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Oscillation results for nonlinear second-order damped dynamic equations

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Abstract

The oscillatory behavior of a class of second-order nonlinear dynamic equations with damping on an arbitrary time scale is considered without requiring explicit sign assumptions on the derivative of the nonlinearity. Several sufficient conditions for the oscillation of solutions are presented using the Riccati transformation and integral averaging technique. An illustrative example is provided.

Keywords: Oscillation, second-order, nonlinear dynamic equation, damping term, time scale.
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1. Introduction

In this paper, we study the oscillation of solutions to a second-order nonlinear dynamic equation with a damping term

$$\left(a(t)\psi(x(t))x^{\Delta}(t)\right)^{\Delta} + p(t)x^{\Delta^{\sigma}}(t) + q(t)f\left(x^{\sigma}(t)\right) = 0, \tag{1.1}$$

where $t \in [t_0, \infty)_{\mathbb{T}} := [t_0, \infty) \cap \mathbb{T}$, a, p, $q \in \mathrm{C}_{\mathrm{rd}}([t_0, \infty)_{\mathbb{T}}, (0, \infty))$, $\psi \in \mathrm{C}(\mathbb{R}, (0, \infty))$, $g_1(t) \le \psi(x(t)) \le g_2(t)$, g_1 and g_2 are positive rd-continuous real-valued functions. Analysis of qualitative properties of (1.1) is

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important not only for the sake of further development of the oscillation theory, but for practical reasons too. As a matter of fact, a particular case of (1.1), a second-order damped differential equation

$$\big(a(t)x'(t)\big)' + p(t)x'(t) + q(t)f(x(t)) = 0,$$

has numerous applications in the study of noise, vibration, and harshness of vehicles; see, e.g., the paper by Fu et al. [10].

By a solution of (1.1) we mean a nontrivial function $x \in \mathrm{C}^1_{\mathrm{rd}}([t_0,\infty)_{\mathbb{T}},\mathbb{R})$ which has the property $a(\psi\circ x)x^{\Delta} \in \mathrm{C}^1_{\mathrm{rd}}([t_0,\infty)_{\mathbb{T}},\mathbb{R})$ and satisfies (1.1) on $[t_0,\infty)_{\mathbb{T}}$. Our attention is restricted to those solutions of (1.1) that satisfy $\sup\{|x(t)| : t \in [t_1,\infty)_{\mathbb{T}}\} > 0$ for all $t_1 \in [t_0,\infty)_{\mathbb{T}}$ and we tacitly assume that (1.1) possesses such solutions. A solution of (1.1) is said to be oscillatory if it is neither eventually positive nor eventually negative; otherwise, it is called nonoscillatory. Equation (1.1) is termed oscillatory if all its solutions are oscillatory.

The oscillation theory of dynamic equations on time scales has received considerable interest in the past few years because it plays an important role in unifying the oscillation of differential equations, difference equations, and the so-called q-difference equations, etc. Following Hilger's landmark [12], several authors have expounded on various aspects of the theory of time scales; see, for instance, the paper [3], the monographs [5, 6], and the references cited therein. For completeness, we recall the following concepts related to the notion of time scales. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$ and, since oscillation of solutions is our primary concern, we assume throughout that $\sup\mathbb{T} = \infty$. For instance, the real numbers and the integers are special examples of time scales. On any time scale $\mathbb{T}$, we define the forward and backward jump operators by

$$\sigma(t) := \inf\{s \in \mathbb{T} | s > t\} \quad \text{and} \quad \rho(t) := \sup\{s \in \mathbb{T} | s < t\},$$

where $\inf\emptyset := \sup\mathbb{T}$ and $\sup\emptyset := \inf\mathbb{T}$, $\emptyset$ denotes the empty set. A point $t \in \mathbb{T}$ is said to be left-dense if $t > \inf\mathbb{T}$ and $\rho(t) = t$, right-dense if $t < \sup\mathbb{T}$ and $\sigma(t) = t$, left-scattered if $\rho(t) < t$, and right-scattered if $\sigma(t) > t$. A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous (right-dense continuous) provided that f is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of all such rd-continuous functions is denoted by $\mathrm{C}_{\mathrm{rd}}(\mathbb{T},\mathbb{R})$. The graininess function μ for a time scale $\mathbb{T}$ is defined by $\mu(t) := \sigma(t) - t$, and for any function $f : \mathbb{T} \to \mathbb{R}$ the notation $f^{\sigma}(t)$ denotes $f(\sigma(t))$. We say that a function $p : \mathbb{T} \to \mathbb{R}$ is regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}$. The set of all regressive and rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted in this paper by $\mathcal{R}(\mathbb{T},\mathbb{R})$. If $p \in \mathcal{R}(\mathbb{T},\mathbb{R})$, then the exponential function is defined by

