsymbol{M}_\omega(\cdot,x)}(z) - \boldsymbol{R}^{\boldsymbol{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\boldsymbol{M}_\omega(\cdot,y)}(z)\| \le \delta_\omega \|x - y\|, \tag{5.1}$$

and

$$\begin{cases} L = \delta_\omega \chi_\omega + [1 + \chi_\omega^q(c_q + q\varsigma_\omega) - q\kappa_\omega]^{\frac{1}{q}} < 1, \\ \rho_\omega \nu_\omega \xi_\omega + (\alpha_\omega^q \chi_\omega^q + c_q \rho_\omega^q \mu_\omega^q \gamma_\omega^q + q\rho_\omega \varphi_\omega \mu_\omega^q \gamma_\omega^q)^{\frac{1}{q}} < (1-L)(r_\omega - k_\omega \rho_\omega m_\omega)\tau_\omega^{1-q}, \\ r_\omega > k_\omega \rho_\omega m_\omega; \end{cases} \tag{5.2}$$

And

$$\lim_{n\to\infty}\|e_n(\omega)\|=0,\quad \sum_{n=1}^{\infty}\|e_n(\omega)-e_{n-1}(\omega)\|<\infty,\quad \forall\omega\in\Omega. \tag{5.3}$$

Then the random variable iterative sequences $\{x_n\},\{u_n\},\{v_n\},\ \{z_n\}$ *and* $\{y_n\}:\Omega\to\boldsymbol{X}$ *generated by Algorithm 4.2 converge strongly to random variables* $x^*,u^*,v^*,\ z^*,$ *and* $y^*:\Omega\to\boldsymbol{X}$*, respectively, and* (x^*,u^*,v^*,z^*,y^*) *is a solution of problem (2.1).*

Proof.. From Algorithm 4.2, Lemma 3.7 and (5.1), for any $\omega\in\Omega$, and $0<\varpi<1$, we have

$$\begin{aligned}
&\|x_{n+1}-x_n\|\\
&\le(1-\varpi)\|x_n-x_{n-1}\|+\|e_n-e_{n-1}\|+\varpi\|x_n-x_{n-1}-(y_n-y_{n-1})\|\\
&+\varpi\|\mathbf{R}^{\mathbf{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\mathbf{M}_\omega(z_n,\cdot)}(\mathbf{A}_\omega(y_n)-\rho_\omega\mathbf{F}_\omega(u_n,v_n))\\
&-\mathbf{R}^{\mathbf{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\mathbf{M}_\omega(z_n,\cdot)}(\mathbf{A}_\omega(y_{n-1})-\rho_\omega\mathbf{F}_\omega(u_{n-1},v_{n-1}))\|\\
&+\varpi\|\mathbf{R}^{\mathbf{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\mathbf{M}_\omega(z_n,\cdot)}(\mathbf{A}_\omega(y_{n-1})-\rho_\omega\mathbf{F}_\omega(u_{n-1},v_{n-1}))\\
&-\mathbf{R}^{\mathbf{A}_\omega,\eta_\omega}_{\rho_\omega k_\omega,\mathbf{M}_\omega(z_{n-1},\cdot)}(\mathbf{A}_\omega(y_{n-1})-\rho_\omega\mathbf{F}_\omega(u_{n-1},v_{n-1}))\|\\
&\le(1-\varpi)\|x_n-x_{n-1}\|+\|e_n-e_{n-1}\|+\varpi\{\|x_n-x_{n-1}-(y_n-y_{n-1})\|\\
&+\frac{\tau_\omega^{q-1}}{r_\omega-k_\omega\rho_\omega m_\omega}[\|\mathbf{A}_\omega(y_n)-\mathbf{A}_\omega(y_{n-1})-\rho_\omega(\mathbf{F}_\omega(u_n,v_{n-1})-\mathbf{F}_\omega(u_{n-1},v_{n-1}))\|\\
&+\rho(\omega)\|\mathbf{F}_\omega(u_n,v_n)-\mathbf{F}_\omega(u_n,v_{n-1})\|]+\delta_\omega\|y_n-y_{n-1}\|\}.
\end{aligned}\tag{5.4}$$

