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COMPATIBILITY OF TYPE (P) IN MODIFIED INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT. The object of this paper is to establish unique common fixed point theorems for four self maps satisfying a new contractive condition in a modified intuitionistic fuzzy metric space through compatibility of type (P). A generalization of a result of D Turkoglu et al [J. Apply. Math. Computing (2006)] in the setting of a modified intuitionistic fuzzy metric space follows from them. Modified intuitionistic fuzzy version of Grabiec contraction Principle has also been established. All the results presented in this paper are new. Examples have been constructed in support of the main results of this paper.

1. INTRODUCTION

In [4] Atanassov generalized fuzzy sets by introducing intuitionistic fuzzy sets. Park [15] introduced the concept of intutionstic fuzzy metric space with the help of a continuous t-norm and a continuous t-conorm as a generalization of fuzzy metric space due to George and Veeramani [8] and Kramosil and Michalec [13], which is a milestone in developing fixed point theory in intuitionistic fuzzy metric space.
Recently, Saadati et. al [17] introduced the modified intuitionistic fuzzy metric space and proved some fixed point theorems through compatibility and weak compatibility in it. In [20] D. Turkoglu, C. Alaca, Y. J. Cho and C. Yildiz (2006) introduced the concept of k-contraction in an intuitionistic fuzzy metric space and established some results on it. Also Adibi et al. [1] introduce the notion of compatibility of type (P) in L-fuzzy metric spaces.

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The purpose of this paper is to establish some unique common fixed point theorems for four self maps in a modified intuitionistic fuzzy metric space satisfying a new contractive condition through compatibility of type (P), which turns out to be a generalization of the result of Turkoglu et. al [19] in the setting of modified intuitionistic fuzzy metric space. Modified intuitionistic fuzzy version of Grabiec contraction Principle [9] has also been established in this paper.

2. Preliminaries

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $([0,1], *)$ is an abelian topological monoid with unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all a, b, c and $d \in [0,1]$.

Definition 2.2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-conorm if $([0,1], \diamond)$ is an abelian topological monoid with unit 0 such that $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for all a, b, c and $d \in [0,1]$.

Proposition 2.3. [7] *Consider the set* L^* *and relation* $\leq_{L^*}$ *defined by:*
$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2$ *and* $x_1 + x_2 \leq 1\}$,
$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1$ *and* $x_2 \geq y_2$, *for* $(x_1, x_2), (y_1, y_2) \in L^*$.
Then $(L^*, \leq_{L^*})$ *is a complete lattice.*
We denote $0_{L^*} = (0,1)$ *and* $1_{L^*} = (1,0)$.

Definition 2.4. [6] A triangular norm on L^* is a mapping $\mathcal{T} : (L^*)^2 \to L^*$ satisfying:
(i) $\mathcal{T}(x, 1_{L^*}) = x$, for all $x \in L^*$;
(ii) $\mathcal{T}(x, y) = \mathcal{T}(y, x)$, for all $x, y \in L^*$;
(iii) $\mathcal{T}(x, \mathcal{T}(y, z)) = \mathcal{T}(\mathcal{T}(x, y), z)$, for all $x, y, z \in L^*$;
(iv) If for $x, x', y, y' \in L^*, x \leq_{L^*} x', y \leq_{L^*} y'$ then $\mathcal{T}(x, y) \leq_{L^*} \mathcal{T}(x', y')$.

Definition 2.5. [7]A continuous t-norm $\mathcal{T}$ on L^* is called continuous t-representable if and only if there exists a continuous t-norm $*$ and a continuous t-conorm $\diamond$ on $[0,1]$ such that for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$;
$\mathcal{T}(x, y) = (x_1 * y_1, x_2 \diamond y_2)$.

Definition 2.6. [17] The 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is called a modified intuitionistic fuzzy metric space if X is an arbitrary non-empty set, M and N are fuzzy sets from $X^2 \times (0, \infty)$ to $[0,1]$ such that $M(x, y, t) + N(x, y, t) \leq 1$, for all $x, y \in X, \mathcal{T}$ is a continuous t-representable and $\mathcal{M}_{M,N}$ is a mapping from $X^2 \times (0, \infty)$ to L* defined by $\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t))$ satisfying the following conditions for all $x, y, z \in X$ and for all s and t,
(a) $\mathcal{M}_{M,N}(x, y, t) >_{L^*} 0_{L^*}$;
(b) $\mathcal{M}_{M,N}(x, y, t) = 1_{L^*}$ iff $x = y$;
(c) $\mathcal{M}_{M,N}(x, y, t) = \mathcal{M}_{M,N}(y, x, t)$;
(d) $\mathcal{M}_{M,N}(x, z, t + s) \geq_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(x, y, t) * \mathcal{M}_{M,N}(y, z, s))$;
(e) $\mathcal{M}_{M,N}(x, y, .) : (0, \infty) \to L^*$ is continuous,
$\mathcal{M}_{M,N}$ called an intuitionistic fuzzy metric.

Note: In the sequel we will call $(X, \mathcal{M}_{M,N}, \mathcal{T})$ to be just an intuitionistic fuzzy metric space.

Remark 2.7. [20] In an intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$, M(x, y, .) is non - decreasing and N(x, y, .) is non - increasing for all $x, y\ inX$. Hence $(X, \mathcal{M}_{M,N,\mathcal{T}})$ is non - decreasing function for all x, y in X.