$$e_p(t,s) := \exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\right) \quad \text{for} \quad s,t \in \mathbb{T},$$

where the cylinder transformation $\xi_h(z)$ is defined by

$$\xi_h(z) := \begin{cases} \frac{\mathrm{Log}(1+zh)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0, \end{cases}$$

where Log is the principal logarithm function. It can be seen immediately from the latter formula that the exponential function never vanishes; however, in contrast to the case $\mathbb{T} = \mathbb{R}$, the exponential function could possibly attain negative values. As an example, consider the problem $y^{\Delta} = -2y$, $y(0) = 1$ for $\mathbb{T} = \mathbb{Z}$. It is well known (see [5, Theorem 2.33]) that if $p \in \mathcal{R}(\mathbb{T},\mathbb{R})$, then $e_p(\cdot,t_0)$ is a solution of the initial value problem $y^{\Delta} = p(t)y$, $y(t_0) = 1$.

In what follows, let us briefly comment on a number of closely related results which motivated our study. In the special case when $\mathbb{T} = \mathbb{R}$, equation (1.1) reduces to a second-order differential equation

$$\big(a(t)\psi(x(t))x'(t)\big)' + p(t)x'(t) + q(t)f(x(t)) = 0,$$

which was studied by Kirane and Rogovchenko [13] and Rogovchenko and Tuncay [17] who established several oscillation criteria. For oscillation of damped dynamic equations on time scales, we refer the reader to the papers [1, 2, 4, 7, 8, 9, 11, 15, 16, 18, 19] and the references cited therein. Erbe and Peterson [7, 8] investigated a second-order nonlinear damped dynamic equation

$$x^{\Delta\Delta} + p(t)x^{\Delta^\sigma} + q(t)(f \circ x^\sigma) = 0 \tag{1.2}$$

under the assumptions that

$$f'(u) \geq \frac{f(u)}{u} \geq \lambda > 0 \quad \text{for all } |u| \geq K > 0 \tag{1.3}$$

and

$$f'(u) \geq \frac{f(u)}{u} > 0 \quad \text{for all } u \neq 0, \tag{1.4}$$

respectively. Assuming

$$f'(u) > 0 \quad \text{and} \quad uf(u) > 0 \quad \text{for all } u \neq 0 \tag{1.5}$$

instead of (1.3) and (1.4), Bohner et al. [4] improved results of [7, 8]. We conclude by mentioning that Saker et al. [18] studied another particular case of (1.1) assuming that $\psi(x(t)) = 1$ and

$$f \in \mathrm{C}(\mathbb{R}, \mathbb{R}), \quad f'(u) \geq K \quad \text{for all } u \neq 0 \text{ and for some } K > 0. \tag{1.6}$$

Note that conditions (1.3), (1.4), (1.5), and (1.6) cannot be applied to some f, for instance, by letting

$$f(u) = u\left(1 + \frac{18}{2 + u^2}\right),$$

we have

$$f'(u) = \frac{(u^2 - 4)(u^2 - 10)}{(2 + u^2)^2}.$$

It should be noted that research in this paper was strongly motivated by the contributions of Bohner et al. [4], Erbe and Peterson [7, 8], Philos [14], and Saker et al. [18]. Our principal goal is to analyze the oscillatory behavior of solutions to (1.1) in the cases where

$$f \in \mathrm{C}(\mathbb{R}, \mathbb{R}), \quad f(u) \geq ku \quad \text{for all } u \neq 0 \text{ and for some } k > 0, \tag{1.7}$$

and

$$\int_{t_0}^{\infty} \frac{\Delta t}{a(t)g_2(t)e_{p/(ag_1)^\sigma}(t, t_0)} = \infty. \tag{1.8}$$

As usual, all functional inequalities considered in the sequel are supposed to hold for all t large enough.