Since $\mathbf{P}_\omega$ is the $\hat{H}$-Lipschitz continuous with χ_ω and is $(\varsigma_\omega,\kappa_\omega)$-relaxed cocoercive in the second argument of $\mathbf{P}_\omega(\cdot)$, and by Lemma (3.8)[38] and Algorithm 4.2, we obtain

$$\begin{aligned}
&\|x_n-x_{n-1}-(y_n-y_{n-1})\|^q=\|x_n-x_{n-1}\|^q\\
&-q\langle y_n-y_{n-1},j_q(x_n-x_{n-1})\rangle+c_q\|y_n-y_{n-1}\|^q\\
&\le\|x_n-x_{n-1}\|^q+c_q(1+n^{-1})^q\hat{H}^q(\mathbf{P}_\omega(x_n),\mathbf{P}_\omega(x_{n-1}))\\
&-q(-\varsigma_\omega\|y_n-y_{n-1}\|^q+\kappa_\omega\|x_n-x_{n-1}\|^q)\\
&\le[1+(1+n^{-1})^q\chi_\omega^q(c_q+q\varsigma_\omega)-q\kappa_\omega]\|x_n-x_{n-1}\|^q.
\end{aligned}\tag{5.5}$$

Since $\mathbf{F}_\omega$ is $\hat{H}$-Lipschitz continuous with (μ_ω,ν_ω), and is $(\varphi_\omega,\psi_\omega)$-$\mathbf{S}_\omega$-relaxed cocoercive with respect to $\mathbf{A}_\omega\mathbf{P}_\omega$ in the second argument of $\mathbf{F}_\omega(\cdot,\cdot)$ and $\mathbf{S}_\omega$ is $\hat{H}$-Lipschitz continuous with

γ_ω, we have

$$\begin{aligned}
&\|\mathbf{A}_\omega(y_n) - \mathbf{A}_\omega(y_{n-1}) - \rho_\omega(\mathbf{F}_\omega(u_n, v_{n-1}) - \mathbf{F}_\omega(u_{n-1}, v_{n-1}))\|^q \\
&\le \|\mathbf{A}_\omega(y_n - \mathbf{A}_\omega(y_{n-})\|^q + c_q\rho_\omega^q\|\mathbf{F}_\omega(u_n, v_{n-1}) - \mathbf{F}_\omega(u_{n-1}, v_{n-1})\|^q \\
&-q\langle\rho_\omega(\mathbf{F}_\omega(u_n, v_{n-1}) - \mathbf{F}_\omega(u_{n-1}, v_{n-1})), J_q(\mathbf{A}_\omega(y_n) - \mathbf{A}_\omega(y_{n-1}))\rangle \\
&\le \alpha_\omega^q\|y_n - y_{n-1}\|^q + c_q\rho_\omega^q\mu_\omega^q\|u_n - u_{n-1}\|^q \\
&-q\rho_\omega(-\varphi_\omega\|\mathbf{F}_\omega(u_n, v_{n-1}) - \mathbf{F}_\omega(u_{n-1}, v_{n-1})\|^q + \psi_\omega\|x_n - x_{n-1}\|^q) \\
&\le \alpha_\omega^q\|y_n - y_{n-1}\|^q + c_q(1+n^{-1})^q\rho_\omega^q\mu^q(\omega)\hat{H}^q(\mathbf{S}_\omega(x_n), \mathbf{S}_\omega(x_{n-1})) \\
&+q\rho_\omega\varphi_\omega\gamma_\omega^q\|u_n - u_{n-1}\|^q - q\rho_\omega\psi_\omega\|x_n - x_{n-1}\|^q \\
&\le [\alpha_\omega^q(1+n^{-1})^q\chi_\omega^q + c_q(1+n^{-1})^q\rho_\omega^q\mu_\omega^q\gamma_\omega^q \\
&+q\rho_\omega\varphi_\omega\mu_\omega^q(1+n^{-1})^q\gamma_\omega^q]\|x_n - x_{n-1}\|^q. \qquad (5.6)
\end{aligned}$$

Further, By assumptions

$$\|\mathbf{F}_\omega(u_n, v_n) - \mathbf{F}_\omega(u_n, v_{n-1})\| \le \nu_\omega\xi_\omega(1+n^{-1})\|x_n - x_{n-1}\|, \qquad (5.7)$$

$$\|y_n - y_{n-1}\| \le (1+n^{-1})\chi_\omega\|x_n - x_{n-1}\|. \qquad (5.8)$$

From (5.4)∼(5.8), It follows that

$$\begin{aligned}
\|x_{n+1} - x_n\| &\le (1 - \varpi + \varpi h_n)\|x_n - x_{n-1}\| + \|e_n - e_{n-1}\| \\
&= \theta_n\|x_n - x_{n-1}\| + \|e_n - e_{n-1}\| \qquad (5.9)
\end{aligned}$$

