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Complex valued rectangular b-metric spaces and an application to linear equations

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Abstract

In this paper, we introduce complex valued rectangular b-metric spaces. We prove an analogue of Banach contraction principle. We also prove a different contraction principle with a new condition and a fixed point theorem in this space. Finally, we give an application of Banach contraction principle to linear equations.

Keywords: Fixed point, Banach contraction principle, rectangular b-metric space.
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1. Introduction

The concept of a metric space was introduced by Frechet [17]. After the Banach contraction principle, because of its various applications, many mathematicians studied existence and uniqueness of fixed points. The Banach contraction principle was also proved in every new generalized metric spaces (see [13]).

The notion of rectangular metric space was introduced by Branciari [9]. He proved an analogue of the Banach contraction principle in this space, then various fixed point theorems were given for different contractions on rectangular metric spaces (see [2, 3, 4, 12, 16, 18, 19, 20, 25]).

Recently, many researchers have obtained fixed point results for b-metric spaces. Bakhtin [6] introduced the concept of b-metric spaces as a generalization of metric spaces. He also proved the Banach contraction principle in b-metric spaces. Many follwoing papers were studied in b-metric spaces (see [8, 10, 11, 22]).

On the other hand, Azam *et al.* [5] defined complex valued metric spaces and gave common fixed point results for mappings. Rao *et al.* [23] introduced the complex valued b-metric spaces. Mukheimer [21] generalized some results in [5]. Many researchers [1, 7, 14, 15, 24, 26] obtained fixed point results for complex valued metric spaces.

In this study, we have presented the notion of complex valued rectangular b-metric space. We give new definitions and two lemmas without proofs. Two contraction principles are then proved in this new space. Finally, we discuss an application to linear equations.

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2. Preliminaries

In this section, we give the required background information before our main results.

Definition 2.1 ([6])**.** Let X be a nonempty set and $d : X \times X \to [0, \infty)$ be a mapping. We say that d is a b-metric on X if,

(b1) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;
(b2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(b3) there exists a real number $s \geq 1$ such that $d(x, y) \leq s[d(x, z) + d(z, y)]$ for all $x, y, z \in X$,

and the pair (X, d) is called a b-metric space.

Definition 2.2 ([9])**.** Let X be a nonempty set and the mapping $d : X \times X \to [0, \infty)$ satisfies:

(R1) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;
(R2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(R3) $d(x, y) \leq d(x, u) + d(u, v) + d(v, y)$ for all $x, y \in X$ and all distinct points $u, v \in X \backslash \{x, y\}$.

Then d is called a rectangular metric on X and (X, d) is called a rectangular metric space.

Definition 2.3 ([19])**.** Let X be a nonempty set and the mapping $d : X \times X \to [0, \infty)$ satisfies:

(Rb1) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;
(Rb2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(Rb3) there exists a real number $s \geq 1$ such that $d(x, y) \leq s[d(x, u) + d(u, v) + d(v, y)]$ for all $x, y \in X$ and all distinct points $u, v \in X \backslash \{x, y\}$.

Then d is called a rectangular b-metric on X and (X, d) is called a rectangular b-metric space.

Definition 2.4 ([19])**.** Let (X, d) be a rectangular b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$.

- The sequence $\{x_n\}$ is said to be convergent in (X, d) and converges to x if for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) < \epsilon$ for all $n > n_0$ and is denoted by $\lim\limits_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

- The sequence $\{x_n\}$ is called Cauchy sequence in (X, d) if for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x_{n+p}) < \epsilon$ for all $n > n_0$, $p > 0$ or equivalently, if $\lim\limits_{n\to\infty} d(x_n, x_{n+p}) = 0$ for all $p > 0$.

- (X, d) is said to be a complete rectangular b-metric space if every Cauchy sequence in X converges to some $x \in X$.

The complex metric space was initiated by Azam *et al.* [5]. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leq Re(z_2) \text{ and } Im(z_1) \leq Im(z_2).$$

It follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

(C_1) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(C_2) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(C_3) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$,
(C_4) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

Particularly, we write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of $(C_2), (C_3)$ and (C_4) is satisfied and we write $z_1 \prec z_2$ if only (C_4) is satisfied. The following statements hold:

(1) If $a, b \in \mathbb{R}$ with $a \leq b$, then $az \prec bz$ for all $z \in \mathbb{C}$.
(2) If $0 \precsim z_1 \precnsim z_2$, then $|z_1| < |z_2|$.
(3) If $z_1 \precsim z_2$ and $z_2 \prec z_3$, then $z_1 \prec z_3$.