Definition 2.8. [17] Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be an intuitionistic metric space.
(a) A sequence $\{x_n\}$ in X is said to Cauchy sequence in X if for each $0 < \epsilon < 1$ there exists $n_0 \in N$ such that
$\mathcal{M}_{M,N}(x_n, x_m, t) >_{L^*} (1-\epsilon, \epsilon)$, for each $m, n \geq n_0$ and for all t.
(b) A sequence $\{x_n\}$ in X is said to converge to a point $x \in X$ denoted by, $x_n \to x$ or $lim_{n\to\infty} x_n = x$, if $lim_{n\to\infty}\mathcal{M}_{M,N}(x_n, x, t) \to 1_{L^*}$, for all t, whenever $n \to \infty$.

Definition 2.9. [17]An intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be complete if and only if every Cauchy sequence in X converges to a point of it.

Definition 2.10. A pair (A, S) of self mappings of an intuitionistic fuzzy metric space is said to be compatible of type (P) if
$lim_{n\to\infty}\mathcal{M}_{M,N}(A^2x_n, S^2x_n, t) = 1_{L^*}$,
when ever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = x \in X$.

Definition 2.11. [2, 17] A pair (A, S) of self mappings of an intuitionistic fuzzy metric space X is said to be weak compatible or coincidentally commuting if A and S commute at their coincidence points i.e. if for some $x \in X, Ax = Sx$ then $ASx = SAx$.

Proposition 2.12. [1]*Let* $(X, \mathcal{M}_{M,N}, \mathcal{T})$ *be a modified intuitionistic fuzzy metric space . For each* $\lambda \in (0, 1)$, *define map* $E_\lambda : X^2 \to R^+ \cup \{0\}$ *by*
$E_\lambda(x, y) = Inf\{t > 0 : \mathcal{M}_{M,N}(x, y, t) >_{L^*} (1-\lambda, \lambda)\}$, *then*
(a) For each $\lambda \in (0, 1)$, *we have a* $\mu \in (0, 1)$ *such that*
$E_\lambda(x_1, x_n) \leq E_\mu(x_1, x_2) + E_\mu(x_2, x_3) + \ldots + E_\mu(x_{n-1}, x_n)$,
for any $x_1, x_2, x_3, \ldots x_n \in X$.
(b) The sequence $\{x_n\}_{n\in N}$ *in* X*is convergent to* x *if and only if* $E_\lambda(x_n, x) \to 0$.
Also the sequence $\{x_n\}_{n\in N}$ *is a Cauchy sequence in* X *if and only if it is a Cauchy sequence with respect to* E_λ.

Proposition 2.13. *Let* $(X, \mathcal{M}_{M,N}, \mathcal{T})$ *be an intuitionistic fuzzy metric space. If for a sequence* $\{x_n\}$ *in* X, *there exists* $k \in (0, 1)$ *such that*
$\mathcal{M}_{M,N}(x_n, x_{n+1}, kt) \geq_{L^*} \mathcal{M}_{M,N}(x_{n-1}, x_n, t)$, *for all* n *and for all* t,
then $\{x_n\}$ *is a Cauchy sequence in* X.

Proof. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be an intuitionistic fuzzy metric space. Let for a sequence $\{x_n\}$ in X , there exists $k \in (0, 1)$ such that
$\mathcal{M}_{M,N}(x_n, x_{n+1}, kt) \geq_{L^*} \mathcal{M}_{M,N}(x_{n-1}, x_n, t)$, for all n and t,
then

$$\begin{aligned}\mathcal{M}_{M,N}(x_n, x_{n+1}, t) &\geq_{L^*} \mathcal{M}_{M,N}(x_{n-1}, x_n, \tfrac{t}{k}) \\ &\geq_{L^*} \mathcal{M}_{M,N}(x_{n-2}, x_{n-1}, \tfrac{t}{k^2}) \\ &\dots \\ &\geq_{L^*} \mathcal{M}_{M,N}(x_0, x_1, \tfrac{t}{k^n}), foralln.\end{aligned}$$

Now

$$\begin{aligned}E_\lambda(x_{n+1}, x_n) &= Inf\{t > 0 : \mathcal{M}_{M,N}(x_{n+1}, x_n, t) \geq_{L^*} (1-\lambda, \lambda)\} \\ &\leq Inf\{t > 0 : \mathcal{M}_{M,N}(x_1, x_0, \tfrac{t}{k^n}) \geq_{L^*} (1-\lambda, \lambda)\} \\ &= Inf\{k^n t > 0 : \mathcal{M}_{M,N}(x_1, x_0, t) \geq_{L^*} (1-\lambda, \lambda)\} \\ &= k^n Inf\{t > 0 : \mathcal{M}_{M,N}(x_1, x_0, t) \geq_{L^*} (1-\lambda, \lambda)\} \\ &= k^n E_\lambda(x_0, x_1).\end{aligned}$$

$E_\lambda(x_{n+1}, x_n) \leq k^n E_\lambda(x_0, x_1)$ ... (A)

Again from Proposition 2.12, for $\lambda \in (0,1)$, there exists $\mu \in (0,1)$ such that

$$\begin{aligned}E_\lambda(x_n, x_{n+p}) &\leq E_\mu(x_n, x_{n+1}) + E_\mu(x_{n+1}, x_{n+2}) + ... + E_\mu(x_{n+p-1}, x_{n+p}) \\ &\leq k^n E_\mu(x_0, x_1) + k^{n+1} E_\mu(x_0, x_1) + \ldots + k^{n+p-1} E_\mu(x_0, x_1), using(A) \\ &= (k^n + k^{n+1} + \ldots + k^{n+p-1}) E_\mu(x_0, x_1), \\ &= \tfrac{k^n}{1-k} E_\mu(x_0, x_1), as 0 < k < 1,\end{aligned}$$

which tends to 0, as $n \to \infty$. Hence $\{x_n\}$ is a Cauchy sequence in X. □

Proposition 2.14. *In an intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$, if for some x, y in X there exists $k \in (0,1)$ such that*
$\mathcal{M}_{M,N}(x, y, kt) \geq_{L^*} \mathcal{M}_{M,N}(x, y, t)$, for all t,
then $x = y$.