2. Oscillation results

Theorem 2.1. *Let conditions* (1.7) *and* (1.8) *be satisfied. If there exists a function* $\delta \in \mathrm{C}^1_{\mathrm{rd}}([t_0, \infty)_{\mathbb{T}}, (0, \infty))$ *such that*

$$\limsup_{t \to \infty} \int_{t_0}^{t} \left[k\delta(s)q(s) - \frac{a(s)g_2(s)\varphi^2(s)}{4\delta(s)}\right] \Delta s = \infty, \tag{2.1}$$

where

$$\varphi(t) := \delta^\Delta(t) - \frac{\delta(t)p(t)}{(ag_2)^\sigma(t)},$$

then equation (1.1) *is oscillatory.*

Proof. Suppose to the contrary that x is a nonoscillatory solution of (1.1). Without loss of generality, we may assume that there exists a $t_1 \in [t_0, \infty)_{\mathbb{T}}$ such that $x(t) > 0$ and $x^\sigma(t) > 0$ for $t \in [t_1, \infty)_{\mathbb{T}}$. It follows from (1.1) and (1.7) that, for $t \in [t_1, \infty)_{\mathbb{T}}$,

$$\left(a(t)\psi(x(t))x^\Delta(t)\right)^\Delta + p(t)x^{\Delta^\sigma}(t) = -q(t)f\left(x^\sigma(t)\right) \le -kq(t)x^\sigma(t) < 0. \tag{2.2}$$

Define a function $y := a(\psi \circ x)x^\Delta$. By virtue of (2.2),

$$y^\Delta(t) + \frac{p(t)}{a^\sigma(t)\psi(x^\sigma(t))}y^\sigma(t) < 0.$$

Then, we deduce from [5, Theorem 2.33] that

$$\left(e_{p/(a^\sigma(\psi\circ x^\sigma))}(\cdot, t_0)y\right)^\Delta < 0. \tag{2.3}$$

Hence, $e_{p/(a^\sigma(\psi\circ x^\sigma))}(\cdot, t_0)y$ is strictly decreasing, and so x^Δ is of one sign. That is, there exists a $t_2 \in [t_1, \infty)_{\mathbb{T}}$ such that either $x^\Delta(t) > 0$ or $x^\Delta(t) < 0$ for $t \in [t_2, \infty)_{\mathbb{T}}$. We consider each of two cases separately.

Case 1. Assume first that $x^\Delta(t) < 0$ for $t \in [t_2, \infty)_{\mathbb{T}}$. It follows now from (2.3) that there exists a constant $M > 0$ such that, for $t \in [t_2, \infty)_{\mathbb{T}}$,

$$a(t)g_2(t)e_{p/(ag_1)^\sigma}(t, t_0)x^\Delta(t) \le a(t)\psi(x(t))e_{p/(a^\sigma(\psi\circ x^\sigma))}(t, t_0)x^\Delta(t) \le -M,$$

which yields

$$x^\Delta(t) \le -\frac{M}{a(t)g_2(t)e_{p/(ag_1)^\sigma}(t, t_0)}.$$

Integrating the latter inequality from t_2 to t, we conclude that

$$x(t) \le x(t_2) - M\int_{t_2}^{t} \frac{\Delta s}{a(s)g_2(s)e_{p/(ag_1)^\sigma}(s, t_0)},$$

which implies that $\lim_{t\to\infty} x(t) = -\infty$ when using condition (1.8). This contradicts the fact that x is positive.

Case 2. Assume now that $x^\Delta(t) > 0$ for $t \in [t_2, \infty)_{\mathbb{T}}$. Inequality (2.2) implies that, for $t \in [t_2, \infty)_{\mathbb{T}}$,

$$\left(a(t)\psi(x(t))x^\Delta(t)\right)^\Delta < 0. \tag{2.4}$$

For $t \in [t_2, \infty)_{\mathbb{T}}$, define a Riccati substitution by

$$\omega(t) := \delta(t)\frac{a(t)\psi(x(t))x^\Delta(t)}{x(t)}. \tag{2.5}$$

Then $\omega(t) > 0$ and

$$\begin{aligned}
\omega^\Delta(t) &= (a(\psi\circ x)x^\Delta)^\sigma(t)\left(\frac{\delta}{x}\right)^\Delta(t) + \delta(t)\frac{(a(\psi\circ x)x^\Delta)^\Delta(t)}{x(t)}\\
&\le \delta^\Delta(t)\frac{(a(\psi\circ x)x^\Delta)^\sigma(t)}{x^\sigma(t)} - \delta(t)p(t)\frac{(a(\psi\circ x)x^\Delta)^\sigma(t)}{(a(\psi\circ x))^\sigma(t)x(t)}\\
&\quad - \delta(t)\frac{x^\Delta(t)(a(\psi\circ x)x^\Delta)^\sigma(t)}{x(t)x^\sigma(t)} - k\delta(t)q(t)\frac{x^\sigma(t)}{x(t)}
\end{aligned} \tag{2.6}$$