where

$$\begin{aligned}
&\theta_n = 1 - \varpi + \varpi h_n, \\
&h_n = \delta_\omega\chi_\omega(1+n^{-1}) + [1 + (1+n^{-1})^q\chi_\omega^q(c_q + q\varsigma_\omega) - q\kappa_\omega]^{\frac{1}{q}} \\
&+\frac{\tau_\omega^{q-1}}{r_\omega - k_\omega\rho_\omega m_\omega}[\rho_\omega\nu_\omega\xi_\omega(1+n^{-1}) + (\alpha_\omega^q(1+n^{-1})^q\chi_\omega^q \\
&+c_q(1+n^{-1})^q\rho_\omega^q\mu_\omega^q\gamma_\omega^q + q\rho_\omega\varphi_\omega\mu_\omega^q(1+n^{-1})^q\gamma_\omega^q)^{\frac{1}{q}}].
\end{aligned}$$

Letting

$$\begin{aligned}
&\theta = 1 - \lambda + \lambda h \\
&h = \delta_\omega\chi_\omega + [1 + \chi_\omega^q(c_q + q\varsigma_\omega) - q\kappa_\omega]^{\frac{1}{q}} \\
&+\frac{\tau_\omega^{q-1}}{r_\omega - k_\omega\rho_\omega m_\omega}[\rho_\omega\nu_\omega\xi_\omega + (\alpha_\omega^q\chi_\omega^q + c_q\rho_\omega^q\mu_\omega^q\gamma_\omega^q + q\rho_\omega\varphi_\omega\mu_\omega^q\gamma_\omega^q)^{\frac{1}{q}}].
\end{aligned}$$

we have that $h_n \to h$ and $\theta_n \to \theta$ as $n \to \infty$. It follows from condition (5.2) and $0 < \varpi < 1$ that $0 < \theta < 1$ and hence there exists $N_0 > 0$ and $\theta_* \in (\theta, 1)$ such that $\theta_n < \theta_*$ for all $n \ge N_0$. Therefore, by (5.9), we have

$$\|x_{n+1} - x_n\| \le \theta_*\|x_n - x_{n-1}\| + \|e_n - e_{n-1}\|, \forall n \ge N_0.$$

Without loss of generality we assume

$$\|x_{n+1} - x_n\| \le \theta_*\|x_n - x_{n-1}\| + \|e_n - e_{n-1}\|, \forall n \ge 1,$$

Hence, for any $m > n > 0$, we have

$$\|x_m - x_n)\| \leq \sum_{i=n}^{m-1} \|x_{i+1} - x_i\| \leq \sum_{i=n}^{m-1} \theta_*^i \|x_1 - x_0\| + \sum_{i=n}^{m-1} \sum_{j=1}^{i} \theta_*^{i-j} \|e_j - e_{j-1}\|.$$

It follows from condition (4.3) that $\|x_m - x_n\| \to 0$, as $n \to \infty$, and so $\{x_n\}$ is a Cauchy sequence in $\mathbf{X}$. Let $x_n \to x^*$ as $n \to \infty$. By the Lipschitz continuity of $\mathbf{S}_\omega(\cdot), \mathbf{T}_\omega(\cdot), \mathbf{G}_\omega(\cdot)$ and $\mathbf{P}_\omega(\cdot)$, we obtain

$$\begin{aligned}
\|u_{n+1} - u_n\| &\leq (1 + n^{-1})\hat{H}(\mathbf{S}_\omega(x_{n+1}), \mathbf{S}_\omega(x_n)) \leq \gamma_\omega(1 + n^{-1})\|x_{n+1} - x_n\|, \\
\|v_{n+1} - v_n\| &\leq (1 + n^{-1})\hat{H}(\mathbf{T}_\omega(x_{n+1}), \mathbf{T}_\omega(x_n)) \leq \xi_\omega(1 + n^{-1})\|x_{n+1} - x_n\|, \\
\|z_{n+1} - z_n\| &\leq (1 + n^{-1})\hat{H}(\mathbf{G}_\omega(x_{n+1}), \mathbf{G}_\omega(x_n)) \leq \zeta_\omega(1 + n^{-1})\|x_{n+1} - x_n\|, \\
\|y_{n+1} - y_n\| &\leq (1 + n^{-1})\hat{H}(\mathbf{P}_\omega(x_{n+1}), \mathbf{P}_\omega(x_n)) \leq \chi_\omega(1 + n^{-1})\|x_{n+1} - x_n\|.
\end{aligned}$$

