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# A note on well-posedness of Nash-type games problems with set payoff

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## Abstract

In this paper, Nash-type games problems with set payoff (for short, NGPSP) are first introduced. Then, in terms of the measure of noncompactness, some well-posedness results for Nash-type games problems with set payoff are obtained in Banach spaces. 

*Keywords:* Well-posedness, Nash-type games problems, Hausdorff distance, set payoff.
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## 1. Introduction and Preliminaries

It is well known that well-posedness plays an important role in the stability theory for optimization problems. The concept of well-posedness was first introduced by Tykhonov in [14]. Since then, some extensions of this concept for vector optimization problems and minimization problems appeared. For details, we refer readers to [8, 9] and the references therein. Moreover, the concept of well-posedness has been generalized to nonconvex constrained variational problems, variational inequality problems, generalized variational inequality problems and equilibrium problems, see [2, 3, 4, 5, 6, 7, 10, 12, 13, 15, 16, 17] and the reference therein.

Game theory has played an important role in the many fields, such as economics, management and operations research. Games problems have three elements: players, strategies of players and payoffs

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functions. However, it is difficult to know the exact values of payoffs owing to some unexpected situations. We can only give an estimate of values of payoffs. Therefore, games problems with set payoff become an interesting research topic.

Throughout this paper, let $I$ be a finite index set and for each $i \in I$, let $X_i$ be a real Banach space, let $Y_i$ be a real Banach space with proper closed convex cone $C_i$, and $intC_i \neq \emptyset$. For each $i \in I$, $X = \prod_{i\in I} X_i$, $X^i = \prod_{j\in I, j\neq i} X_j$, $F_i : X^i \times X_i \to 2^{Y_i}$ and $S_i : X^i \to 2^{X_i}$. For each $x \in X$, suppose that $x_i$ and $x^i$ denote the $i^{th}$ coordinate of $x$ and the projection of $x$ on $X^i$, respectively. In this paper, we may write $x = (x_i, x^i)$. We introduce the following Nash-type games problems with set payoff:

the game problem is to find $\bar{x} = (\bar{x})_{i\in I} \in X, \bar{y}_i \in F_i(\bar{x}^i, \bar{x}_i)$ such that
for each $i \in I$,

$$\bar{x}_i \in S_i(\bar{x}^i),$$

and

$$y_i - \bar{y}_i \notin -\mathrm{int}C_i,$$

for all $y_i \in F_i(\bar{x}^i, u_i)$ and all $u_i \in S_i(\bar{x}^i)$.

*Remark* 1.1.

(i) If $I = \{1, 2\}$, the problem is to find $\bar{x} = (\bar{x}_1, \bar{x}_2)_{i\in I} \in X, \bar{y}_1 \in F_1(\bar{x}_2, \bar{x}_1), \bar{y}_2 \in F_2(\bar{x}_1, \bar{x}_2)$ such that for each $i \in I, \bar{x}_i \in S_i(\bar{x}^i)$, and
(NGPSP) $\quad y_i - \bar{y}_i \notin -\mathrm{int}C_i$, for all $y_i \in F_i(\bar{x}^i, u_i)$ and all $u_i \in S_i(\bar{x}^i)$,
where $\bar{x}^1 = \bar{x}_2,\ \bar{x}^2 = \bar{x}_2$.

(ii) For each $i \in I$, if $F_i$ is a real-valued function, the problem reduces to the Nash-type game problem with real-valued payoff functions in [13].

Motivated by the earlier work, we introduce the well-posedness for (NGPSP) and some well-posedness results for (NGPSP) are obtained by using the measure of noncompactness.

The rest of the paper is organized as follows. In Section 2, we present the concepts of well-posedness and approximate solution set. In Section 3, we establish some metric characterizations for well-posedness by using the measure of noncompactness.

In this paper, We assume that the set of solutions for (NGPSP) is always nonempty, denoted by $Q$. Let $e_i \in \mathrm{int}C_i, i \in I$. Now, we introduce the definitions of approximate solution sequences and well-posedness for (NGPSP).