due to inequality (2.2). By (2.4) and condition $x^\Delta > 0$, we have

$$x^\Delta(t) \ge \frac{(a(\psi\circ x)x^\Delta)^\sigma(t)}{a(t)\psi(x(t))} \ge \frac{(a(\psi\circ x)x^\Delta)^\sigma(t)}{a(t)g_2(t)} \quad \text{and} \quad x^\sigma(t) \ge x(t).$$

Substituting the latter inequalities into (2.6) and using (2.5), we obtain

$$\begin{aligned}\omega^{\Delta}(t) &\leq -k\delta(t)q(t) + \frac{1}{\delta^{\sigma}(t)}\left(\delta^{\Delta} - \frac{\delta p}{(ag_2)^{\sigma}}\right)(t)\omega^{\sigma}(t) - \frac{\delta(t)}{(\delta^{\sigma}(t))^2 a(t)g_2(t)}(\omega^{\sigma}(t))^2 \\ &= -k\delta(t)q(t) + \frac{\varphi(t)}{\delta^{\sigma}(t)}\omega^{\sigma}(t) - \frac{\delta(t)}{(\delta^{\sigma}(t))^2 a(t)g_2(t)}(\omega^{\sigma}(t))^2.\end{aligned} \tag{2.7}$$

This implies, after completing the square, that

$$\omega^{\Delta}(t) \leq -\left[k\delta(t)q(t) - \frac{a(t)g_2(t)\varphi^2(t)}{4\delta(t)}\right].$$

Integrating the latter inequality from t_2 to t, we deduce that

$$\int_{t_2}^{t}\left[k\delta(s)q(s) - \frac{a(s)g_2(s)\varphi^2(s)}{4\delta(s)}\right]\Delta s \leq \omega(t_2) - \omega(t) \leq \omega(t_2),$$

which contradicts condition (2.1). Therefore, equation (1.1) is oscillatory. □

Remark 2.2. On the basis of Theorem 2.1, one can obtain various oscillation criteria for (1.1), e.g., by letting $\delta(t) = 1$, $\delta(t) = t$, etc. The details are left to the reader.

In the remainder of this section, we employ the integral averaging technique to replace assumption (2.1) with a Philos-type (see [14]) condition. To this end, let $\mathbb{D} := \{(t, s) : t \geq s \geq t_0, t, s \in [t_0,\infty)_{\mathbb{T}}\}$ and $\mathbb{D}_0 := \{(t, s) : t > s \geq t_0, t, s \in [t_0,\infty)_{\mathbb{T}}\}$. The function $H \in \mathrm{C_{rd}}(\mathbb{D}, \mathbb{R})$ is said to belong to the class $\mathcal{P}$ if

$$H(t, t) = 0 \quad \text{for } t \geq t_0, \quad H(t, s) > 0 \quad \text{for } (t, s) \in \mathbb{D}_0,$$

and H has a nonpositive rd-continuous Δ-partial derivative $H^{\Delta_s}(t, s)$ on $\mathbb{D}_0$ with respect to the second variable.

Theorem 2.3. *Let conditions* (1.7) *and* (1.8) *be satisfied. If there exists a function* $\delta \in \mathrm{C}^1_{\mathrm{rd}}([t_0,\infty)_{\mathbb{T}}, (0,\infty))$ *such that, for some* $H \in \mathcal{P}$ *and for all sufficiently large* $t_* \in [t_0,\infty)_{\mathbb{T}}$,

$$\limsup_{t\to\infty} \frac{1}{H(t, t_*)}\int_{t_*}^{t}\left[k\delta(s)q(s)H(t, s) - \frac{a(s)g_2(s)(\delta^{\sigma}(s)A(t, s))^2}{4\delta(s)H(t, s)}\right]\Delta s = \infty, \tag{2.8}$$