Proof. Let for $\lambda \in (0,1)$

$$\begin{aligned}E_\lambda(x, y) &= Inf\{t > 0 : \mathcal{M}_{M,N}(x, y, t) \geq_{L^*} (1-\lambda, \lambda)\} \\ &\leq Inf\{t > 0 : \mathcal{M}_{M,N}(x, y, t/k) \geq_{L^*} (1-\lambda, \lambda)\} \\ &= Inf\{kt > 0 : \mathcal{M}_{M,N}(x, y, t) \geq_{L^*} (1-\lambda, \lambda)\} \\ &= kInf\{t > 0 : \mathcal{M}_{M,N}(x, y, t) \geq_{L^*} (1-\lambda, \lambda)\} \\ &= kE_\lambda(x, y).\end{aligned}$$

Therefore $E_\lambda(x, y) = 0$.
Hence $x = y$. □

Proposition 2.15. *: For $x = (x_1, x_2), y = (y_1, y_2) \in L^*, x\mathcal{T}y \leq_{L^*} x$.*

Proof. For

$$\begin{aligned}x\mathcal{T}y = &\ (x_1 * y_1, x_2 \diamond y_2) \\ &\leq_{L^*} (x_1, x_2), as x_1 * y_1 \leq x_1, x_2 \diamond y_2 \geq x_2, \\ &= x.\end{aligned}$$

Thus

$x\mathcal{T}y \leq_{L^*} x.$ □

Proposition 2.16. *In an intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$, if a pair of self maps is compatible of type (P) then it is weak compatible.*

Proof. Let (A, S) be a compatible pair of type (P) in an intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$. Let for some $x \in X, Ax = Sx$. Taking $x_n = x$, then $A^2x = S^2x$. Also $A^2x = A(Ax) = A(Sx)$ and $S^2x = S(Sx) = S(Ax)$. Therefore $ASx = SAx$. Hence (A, S) is weak compatible. □

Lemma 2.17. *Let A, B, S and T be self mappings of a modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$. satisfying*
(i) $AT(X) \bigcup BS(X) \subseteq ST(X)$;
(ii) $ST = TS$;
(iii) For some $k \in (0, 1)$ there exists continuous real functions $p(t), q(t)$ and $a(t)$ from $(0, \infty)$ to $[0, 1]$ with $p(t) < 1$ and $p(t) + q(t) - a(t) = 1$, for all t, such that for all $x, y \in X$
$\mathcal{M}_{M,N}(Ax, By, kt) + a(t)\mathcal{M}_{M,N}(By, Ty, kt) \geq_{L^*} p(t)\mathcal{M}_{M,N}(Ax, Sx, t) + q(t)\mathcal{M}_{M,N}(Sx, Ty, t).$
For some $x_0 \in X$, we define sequence $\{y_n\}$ by $ATx_{2n} = STx_{2n+1} = y_{2n+1}, BSx_{2n+1} = STx_{2n+2} = y_{2n+2}$. Then $\{y_n\}$ is a Cauchy sequence in X.

Proof. We prove that $\{y_n\}$ is a Cauchy sequence in X. Putting $x = Tx_{2n}$ and $y = Sx_{2n+1}$ in (iii) and as $ST = TS$ we have,

$$\mathcal{M}_{M,N}(ATx_{2n}, BSx_{2n+1}, kt) + a(t)\mathcal{M}_{M,N}(BSx_{2n+1}, TSx_{2n+1}, kt)$$
$$\geq_{L^*} p(t)\mathcal{M}_{M,N}(ATx_{2n}, STx_{2n}, t) + q(t)\mathcal{M}_{M,N}(STx_{2n}, TSx_{2n+1}, t).$$

Thus

$$\mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, kt) + a(t)\mathcal{M}_{M,N}(y_{2n+1}, y_{2n+2}, kt)$$
$$\geq_{L^*} p(t)\mathcal{M}_{M,N}(y_{2n}, y_{2n+1}, t) + q(t)\mathcal{M}_{M,N}(y_{2n+1}, y_{2n}, t).$$

Writing $d_n(t) = \mathcal{M}_{M,N}(y_n, y_{n+1}, t)$, we get
$d_{2n+1}(kt) + a(t)d_{2n+1}(kt) \quad \geq_{L^*} [p(t) + q(t)]d_{2n}(t),$
i. e.
$(1 + a(t))d_{2n+1}(kt) \geq_{L^*} [p(t) + q(t)]d_{2n}(t).$
As $p(t) + q(t) - a(t) = 1$, we have
$d_{2n+1}(kt) \geq_{L^*} d_{2n}(t).$
Similarly, if we take $x = Tx_{2n+2}$ and $y = Sx_{2n+1}$ in (iii) we have,
$(1 - p(t))d_{2n+2}(kt) \geq_{L^*} [q(t) - a(t)]d_{2n+1}(t).$
As $p(t) + q(t) - a(t) = 1$ and $p(t) < 1$, we have
$d_{2n+2}(kt) \geq_{L^*} d_{2n+1}(t)$, for all t and for all n.
Thus for all n we have
$d_{n+1}(kt) \geq_{L^*} d_n(t).$
Hence by Proposition 2.13, $\{y_n\}$ is a Cauchy sequence in X. □