It follows that $\{u_n\}$, $\{v_n\}$, $\{z_n\}$, and $\{y_n\}$ are also Cauchy sequences in $\mathbf{X}$. We can assume that $u_n \to u^*$, $v_n \to v^*$, $z_n \to z^*$, and $y_n \to y^*$ respectively. Note that $u_n \in \mathbf{S}_\omega(x_n)$, we have

$$\begin{aligned}
d(u^*, \mathbf{S}_\omega(x^*)) &\leq \|u^* - u_n\| + d(u_n, \mathbf{S}_\omega(x^*)) \\
&\leq \|u^* - u_n\| + \hat{H}(\mathbf{S}_\omega(x_n), \mathbf{S}_\omega(x^*)) \\
&\leq \|u^* - u_n\| + \gamma_\omega\|x_n - x^*\| \to 0 (n \to \infty).
\end{aligned}$$

Hence $d(u^*, \mathbf{S}_\omega(x^*)) = 0$ and therefore $u^* \in \mathbf{S}_\omega(x^*)$. Similarly, we can prove that $v^* \in \mathbf{T}_\omega(x^*)$, $z^* \in \mathbf{G}_\omega(x^*)$, and $y^* \in \mathbf{P}_\omega(x^*)$).

By the Lipschitz continuity of $\mathbf{S}_\omega(\cdot), \mathbf{T}_\omega(\cdot), \mathbf{G}_\omega(\cdot)$ and $\mathbf{P}_\omega(\cdot)$, and Lemma 4.1, condition (5.1) and $\lim_{n\to\infty} \|e_n(\omega)\| = 0$, we have

$$\begin{aligned}
x^*(\omega) = (1 - \varpi)x^*(\omega) + \varpi[x^*(\omega) - y^*(\omega) \\
+ R^{\mathbf{A}_\omega(y^*(\omega)), \eta_\omega}_{\rho_\omega k_\omega, M_\omega(z^*(\omega), \cdot)}(\mathbf{A}_\omega(y^*(\omega)) + \rho_\omega g_\omega - \rho_\omega \mathbf{F}_\omega(u^*(\omega), v^*(\omega)))].
\end{aligned}$$

By Lemma 3.1, we know that $(x^*, u^*, v^*, z^*, y^*)$ is a solution of problem (2.1). This completes the proof. From Theorem 5.1, we have the following theorem.

Theorem 5.2. *Let $\boldsymbol{A}_\omega, g_\omega, \eta_\omega, \boldsymbol{F}_\omega, \boldsymbol{M}_\omega, \boldsymbol{X}$ be the same as in Theorem 5.1, and $\boldsymbol{S}_\omega$, $\boldsymbol{T}_\omega$, $\boldsymbol{G}_\omega$, $\boldsymbol{P}_\omega$: $\Omega \times \boldsymbol{X} \to CB(\boldsymbol{X})$ be D-Lipschitz continuous with random variables $\gamma_\omega, \xi_\omega, \zeta_\omega, \chi_\omega$, respectively, and let $\boldsymbol{P}_\omega$ be $(\varsigma_\omega, \kappa_\omega)$-relaxed cocoercive in the second argument of $\boldsymbol{P}_\omega(\cdot)$. Let $\boldsymbol{F}_\omega$: $\Omega \times \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X}$ be Lipschitz continuous with random variables (μ_ω, ν_ω), and $\boldsymbol{F}_\omega$ be $(\varphi_\omega, \psi_\omega)$-$\boldsymbol{S}_\omega$-relaxed cocoercive with respect to $\boldsymbol{A}_\omega \boldsymbol{P}_\omega$ in the second argument of $\boldsymbol{F}_\omega(\cdot, \cdot)$. If conditions (5.1)∼(5.3) of Theorem 5.1 hold, then the random variable iterative sequences $\{x_n\}, \{u_n\}, \{v_n\}$, $\{z_n\}$ and $\{y_n\} : \Omega \to \boldsymbol{X}$ generated by Algorithm 4.3 converge strongly to random variables x^*, u^*, v^*, z^* and $y^* : \Omega \to \boldsymbol{X}$, respectively, and $(x^*, u^*, v^*, z^*, y^*)$ is a solution of the problem (2.2).*

Remark 5.3. *For a suitable choice of the mappings $\boldsymbol{A}_\omega, g_\omega, \eta_\omega, \boldsymbol{F}_\omega, \boldsymbol{M}_\omega$, $\boldsymbol{S}_\omega$, $\boldsymbol{T}_\omega$, $\boldsymbol{G}_\omega, \boldsymbol{P}_\omega$ and $\boldsymbol{X}_\omega$, we can obtain several known results [*[4], [11], [16], [25], [31], [35] *et al.] as special cases of Theorem 5.2.*

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