where

$$A(t, s) := H^{\Delta_s}(t, s) + H(t, s)\frac{\varphi(s)}{\delta^{\sigma}(s)}$$

and φ *is defined as in Theorem 2.1, then equation* (1.1) *is oscillatory.*

Proof. As above, we assume that x is an eventually positive solution of (1.1). Proceeding as in the proof of Theorem 2.1, we arrive at the inequality (2.7). Multiplying (2.7) by $H(t, s)$ and integrating the resulting inequality from t_2 to t, we have

$$\begin{aligned}\int_{t_2}^{t} k\delta(s)q(s)H(t, s)\Delta s \leq &-\int_{t_2}^{t} H(t, s)\omega^{\Delta}(s)\Delta s + \int_{t_2}^{t} H(t, s)\frac{\varphi(s)}{\delta^{\sigma}(s)}\omega^{\sigma}(s)\Delta s \\ &- \int_{t_2}^{t} H(t, s)\frac{\delta(s)}{(\delta^{\sigma}(s))^2 a(s)g_2(s)}(\omega^{\sigma}(s))^2\Delta s.\end{aligned}$$

By using integration by parts, we conclude that

$$\int_{t_2}^{t} H(t, s)\omega^{\Delta}(s)\Delta s = -H(t, t_2)\omega(t_2) - \int_{t_2}^{t} H^{\Delta_s}(t, s)\omega^{\sigma}(s)\Delta s.$$

Substitution of the latter equality into (2.9) implies that

$$\begin{aligned}\int_{t_2}^{t} k\delta(s)q(s)H(t,s)\Delta s \quad &\leq \quad H(t,t_2)\omega(t_2) + \int_{t_2}^{t}\left[H^{\Delta_s}(t,s) + H(t,s)\frac{\varphi(s)}{\delta^{\sigma}(s)}\right]\omega^{\sigma}(s)\Delta s \\ &\quad - \int_{t_2}^{t} H(t,s)\frac{\delta(s)}{(\delta^{\sigma}(s))^2 a(s)g_2(s)}(\omega^{\sigma}(s))^2\Delta s.\end{aligned}$$

Using the method of completing the square in the latter inequality, we get

$$\int_{t_2}^{t}\left[k\delta(s)q(s)H(t,s) - \frac{a(s)g_2(s)(\delta^{\sigma}(s)A(t,s))^2}{4\delta(s)H(t,s)}\right]\Delta s \leq H(t,t_2)\omega(t_2),$$

which yields

$$\frac{1}{H(t,t_2)}\int_{t_2}^{t}\left[k\delta(s)q(s)H(t,s) - \frac{a(s)g_2(s)(\delta^{\sigma}(s)A(t,s))^2}{4\delta(s)H(t,s)}\right]\Delta s \leq \omega(t_2),$$

which contradicts condition (2.8). The proof is complete. □

Remark 2.4. The conclusion of Theorem 2.3 remains intact if assumption (2.8) is replaced by the following two conditions

$$\limsup_{t\to\infty}\frac{1}{H(t,t_*)}\int_{t_*}^{t}\delta(s)q(s)H(t,s)\Delta s = \infty$$

and

$$\limsup_{t\to\infty}\frac{1}{H(t,t_*)}\int_{t_*}^{t}\frac{a(s)g_2(s)(\delta^{\sigma}(s)A(t,s))^2}{4\delta(s)H(t,s)}\Delta s < \infty.$$

Remark 2.5. With an appropriate choice of the functions δ and H, one can derive from Theorem 2.3 a number of oscillation criteria for (1.1). For instance, consider a Kamenev-type function $H(t,s)$ defined by $H(t,s) = (t-s)^{n-1}$, $(t,s) \in \mathbb{D}$, where $n > 2$ is an integer. The details are left to the reader.

3. Example

The following example illustrates possible applications of theoretical results obtained in the previous section.

Example 3.1. For $t \in [1,\infty)$, consider a second-order differential equation

$$\left(\frac{1+\sin^2\ln t}{\frac{1}{2}+\sin^2\ln t}\frac{\frac{1}{2}+x^2(t)}{1+x^2(t)}x'(t)\right)' + \frac{1}{t}x'(t) + \frac{1}{t^2}x(t) = 0. \tag{3.1}$$

Let $k = 1$, $g_1(t) = 1/2$, $g_2(t) = 1$, and $\delta(t) = t$. It is not difficult to verify that all assumptions of Theorem 2.1 are satisfied. Hence, equation (3.1) is oscillatory. As a matter of fact, $x(t) = \sin\ln t$ is one such solution.

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