3. Main Results

In this section, we first introduce the complex valued rectangular b-metric spaces.

Definition 3.1. Let X be a nonempty set. Suppose that a mapping $d : X \times X \to \mathbb{C}$ satisfies:

(CRb1) $d(x,y) = 0$ if and only if $x = y$ for all $x, y \in X$;
(CRb2) $d(x,y) = d(y,x)$ for all $x, y \in X$;
(CRb3) there exists a real number $s \geq 1$ such that $d(x,y) \precsim s[d(x,u) + d(u,v) + d(v,y)]$ for all $x, y \in X$ and all distinct points $u, v \in X \backslash \{x, y\}$.

Then d is called a complex valued rectangular b-metric on X and (X,d) is called a complex valued rectangular b-metric space.

Example 3.2. Let $X = A \cup B$, where $A = \{\frac{1}{n} : n \in \mathbb{N}\}$ and $B = \mathbb{Z}^+$ and $d : X \times X \to \mathbb{C}$ be defined as follows:

$$d(x,y) = d(y,x)$$

for all $x, y \in X$ and

$$d(x,y) = \begin{cases} 0, & \text{if } x = y; \\ 2t, & \text{if } x, y \in A; \\ \frac{t}{2n}, & \text{if } x \in A \text{ and } y \in \{2,3\}; \\ t, & \text{otherwise,} \end{cases}$$

where $t > 0$ is a constant. Then (X,d) is a complex valued rectangular b-metric space with coefficient $s = 2 > 1$.

Definition 3.3. Let (X,d) be a complex valued rectangular b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$.

(a) The sequence $\{x_n\}$ is said to be complex valued convergent in (X,d) and converges to x if for every $\epsilon \succ 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) \prec \epsilon$ for all $n > n_0$ and is denoted by $x_n \to x$ as $n \to \infty$.

(b) The sequence $\{x_n\}$ is called complex valued Cauchy sequence in (X,d) if $\lim\limits_{n\to\infty} d(x_n, x_{n+p}) = 0$ for all $p \succ 0$.

(c) (X,d) is said to be a complex valued complete rectangular b-metric space if every complex valued Cauchy sequence in X converges to some $x \in X$.

Since the following two lemmas are the analogues of the lemmas in [5], we state these for complex valued rectangular b-metric spaces without their proofs.

Lemma 3.4. *Let (X,d) be a complex valued rectangular b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

Lemma 3.5. *Let (X,d) be a complex valued rectangular b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.*

We now prove the Banach contraction principle in complex valued rectangular b-metric spaces.

Theorem 3.6. *Let (X,d) be a complex valued complete rectangular b-metric space with coefficient $s > 1$ and $T : X \to X$ be a mapping satisfying:*

$$d(Tx, Ty) \precsim \alpha d(x,y) \tag{3.1}$$

for all $x, y \in X$, where $\alpha \in [0, \frac{1}{s}]$. Then T has a unique fixed point.

Proof. Let T satisfy (3.1), $x_0 \in X$ be an arbitrary point and define the sequence $\{x_n\}$ by $x_n = T^n x_0$. From (3.1), we get