**Definition 1.2.** A sequence $\{x_n = (x_{ni}, x_n^i)\}_{i\in I} \subseteq X$ is called an approximate solution sequence for (NGPSP) if there exists $\{\epsilon_n\} \subseteq R^1_+$ with $\epsilon_n \to 0$ such that

$$d(x_{ni}, S_i(x_n^i)) \leq \epsilon_n, \qquad \forall n \in N, \quad i \in I,$$

and

$$\exists \bar{y}_{ni} \in F_i(x_n^i, x_{ni}) \text{ such that } y_{ni} - \bar{y}_{ni} + \epsilon_n e_i \notin -\mathrm{int}C_i,\ \forall y_{ni} \in F_i(x_n^i, u_{ni})\ \forall u_{ni} \in S_i(x_n^i)\ \forall n \in N,\ i \in I.$$

**Definition 1.3.** (NGPSP) is said to be well-posed [respectively, well-posed in the generalized sense] if there exists a unique Nash game $\bar{x}$ and every approximate solution sequence strongly converge to $\bar{x}$[respectively, if $Q \neq \emptyset$ and every approximate solution sequence has a subsequence which strongly converges to some point of $Q$].

*Remark* 1.4. (NGPSP) is well-posed in the generalized sense implies that the solution set $Q$ is nonempty and compact.

**Definition 1.5** ([1])**.** Let $X$ and $Y$ be two normed spaces. A set-valued mapping $G$ from $X$ to $Y$ is called

(i) closed if for every $x \in X$, for every sequence $\{x_n\}$ converging to $x$, and for every sequence $\{y_n\}$ converging to a point $y$, such that $y_n \in G(x_n)$, $\forall n \in N$, one has $y \in G(x)$;

(ii) subcontinuous on $X$ if for every convergent sequence $\{x_n\}$ converging to $x$, every sequence $\{y_n\}$ with $y_n \in G(x_n)$, $\{y_n\}$ has a convergent subsequence;

(iii) upper semicontinuous (u.s.c.) if for every $x \in X$, if for any neighborhood $N(G(x))$ of $G(x)$, there exists a neighborhood $N(x)$ of $x$ such that

$$G(x) \subset N(G(x)), \quad \forall x \in N(x);$$

(iv) lower semicontinuous (l.s.c.) if for every $x \in X$, for every sequence $\{x_n\}$ converging to $x$, and for every $y \in G(x)$, there exists a sequence $\{y_n\}$ converging to $y$, such that $y_n \in G(x_n)$ $\forall n \in N$;

(v) continuous at $x_0 \in X$ if $G$ is both u.s.c. and l.s.c. at $x_0$.

## 2. Main results

In this section, we shall give characterizations for well-posedness of (NGPSP).

First, we present the Kuratowski measure of noncompactness for a nonempty subset $B$ of $X$ ([9]) defined by

$$\alpha(B) = \inf\{\varepsilon > 0 : B \subset \bigcup_{i=1}^{n} B_i, \text{for every } B_i, \text{diam} B_i < \epsilon\},$$

where $diam(B_i)$ be the diameter of $B_i$ defined by

$$diam(B_i) = \sup\{d(x_1, x_2) : x_1, x_2 \in B_i\}.$$

**Definition 2.1** ([11])**.** Let $(X, d)$ be a metric space. Given two nonempty subsets $A$ and $B$ of X, the Hausdorff distance between $A$ and $B$ is defined as

$$H(A, B) = \max\{e(A, B), e(B, A)\},$$

where $e(A, B) = \sup\{d(a, B) : a \in A\}$ and $d(a, B) = \inf_{b\in B} d(a, b)$.

In order to study the well-posedness for (NEP), we introduce the approximate solution set for (NGPSP) which is defined by

$Q_\epsilon = \{\bar{x} = (\bar{x}_i, \bar{x}^i) : d(\bar{x}_i, S_i(\bar{x}^i)) \le \epsilon, \exists \bar{y}_i \in F_i(\bar{x}^i, \bar{x}_i)$ and $y_i - \bar{y}_i + \epsilon e_i \notin -\text{int}C_i$, for all $y_i \in F_i(\bar{x}^i, u_i)$ and all $u_i \in S_i(\bar{x}^i)$, $\forall i \in I\}$, where $\epsilon > 0$.

For the sake of simplicity, we restrict ourselves to $I = \{1, 2\}$. But the result can be easily obtained for finite players.