3. MAIN RESULTS

Theorem 3.1. *Let A, B, S and T be self mappings of a complete intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying (i), (ii) and :*
(iv) pairs (A, S) and (B, T) are compatible of type (P);
(v) For $k \in (0, 1)$ there exist continuous functions $p(t), q(t), a(t)$ from $(0, \infty)$ to $[0, 1]$ with $p(t) < 1, q(t) < 1$ or else $q(t) = 1(constant)$, for all t with $p(t) + q(t) - a(t) = 1$, for all t, such that for all $x, y \in X$,

$$\begin{aligned}&\mathcal{M}_{M,N}(Ax, By, kt) + a(t)\mathcal{M}_{M,N}(By, Ty, kt)\\&\geq_{L^*} p(t)\mathcal{M}_{M,N}(Ax, Sx, t) + q(t)\mathcal{M}_{M,N}(Sx, Ty, t).\end{aligned}$$

(vi) either self maps S and T are continuous or else B and S are continuous or else A and T are continuous.
Then the maps A, B, S and T have a unique common fixed point in X.

Proof.: Let $x_0 \in X$ be any arbitrary point in X.Construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_{2n+1} = ATx_{2n} = STx_{2n+1}, y_{2n+2} = BSx_{2n+1} = STx_{2n+2}$, for $n = 0, 1, 2 \ldots.$ Then by Lemma 2.17, $\{y_n\}$ in a Cauchy sequence in X, which is complete. Hence $\{y_n\} \to z \in X$. Also
$ATx_{2n} \to z$ and $STx_{2n} \to z$.
$BSx_{2n+1} \to z$ and $STx_{2n+1} \to z$.
Putting $Tx_{2n} = v_n, Sx_{2n+1} = w_{n+1}$ and as $ST = TS$ we get

$$Av_n \to z \quad \text{and} \quad Sv_n \to z. \tag{3.1}$$

$$Bw_{n+1} \to z \quad \text{and} \quad Sw_{n+1}\} \to z. \tag{3.2}$$

Case I: Self maps S and T are continuous:
As S is continuous, we have

$$SAv_n \to Sz, \text{ and } S^2v_n \to Sz. \tag{3.3}$$

And as (A, S) is compatible of type (P) we have,

$$A^2v_n \to Sz. \tag{3.4}$$

Step I: Putting $x = Av_n$ and $y = w_{n+1}$ in (v) we have,

$$\begin{aligned}&\mathcal{M}_{M,N}(A^2v_n, Bw_{n+1}, kt) + a(t)\mathcal{M}_{M,N}(Bw_{n+1}, Tw_{n+1}, kt)\\&\geq_{L^*} p(t)\mathcal{M}_{M,N}(A^2v_n, SAv_n, t) + q(t)\mathcal{M}_{M,N}(SAv_n, Tw_{n+1}, t).\end{aligned}$$

Taking limit as $n \to \infty$, using (3.2), (3.3) and (3.4)we get,
$\mathcal{M}_{M,N}(Sz, z, kt) + a(t)\mathcal{M}_{M,N}(z, z, kt) \geq_{L^*} p(t)\mathcal{M}_{M,N}(Sz, Sz, t) + q(t)\mathcal{M}_{M,N}(Sz, z, t)$,

i. e.

$$\mathcal{M}_{M,N}(Sz, z, kt) + a(t)1_{L^*} \geq_{L^*} p(t)1_{L^*} + q(t)\mathcal{M}_{M,N}(Sz, z, t). \tag{3.5}$$

As $0 < k < 1$ we have
$\mathcal{M}_{M,N}(Sz, z, t) + a(t)1_{L^*} \geq_{L^*} p(t)1_{L^*} + q(t)\mathcal{M}_{M,N}(Sz, z, t)$,
which gives
$(1 - q(t))\mathcal{M}_{M,N}(Sz, z,) \geq_{L^*} [p(t) - a(t)]1_{L^*}$.
If $q(t) < 1$ and as $p(t) + q(t) - a(t) = 1$, we get,
$\mathcal{M}_{M,N}(Sz, z, t) \geq_{L^*} 1_{L^*}$;
which gives $Sz = z$. If $q(t) = 1$, then as $p(t) + q(t) - a(t) = 1$ we have $p(t) = a(t)$. Hence from (3.5)we have
$\mathcal{M}_{M,N}(Sz, z, kt) \geq_{L^*} \mathcal{M}_{M,N}(Sz, z, t)$.
Therefore by Proposition 2.14, we have $Sz = z$. Thus in both the cases $Sz = z$.
Step II: Putting $x = z$ and $y = w_{n+1}$in (v) we have,

$$\begin{aligned} &\mathcal{M}_{M,N}(Az, Bw_{n+1}, kt) + a(t)\mathcal{M}_{M,N}(Bw_{n+1}, Tw_{n+1}, kt) \\ &\geq_{L^*} p(t)\mathcal{M}_{M,N}(Az, Sz, t) + q(t)\mathcal{M}_{M,N}(Sz, Tw_{n+1}, t). \end{aligned}$$