$$d(x_n, x_{n+1}) \precsim \alpha d(x_{n-1}, x_n). \tag{3.2}$$

Using again (3.1), we have

$$d(x_{n-1}, x_n) \precsim \alpha d(x_{n-2}, x_{n-1}),$$

and by (3.2), we get

$$d(x_n, x_{n+1}) \precsim \alpha^2 d(x_{n-2}, x_{n-1}).$$

If we continue this process, we obtain

$$d(x_n, x_{n+1}) \precsim \alpha^n d(x_0, x_1). \tag{3.3}$$

Using $(CRb3)$ and (3.3) for all $n, m \in \mathbb{N}$ with $n < m$,

$$\begin{aligned}
d(x_n, x_m) &\precsim s[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_m)] \\
&\precsim s[d(x_n, x_{n+1})] + s^2[d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_m)] \\
&\precsim s[d(x_n, x_{n+1})] + s^2[d(x_{n+1}, x_{n+2})] + s^3[d(x_{n+2}, x_{n+3})] + \ldots + s^{m-n} d(x_{m-1}, x_m)] \\
&\precsim (s\alpha^n + s^2\alpha^{n+1} + s^3\alpha^{n+2} + \ldots + s^{m-n}\alpha^{m-1}) d(x_0, x_1) \\
&\precsim s\alpha^n[1 + s\alpha + (s\alpha)^2 + (s\alpha)^3 + \ldots + (s\alpha)^{m-n-1}] d(x_0, x_1) \\
&\precsim \frac{s\alpha^n}{1 - s\alpha} d(x_0, x_1).
\end{aligned}$$

Thus, we have

$$|d(x_n, x_m)| \leq \frac{s\alpha^n}{1 - s\alpha} |d(x_0, x_1)|.$$

Since $\alpha \in [0, \frac{1}{s})$ where $s > 1$, taking limits as $n \to \infty$, then

$$\frac{s\alpha^n}{1 - s\alpha} |d(x_0, x_1)| \to 0.$$

This means that

$$|d(x_n, x_m)| \to 0.$$

So $\{x_n\}$ is complex valued Cauchy sequence by Lemma 3.5. Completeness of (X, d) gives us that there is an element $u \in X$ such that $\{x_n\}$ is complex valued convergent to u.

We show that u is a fixed point of T, i.e., $Tu = u$. For any $n \in \mathbb{N}$, we get

$$\begin{aligned}
d(u, Tu) &\precsim s[d(u, x_n) + d(x_n, x_{n+1}) + d(x_{n+1}, Tu)] \\
&= s[d(u, x_n) + d(x_n, x_{n+1}) + d(Tx_n, Tu)] \\
&\precsim s[d(u, x_n) + d(x_n, x_{n+1}) + \alpha d(x_n, u)].
\end{aligned}$$

Since x_n converges to u as $n \to \infty$, it follows from the last inequality that $d(u, Tu) = 0$, i.e., $Tu = u$.

Finally, we prove the uniqueness. Let $w \neq u$ be another fixed point of T. Using (3.1),

$$d(z, w) = d(Tz, Tw) \precsim \alpha d(z, w).$$

and

$$|d(z, w)| \leq \alpha |d(z, w)|.$$

Since $\alpha \in [0, \frac{1}{s})$, we have $|d(z, w)| \leq 0$. Thus, $u = w$ and so u is a unique fixed point of T. □

Now we give a different contraction principle with a new condition.

Theorem 3.7. *Let (X,d) be a complete complex valued rectangular b-metric space and $T : X \to X$ be a continuous mapping such that for some function $\phi : X \to \mathbb{C}$, the following condition hold:*

$$d(x,T(x)) \precsim \frac{1}{s^m}(\phi(x) - \phi(T(x))) \tag{3.4}$$

for all $x \in X$ and all integers $m \geq 0$ where $s > 1$ is an integer. Then $\{T^n(x)\}$ converges to a fixed point of T for all $x \in X$.

Proof. For any fixed $x \in X$, let $x_n = T^n(x)$, $n \in \mathbb{N}$. From (3.4), we obtain

$$0 \precsim \frac{1}{s^m}(\phi(x) - \phi(T(x))) \quad \Leftrightarrow \quad \phi(x) \precsim \phi(T(x))$$

for all $x \in X$ and so,

$$\phi(x_{n+1}) = \phi(T^{n+1}(x)) = \phi(T(T^n(x))) = \phi(T(x_n)) \precsim \phi(x_n).$$

Since we conclude that $\{\phi(T^n(x))\}$ is monotonically decreasing and bounded below, we have $\lim_{n\to\infty} \phi(T^n(x)) = a \geq 0$. If $m, n \in \mathbb{N}$ and $m > n$, then by the axiom $(CRb3)$ and (3.4)