Taking limit as $n \to \infty$, using (3.2) and $Sz = z$ we get,
$\mathcal{M}_{M,N}(Az, z, kt) + a(t)\mathcal{M}_{M,N}(z, z, kt) \geq_{L^*} p(t)\mathcal{M}_{M,N}(Az, z, t) + q(t)\mathcal{M}_{M,N}(z, z, t)$,
i. e.
$\mathcal{M}_{M,N}(Az, z, kt) + a(t)_{L^*} \geq_{L^*} p(t)\mathcal{M}_{M,N}(Az, z, t) + q(t)1_{L^*}$.
As $0 < k < 1, p(t) < 1$ we have
$\mathcal{M}_{M,N}(Az, z, t) \geq_{L^*} \frac{q(t)-a(t)}{1-p(t)}1_{L^*}$.
As $p(t) + q(t) - a(t) = 1$, we get
$\mathcal{M}_{M,N}(Az, z, t) \geq_{L^*} 1_{L^*}$, for all t.
Thus $Az = z$. Therefore $Az = Sz = z$.
Step III: As T is continuous, we have

$$TBw_{n+1} \to Tz \text{ and } T^2w_{n+1} \to Tz. \tag{3.6}$$

And as (B, T) is compatible of type (P) we have,

$$B^2w_{n+1} \to Tz. \tag{3.7}$$

Putting $x = v_n$ and $y = Bw_{n+1}$ in (v) we have,

$$\begin{aligned} &\mathcal{M}_{M,N}(Av_n, B^2w_{n+1}, kt) + a(t)\mathcal{M}_{M,N}(B^2w_{n+1}, TBw_{n+1}, kt) \\ &\geq_{L^*} p(t)\mathcal{M}_{M,N}(Av_n, Sv_n, t) + q(t)\mathcal{M}_{M,N}(Sv_n, TBw_{n+1}, t). \end{aligned}$$

Taking limit as $n \to \infty$, using (3.1), (3.6) and (3.7) we get,
$\mathcal{M}_{M,N}(z, Tz, kt) + a(t)\mathcal{M}_{M,N}(Tz, Tz, kt) \geq_{L^*} p(t)\mathcal{M}_{M,N}(z, z, t) + q(t)\mathcal{M}_{M,N}(z, Tz, t)$,
i. e.

$$\mathcal{M}_{M,N}(z, Tz, kt) + a(t)1_{L^*} \geq_{L^*} p(t)1_{L^*} + q(t)\mathcal{M}_{M,N}(z, Tz, t) \tag{3.8}$$

As $0 < k < 1$, we get,
$\mathcal{M}_{M,N}(z, Tz, t) + a(t)1_{L^*} \geq_{L^*} p(t)1_{L^*} + q(t)\mathcal{M}_{M,N}(z, Tz, t)$.

If $q(t) < 1$, then
$\mathcal{M}_{M,N}(z, Tz, t) \geq_{L^*} \frac{p(t)-a(t)}{1-q(t)} 1_{L^*}$;
and p(t) + q(t) - a(t) = 1 implies
$\mathcal{M}_{M,N}(z, Tz, t) \geq_{L^*} 1_{L^*}$,
which gives $Tz = z$.
On the other hand if $q(t) = 1$, for all t , then as $p(t) + q(t) - a(t) = 1$, we have $p(t) = a(t)$.
Hence from (3.8)we have
$\mathcal{M}_{M,N}(z, Tz, kt) \geq_{L^*} \mathcal{M}_{M,N}(z, Tz, t)$.
Therefore by Proposition 2.14, we have $Tz = z$. Thus in both the cases $Tz = z$. Hence $Az = Sz = Tz = z$.
Step IV : Putting $x = v_n$ and $y = z$ in (v) we have,
$\mathcal{M}_{M,N}(Av_n, Bz, kt) + a(t)\mathcal{M}_{M,N}(Bz, Tz, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(Av_n, Sv_n, t) + q(t)\mathcal{M}_{M,N}(Sv_n, Tz, t)$.
Letting $n \to \infty$, using (3.1) and $Bz = Tz = z$ we get
$\mathcal{M}_{M,N}(z, Bz, kt) + a(t)\mathcal{M}_{M,N}(Bz, z, kt) \geq_{L^*} p(t)\mathcal{M}_{M,N}(z, z, t) + q(t)\mathcal{M}_{M,N}(z, z, t)$,
so
$(1 + a(t))\mathcal{M}_{M,N}(Bz, z, kt) \geq_{L^*} (p(t) + q(t))1_{L^*}$.
As $p(t) + q(t) - a(t) = 1, a(t) > 0$ we get,
$\mathcal{M}_{M,N}(Bz, z, kt) \geq_{L^*} 1_{L^*}$.
Therefore
$\mathcal{M}_{M,N}(Bz, z, kt) = 1$, for all t,
and so $Bz = z$. Thus

$$Tz = Bz = z. \tag{3.9}$$

Hence $Az = Bz = Sz = Tz = z$ in this case.
Case II: Self maps S and B are continuous:
By case I (step I and step II), as S is continuous and (A, S) is compatible of type (P)we get $Az = Sz = z$. As B is continuous we have
$B^2 w_{n+1} \to Bz$ and $BTw_{n+1} \to Bz$.
And as (B, T) is compatible of type (P) we have,
$T^2 w_{n+1} \to Bz$.
Step V: Putting $x = v_n$ and $y = Tw_{n+1}$ in (v) we have,

$$\begin{aligned}&\mathcal{M}_{M,N}(Av_n, BTw_{n+1}, kt) + a(t)\mathcal{M}_{M,N}(BTw_{n+1}, T^2 w_{n+1}, kt)\\ &\geq_{L^*} p(t)\mathcal{M}_{M,N}(Av_n, Sv_n, t) + q(t)\mathcal{M}_{M,N}(Sv_n, T^2 w_{n+1}, t).\end{aligned}$$