$$\begin{aligned}
d(x_n, x_m) &\precsim s[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_m)] \\
&= s[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})] + s[d(x_{n+2}, x_m)] \\
&\precsim s[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})] + s^2[d(x_{n+2}, x_{n+3}) + d(x_{n+3}, x_{n+4}) + d(x_{n+4}, x_m)] \\
&\vdots \\
&\precsim s[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})] + \ldots + s^{\frac{m-n-1}{2}}[d(x_{m-3}, x_{m-2}) + d(x_{m-2}, x_{m-1})] \\
&\quad + s^{\frac{m-n-1}{2}}[d(x_{m-1}, x_m)] \\
&\precsim \frac{s}{s^m}[\phi(x_n) - \phi(x_{n+1}) + \phi(x_{n+1}) - \phi(x_{n+2})] + \frac{s^2}{s^m}[\phi(x_{n+2}) - \phi(x_{n+3}) + \phi(x_{n+3}) - \phi(x_{n+4})] \\
&\quad + \frac{s^{\frac{m-n-1}{2}}}{s^m}[\phi(x_{m-3}) - \phi(x_{m-2}) + \phi(x_{m-2}) - \phi(x_{m-1})] + \frac{s^{\frac{m-n-1}{2}}}{s^m}[\phi(x_{m-1}) - \phi(x_m)] \\
&= \frac{s}{s^m}\phi(x_n) + [\frac{-s+s^2}{s^m}]\phi(x_{n+2}) + \ldots + [\frac{-s^{\frac{m-n-4}{2}} + s^{\frac{m-n-2}{2}}}{s^m}]\phi(x_{m-2}) + [\frac{s^{\frac{m-n-1}{2}}}{s^m}]\phi(x_m).
\end{aligned}$$

Using the fact that $\lim_{n\to\infty} \phi(x_n) = a$, we have

$$\begin{aligned}
&= \frac{s}{s^m}a + [\frac{-s+s^2}{s^m}]a + \ldots + [\frac{-s^{\frac{m-n-4}{2}} + s^{\frac{m-n-2}{2}}}{s^m}]a + [\frac{s^{\frac{m-n-1}{2}}}{s^m}]a \\
&= [\frac{s - s + s^2 + \ldots + -s^{\frac{m-n-4}{2}} + s^{\frac{m-n-2}{2}} + s^{\frac{m-n-1}{2}}}{s^m}]a \\
&= [\frac{s^{\frac{m-n-2}{2}} + s^{\frac{m-n-1}{2}}}{s^m}]a \\
&= [s^{\frac{-m-n-2}{2}} + s^{\frac{-m-n-1}{2}}]a.
\end{aligned}$$

Thus, we have

$$|d(x_n, x_m)| \leq [s^{\frac{-m-n-2}{2}} + s^{\frac{-m-n-1}{2}}]a.$$

Since $s > 1$, taking limits as $n \to \infty$, then

$$|d(x_n, x_m)| \to 0.$$

Thus $\lim_{m,n\to\infty} d(x_n, x_m) = 0$ and $\{T^n(x)\}$ is a Cauchy sequence in X by Lemma 3.5. Since X is complex valued complete rectangular b-metric, there exists a point $u \in X$ such that $\lim_{n\to\infty} T^n(x) = u$ and from continuity of T, $u = T(u)$. □

Theorem 3.8. *Let (X,d) be a complex valued complete rectangular b-metric space and $\psi:(0,\infty)\to(0,\infty)$ be monotone nondecreasing such that $\lim\limits_{n\to\infty}\psi^n(t)=0$ for all $t>0$. If $T:X\to X$ satisfies*

$$d(T(x),T(y))\precsim\psi(d(x,y)) \quad \text{for all} \quad x,y\in X, \tag{3.5}$$

then it has a unique fixed point u and $\lim\limits_{n\to\infty}d(T^n(x),u)=0$ for all $x\in X$.

Proof. For any $x\in X$, let $x_n=T^n(x)$ where $n\in\mathbb{N}$. If $x_1=T(x)=x$, then x would be a fixed point of T. So we assume $x_1=T(x)\neq x$. Since

$$\begin{aligned} d(x_n,x_{n+1}) &\precsim \psi(d(x_{n-1},x_n))\\ &\precsim \psi^2(d(x_{n-2},x_{n-1}))\\ &\vdots\\ &\precsim \psi^n(d(x,T(x)))=\psi^n(d(x,x_1)), \end{aligned}$$

we get

$$0\precsim\lim_{n\to\infty}d(x_n,x_{n+1})=\lim_{n\to\infty}d(x_n,x_{n+1})\precsim\lim_{n\to\infty}\psi^n(d(x,x_1))=0.$$

Therefore we have

$$\lim_{n\to\infty}d(x_n,x_{n+1})=0. \tag{3.6}$$

We must show that $\{x_n\}$ is a Cauchy sequence. We can consider $\psi(\epsilon)<\frac{\epsilon}{2s}$ for any $\epsilon>0$ and $s>0$ because $\psi^n(t)\to 0$ for all $t>0$. For any $\epsilon>0$, by (3.6), the following statement could be chosen as follows:

$$d(x_n,x_{n+1})\precsim\frac{\epsilon}{4s}$$

where $s>0$ is a real number.