By similar reasoning as given in step III, we get $Bz = z$. Thus $Az = Sz = Bz = z$. Therefore $Az = Bz = Sz = z$. Also $BSz = z$. As $BS(X) \subseteq ST(X)$, there exists $v \in X$ such that $z = BSz = STv$. As $ST = TS$ we have $z = BSz = STv = TSv$.
Step VI: Putting $x = z$ and $y = Sv$ in (v) we have,
$\mathcal{M}_{M,N}(Az, BSv, kt) + a(t)\mathcal{M}_{M,N}(BSv, TSv, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(Az, Sz, t) + q(t)\mathcal{M}_{M,N}(Sz, TSv, t)$,
i. e.
$\mathcal{M}_{M,N}(z, BSv, kt) + a(t)\mathcal{M}_{M,N}(BSv, z, kt)$

$\geq_{L^*} p(t)\mathcal{M}_{M,N}(z,z,t) + q(t)\mathcal{M}_{M,N}(z,z,t)$,
i. e.
$[1 + a(t)]\mathcal{M}_{M,N}(z, BSv, kt) \geq_{L^*} [p(t) + q(t)]1_{L^*}$,
so as $a(t) > 0$ we have
$\mathcal{M}_{M,N}(z, BSv, kt) \geq_{L^*} \frac{p(t)+q(t)}{1+a(t)} 1_{L^*}$;
and $p(t) + q(t) - a(t) = 1$ gives
$\mathcal{M}_{M,N}(z, BSv, kt) \geq_{L^*} 1_{L^*}$.
Thus $BSv = z$. Therefore $BSv = TSv = z$. As (B, T) is compatible of type (P) so is weak compatible and so we have $Bz = Tz$. Therefore in this case also $Az = Bz = Sz = Tz = z$.
Case III: Self maps A and T are continuous:
As T is continuous, by case I (step III and IV), we get $Bz = Tz = z$. As A is continuous, we have

$$A^2 v_n \to Az \text{ and } ASv_n \to Az. \tag{3.10}$$

And as (A, S) is compatible of type (P) we have,

$$S^2 v_n \to Az. \tag{3.11}$$

Step VII: Putting $x = Sv_n$ and $y = w_{n+1}$ in (v) we have,
$\mathcal{M}_{M,N}(ASv_n, Bw_{n+1}, kt) + a(t)\mathcal{M}_{M,N}(Bw_{n+1}, Tw_{n+1}, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}ASv_n, S^2v_n, t) + q(t)\mathcal{M}_{M,N}(S^2v_n, Tw_{n+1}, t)$.
Taking limit as $n \to \infty$, using (3.2), (3.10), (3.11) we get,
$\mathcal{M}_{M,N}(Az, z, kt) + a(t)\mathcal{M}_{M,N}(z, z, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(Az, Az, t) + q(t)\mathcal{M}_{M,N}(Az, z, t)$.
By similar reasoning as given in step I, we get $Az = z$. Thus $Az = Bz = Tz = z$. Also $ATz = z$. As $AT(X) \subseteq ST(X)$, there exists $w \in X$ such that $z = ATz = STw$.
Step VIII: Putting x = Tw and y = z in (v) we have,
$\mathcal{M}_{M,N}(ATw, Bz, kt) + a(t)\mathcal{M}_{M,N}(Bz, Tz, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(ATw, STw, t) + q(t)\mathcal{M}_{M,N}(STw, Tz, t)$.
Using $Bz = Tz = STw = z$ we get,
$\mathcal{M}_{M,N}(ATw, z, kt) + a(t)1_{L^*} \geq_{L^*} p(t)\mathcal{M}_{M,N}(ATw, z, t) + q(t)1_{L^*}$.
By similar reasoning as given in step II we have $ATw = z$. Thus $ATw = STw = z$. As (A, S) is compatible of type (P) so is weak compatible and so we have $Az = Sz$. Therefore in this case also $Az = Bz = Sz = Tz = z$. Thus z is a common fixed point of four self maps A, B, S and T, in all the three cases.
Uniqueness: Let u be another common fixed point of A, B, S and T i.e. $Au = Bu = Su = Tu = u$. Putting $x = z$ and $y = u$ in (v) we get,
$\mathcal{M}_{M,N}(Az, Bu, kt) + a(t)\mathcal{M}_{M,N}(Bu, Tu, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(Az, Sz, t) + q(t)\mathcal{M}_{M,N}(Sz, Tu, t)$,
i. e.
$\mathcal{M}_{M,N}(z, u, kt) + a(t)\mathcal{M}_{M,N}(u, u, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(z, z, t) + q(t)\mathcal{M}_{M,N}(z, u, t)$.
By similar reasoning as given in step I, we get $z = u$. Therefore u is the unique common fixed point of four self maps A, B, S and T. □

Theorem 3.2. *Let A, B, S and T be self mappings of a complete intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying (i), (ii) (iv), (v) and :*
(vii) either S is onto or else T is onto;
(viii) self maps A and B are continuous.
Then the maps A, B, S and T have a unique common fixed point in X.