Consider the set $B_\epsilon[x_n]=\{x\in X: d(x,x_{n+1})\precsim\epsilon\}$. If $z\in B_\epsilon[x_{n-1}]$, then $d(z,x_{n-1})\precsim\epsilon$ and

$$\begin{aligned} d(T(z),x_n) &\precsim s[d(T(z),T(x_{n-1}))+d(T(x_{n-1}),T(x_n))+d(T(x_n),x_n)]\\ &\precsim s[\psi(d(z,x_{n-1}))+d(x_n,x_{n+1})+d(x_{n+1},x_n)]\\ &= s[\psi(d(z,x_{n-1}))+2d(x_n,x_{n+1})]\\ &\precsim s[\psi(\epsilon)+2(\frac{\epsilon}{4s})]\\ &\precsim s[\frac{\epsilon}{2s}+\frac{\epsilon}{2s}]\\ &=\epsilon. \end{aligned}$$

As a result, $T(z)\in B_\epsilon[x_n]$ and $d(x_m,x_n)\precsim\epsilon$ for all $m\geq n$. So $\{x_n\}$ is a Cauchy sequence. By the completeness of X,

$$\lim_{n\to\infty}T^n(x)=u\in X$$

and so $\lim\limits_{n\to\infty}d(T^n(x),u)=0$ for all $x\in X$. Since T is continuous, we have $T(u)=u$.

To show the uniqueness, assume that $u,x\in X$ $(u\neq x)$ are fixed points of T. Let's apply (3.6) n times.

$$0\precsim d(x,u)=d(T(x),T(u))\precsim\psi(d(x,u))\precsim\ldots\precsim\psi^n(d(x,u)).$$

If we take limit as $n\to\infty$, since $\lim\limits_{n\to\infty}\psi^n(t)=0$ for all $t>0$, we get

$$d(x,u)=0 \quad\Leftrightarrow\quad x=u.$$

□

4. An application to Linear Equations

In this section we give an application using Theorem 3.6.

Theorem 4.1. *Let $X = \mathbb{C}^n$ be a complex valued rectangular b-metric space with the metric*

$$d_\infty(x,y) = \max_{1\le i\le n} |x_i - y_i|$$

where $x, y \in X$. If

$$\sum_{j=1}^{n} |\alpha_{ij}| \le \alpha < 1 \quad \textit{for all } i = 1,2,\ldots,n,$$

then the linear system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \tag{4.1}$$

of n linear equations in n unknowns has a unique solution.

Proof. Since every complex valued metric space is a complex valued rectangular metric space and every complex valued rectangular metric is a complex valued rectangular b-metric, it is easy to show that (X, d_∞) is complex valued complete rectangular b-metric space. So we need to prove that the mapping $T : X \to X$ given by

$$T(x) = Ax + b$$

where $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, $b = (b_1, b_2, \ldots, b_n) \in \mathbb{R}^n$ and

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$$

is a contraction. Since

$$\begin{aligned} d_\infty(Tx,Ty) &= \max_{1\le i\le n} |\sum_{j=1}^{n} \alpha_{ij}(x_j - y_j)| \\ &\precsim \max_{1\le i\le n} \sum_{j=1}^{n} |\alpha_{ij}||x_j - y_j| \\ &\precsim \max_{1\le i\le n} (\max_{1\le j\le n} |x_j - y_j|) \sum_{j=1}^{n} |\alpha_{ij}| \\ &= \max_{1\le i\le n} \sum_{j=1}^{n} |\alpha_{ij}| d_\infty(x,y) \\ &\precsim \alpha d_\infty(x,y), \end{aligned}$$

we conclude that T is a contraction mapping. By Theorem 3.6, the linear equation system (4.1) has a unique solution. □

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