Proof. Proceeding as in Theorem 3.1 we have (3.1) and (3.2). As A is continuous, from step VII of Theorem 3.1, $Az = z$. As B is continuous, from step V of Theorem 3.1 we have $Bz = z$. Thus $Az = Bz = z$.
If T is onto then $z = Az \in AT(X) \subseteq ST(X)$. If S is onto then $z = Bz \in BS(X) \subseteq ST(X)$. Thus in both the cases $z \in ST(X)$. Therefore there exists $w \in X$ such that $z = STw$. As $ST = TS$ we have $z = STw = TSw$.
Step IX: Putting $x = v_n$ and $y = Sw$ in (v) we have,
$\mathcal{M}_{M,N}(Av_n, BSw, kt) + a(t)\mathcal{M}_{M,N}(BSw, TSw, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(Av_n, Sv_n, t) + q(t)\mathcal{M}_{M,N}(Sv_n, TSw, t)$.
Letting $n \to \infty$ and using $TSw = z$ we get
$\mathcal{M}_{M,N}(z, BSw, kt) + a(t)\mathcal{M}_{M,N}(BSw, z, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(z, z, t) + q(t)\mathcal{M}_{M,N}(z, z, t)$,
i.e.
$[1 + a(t)]\mathcal{M}_{M,N}(BSw, z, kt) \geq_{L^*} [p(t) + q(t)]1_{L^*}$.
By similar reasoning as given in step VI of Theorem 3.1, we have $BSw = z$. Therefore $TSw = BSw = z$. As (B, T) is compatible of type (P) so is weak compatible and thus we have $Tz = Bz = z$. Hence $Az = Bz = Tz = z$.
Step X: Putting $x = Tw$ and $y = w_{n+1}$ in (v) we have,
$\mathcal{M}_{M,N}(ATw, Bw_{n+1}, kt) + a(t)\mathcal{M}_{M,N}(Bw_{n+1}, Tw_{n+1}, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(ATw, STw, t) + q(t)\mathcal{M}_{M,N}(STw, Tw_{n+1}, t)$.
Letting $n \to \infty$ and using $STw = z$ we get
$\mathcal{M}_{M,N}(ATw, z, kt) + a(t)\mathcal{M}_{M,N}(z, z, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(ATw, z, t) + q(t)\mathcal{M}_{M,N}(z, z, t)$.
By similar reasoning as given in step V of Theorem 3.1 we get $ATw = z$. Therefore $z = ATw = STw$. As (A, S) is compatible of type (P) so is weak compatible and thus we have $Az = Sz = z$. Hence $Az = Bz = Sz = Tz = z$. Thus in both the cases z is a common fixed point of four self maps A, B, S and T. Uniqueness of the fixed point follows from Theorem 3.1.

□

Example 3.3. (Theorem 3.2) Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be the complete intuitionistic fuzzy metric space, where $X = [0, 1]$ and for $a = (a_1, a_2), b = (b_1, b_2)$ in L^* define

$$\mathcal{T}(a, b) = (a_1 b_1, \min\{a_2 + b_2, 1\})$$

and

$$\mathcal{M}_{M,N}(x,y,t) = \left(\frac{t}{t+|x-y|}, \frac{|x-y|}{t+|x-y|}\right),$$

for all $x, y \in X$ and for all t. Define self maps A, B, S and T on X as follows:
$A(x) = B(x) = \frac{1}{4}, T(x) = x$ for all x in X
$S(x) = \{ \begin{array}{ll} x, & \text{if x is rational} \\ \frac{1}{4}, & \text{if x is irrational.} \end{array}$
Then the maps A and B are continuous and T is surjective. Also pairs (A, S) and (B, T) are compatible of type (P).Clearly the containment condition (i) of the Theorem 3.2 is satisfied. Further the contractive condition (v) holds for $k = \frac{1}{2}$. Thus all the conditions of Theorem 3.2 are satisfied and $x = \frac{1}{4}$ is the unique common fixed point of four self maps A, B, S and T.

Theorem 3.4. *Let A, B, S and T be self mappings of a complete intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying (i), (ii) (iv), (viii) and :*
(ix) For some $k \in (0,1)$ there exists continuous real functions $p(t), q(t), a(t)$ from $(0,\infty)$ to $[0,1]$ with $p(t) < 1, q(t) < 1$ or else $q(t) = 1$, for all t with $p(t) + q(t) - a(t) = 1$, such that for all $x, y \in X$ and for all t
$\mathcal{M}_{M,N}(Ax, By, kt)\mathcal{T}\mathcal{M}_{M,N}(Sx, Ty, kt) + a(t)\mathcal{M}_{M,N}(By, Ty, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(Ax, Sx, t) + q(t)\mathcal{M}_{M,N}(Sx, Ty, t).$
Then the maps A, B, S and T have a unique common fixed point in X.

Proof. As
$\mathcal{M}_{M,N}(Ax, By, kt)\mathcal{T}\mathcal{M}_{M,N}(Sx, Ty, kt) \leq_{L^*} \mathcal{M}_{M,N}(Ax, By, kt),$
by Proposition 2.14, it follows that condition (xi) implies condition (v). Hence by step V and VII we have $Az = Bz = z$.
Step XI: Putting $x = v_n, y = z$ in (ix) we have,
$\mathcal{M}_{M,N}(Av_n, Bz, kt)\mathcal{T}\mathcal{M}_{M,N}(Sv_n, Tz, kt) + a(t)\mathcal{M}_{M,N}(Bz, Tz, kt)$
$\geq_{L^*} p(t)\mathcal{M}_{M,N}(Av_n, Sv_n, t) + q(t)\mathcal{M}_{M,N}(Sv_n, Tz, t).$
Letting $n \to \infty$ and using $Bz = z$ we get
$\mathcal{M}_{M,N}(z, z, kt)\mathcal{T}\mathcal{M}_{M,N}(z, Tz, kt) + a(t)\mathcal{M}_{M,N}(z, Tz, kt) \geq_{L^*} p(t)\mathcal{M}_{M,N}(z, z, t) + q(t)\mathcal{M}_{M,N}(z, Tz, t),$
i.e.
$1_{L^*}\mathcal{T}\mathcal{M}_{M,N}(z, Tz, kt) + a(t)\mathcal{M}_{M,N}(z, Tz, kt) \geq_{L^*} p(t)1_{L^*} + q(t)\mathcal{M}_{M,N}(z, Tz, t).$
Therefore
$[1 + a(t)]\mathcal{M}_{M,N}(z, Tz, kt) \geq_{L^*} [p(t) + q(t)]\mathcal{M}_{M,N}(z, Tz, t).$
As $p(t) + q(t) - a(t) = 1$ and $a(t) > 0$ we have
$\mathcal{M}_{M,N}(z, Tz, kt) \geq_{L^*} \mathcal{M}_{M,N}(z, Tz, t).$
Hence by Proposition 2.13, we have $Tz = z$. Thus $Az = Bz = Tz = z$. By step VIII, in Theorem 3.1 we get $Sz = z$. Therefore $Az = Bz = Sz = Tz = z$ i. e. z is a common fixed point of four self maps A, B, S and T. Uniqueness of the fixed point follows from Theorem 3.1. □

Following is a more complete result:

Theorem 3.5. *Let A, B, S and T be self mappings of a complete intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying (i), (ii),(ix) and :*
- *pairs (A, S) and (B, T) are compatible of type (P);*
- *one map from each of the two compatible pairs of type (P) is continuous.*

Then the maps A, B, S and T have a unique common fixed point in X.

Proof. Result follows from Theorem 3.4 and cases I, II and III of Theorem 3.1 as the contractive condition (ix) implies contractive condition (v). □

Taking $B = A, T = S$ and $p(t) = a(t) = 0, q(t) = 1$ Theorem 3.1 we get,

Corollary 3.6. *: Let A and S be self mappings of a complete intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying:*

(x)$AS(X) \subseteq S^2(X)$;
(xi) the pair (A, S) is compatible of type (P);
(xii) Self map S is continuous;
(xiii) there exist $k \in (0, 1)$ such that for all $x, y \in X$ and for all t,
$\mathcal{M}_{M,N}(Ax, Ay, kt) \geq_{L^*} \mathcal{M}_{M,N}(Sx, Sy, t)$.
Then the self maps A and S have a unique common fixed point in X.

In [20] D. Turkoglu et. al (2006) proved the following:
Theorem 2 [20]: Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and $A, S : X \to X$ be mappings satisfying the following conditions:

- $A(X) \subseteq S(X)$;
- S is continuous;
- there exists $0 < k < 1$ such that for all $x, y \in X$, and for all $t > 0$
  $M(S(x), S(y), kt) \geq M(A(x), A(y), t), N(S(x), S(y), kt) \leq N(A(x), A(y), t)$.

Then A and S have a unique common fixed point in X provided A and S commute on X.

Remark 3.7. :Above result follows from Corollary 3.6 as $AS = SA$ gives $AS(X) = SA(X) \subseteq SS(X)$. Also as S is continuous and $Ax_n \to x, Sx_n \to x$ give $ASx_n = SAx_n \to Sx, S^2x_n \to Sx$. Taking $x = Ax_n, y = Sx_n$ in (xii) we get
$\mathcal{M}_{M,N}(A^2x_n, ASx_n, kt) \geq_{L^*} \mathcal{M}_{M,N}(SAx_n, S^2x_n, t)$.
Letting $n \to \infty$ we get
$lim_{n\to\infty}\mathcal{M}_{M,N}(A^2x_n, ASx_n, kt) \geq_{L^*} \mathcal{M}_{M,N}(Sx, Sx, t) = 1_{L^*}$,
i. e.
$lim_{n\to\infty}\mathcal{M}_{M,N}(A^2x_n, Sx, kt) = 1_{L^*}$,
implies $A^2x_n \to Sx$. Hence
$lim_{n\to\infty}\mathcal{M}_{M,N}(A^2x_n, S^2x_n, t) = 1_{L^*}$.

Thus (A, S) is compatible of type (P). Hence A and S have a unique common fixed point in X.

Example 3.8. (of Corollary 3.6): Let $X = \{\frac{1}{n}\}_{n\in N} \cup \{0\}$. Define $\mathcal{M}_{M,N}$ and $\mathcal{T}$ as in Example 3.3. Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is a complete intuitionistic fuzzy metric space. Define self maps A and S on X as follows:
$A(0) = 0, A(\frac{1}{n}) = \frac{1}{2n+3}$ and $S(0) = 0, S(\frac{1}{n}) = \frac{1}{n+1}$, for all n. Then $AS(1) = A(\frac{1}{2}) = \frac{1}{7}$ and $SA(1) = S(\frac{1}{5}) = \frac{1}{6}$. Therefore $AS \neq SA$. Hence (A, S) is non-commuting. Clearly pair (A, S) is compatible of type (P), as every sequence in X converges to 0. Hence the pair (A, S) is non-commuting yet it is compatible of type (P). Moreover $AS(X) \subseteq S^2(X)$. Also the contractive condition (xiii) holds for $k = \frac{1}{2}$. Thus all the conditions of corollary 3.6 are satisfied and $x = 0$ is the unique common fixed point of the self maps A and S.

Taking $S = I$, the identity map in Corollary 3.6 we get the following intuitionistic fuzzy metric space version of Grabiec's result [9]:

Corollary 3.9. *: Let A be a self mapping of a complete intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying :*
$\mathcal{M}_{M,N}(Ax, Ay, kt) \geq_{L^*} \mathcal{M}_{M,N}(x, y, t)$, for all t and for some $k \in (0, 1)$.
Then A has a unique fixed point in X.

Proof. Result follows from Corollary 3.6 as $AI(X) = A(X) \subseteq X = I^2(X)$ and $AI = IA$. ☐

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