**Theorem 2.2.** *If (NGPSP) is well-posed ,then*

$$Q_\epsilon \ne \emptyset, \ \forall \epsilon > 0 \quad \textit{and} \quad \lim_{\epsilon \to 0} diam Q_\epsilon = 0. \tag{2.1}$$

*Moreover, if the following conditions are satisfied:*

(i) $F_i : X^i \times X_i \to 2^{Y_i}$ *is continuous with compact values;*

(ii) $S_i : X^i \to 2^{X_i}$ *is closed, subcontinuous and lower semicontinuous,*

*then the converse holds.*

*Proof.* If (NGPSP) is well-posed, then the set $Q$ is a singleton. $Q_\epsilon$ is always nonempty for every $\epsilon$, since $Q \subseteq Q_\epsilon$. By contradiction, assume that $\lim_{\epsilon_n \to 0} diam Q_{\epsilon_n} > \beta > 0$, for some sequence $\{\epsilon_n\} \subseteq R_1^+$. We could find two sequences $\{(x_n)_{i\in I}\}$ and $\{(z_n)_{i\in I}\}$ satisfying $x_n \in Q_{\epsilon_n}, z_n \in Q_{\epsilon_n}$ and $\| x_n - z_n \| > \beta$, for $n$ large enough. Since $\{x_n\}, \{z_n\}$ are approximate solutions for (NGPSP) and should converge to the unique solution, we get a contradiction with the assumption.

Now, we assume that (2.1) holds and $\{x_n = (x_{n1}, x_{n2})\} \subseteq X$ is an approximate solution sequence for (NGPSP). Then there exists a sequence $\{t_n\} \subseteq R_1^+$ decreasing to 0, such that

$$d(x_{ni}, S_i(x_n^i)) \le t_n, \qquad \forall n \in N, \quad i = 1, 2 \tag{2.2}$$

and

$$\begin{aligned} &\exists \bar{y}_{ni} \in F_i(x_n^i, x_{ni}) \text{ s.t. } y_{ni} - \bar{y}_{ni} + t_n e_i \notin -\text{int}C_i, \\ &\forall y_{ni} \in F_i(x_n^i, u_{ni}) \textit{ and } \forall u_{ni} \in S_i(x_n^i) \forall n \in N, \ i = 1, 2. \end{aligned} \tag{2.3}$$

According to the assumption, we conclude that $\{x_n\}$ is a Cauchy sequence and has to converge to some point $\tilde{x} = (\tilde{x}_1, \tilde{x}_2) \in X$. Next, we only need to show that $\tilde{x} \in Q$.

From (2.2), we get that $d(x_{n1}, S_1(x_{n2})) \le t_n$. Let $\eta_{n1} \in S_1(x_{n2})$ such that $\| \eta_{n1} - x_{n1} \| \le t_n + \frac{1}{n}$, for each $n \in N$. Since $S_1$ is $(\tau, \sigma)$-closed and $(\tau, \sigma)$- subcontinuous, the sequence $\{\eta_{n1}\}$ has a subsequence, still denoted by $\{\eta_{n1}\}$, which converges to a point $\eta_1 \in S_1(\tilde{x}_2)$. It follows that

$$\begin{aligned} \| (x_{n_1} - \tilde{x}_1) - (\eta_{n_1} - \eta_1) \| &= \| (x_{n_1} - \eta_{n_1}) - (\tilde{x}_1 - \eta_1) \| \\ &= \| (\tilde{x}_1 - \eta_1) - (x_{n_1} - \eta_{n_1}) \| \\ &\ge \| \tilde{x}_1 - \eta_1 \| - \| x_{n_1} - \eta_{n_1} \| . \end{aligned}$$

We have $\| \tilde{x}_1 - \eta_1 \| \le \| x_{n_1} - \eta_{n_1} \| + \| (x_{n_1} - \tilde{x}_1) - (\eta_{n_1} - \eta_1) \|$.

Therefore, $\| \tilde{x}_1 - \eta_1 \| \le \| x_{n_1} - \eta_{n_1} \| + \| ((x_{n_1} - \tilde{x}_1) - (\eta_{n_1} - \eta_1) \| \le t_n + \frac{1}{n} = 0$. Thus, $\tilde{x}_1 \in S_1(\tilde{x}_2)$. Similarly, we can get that $\tilde{x}_2 \in S_2(\tilde{x}_1)$.

For each $i \in I$, in the light of (2.3) and assumption (i), the sequence $\{\bar{y}_{ni}\}$ have subsequences which converge to $\bar{y}_i \in F_i(\tilde{x}^i, \tilde{x}_i)$. For all $u_i \in S_i(\tilde{x}^i)$, since $S_i$ is $(\tau, \sigma)$-lower semicontinuous, there exists a sequence $\{u_{ni}\}$ converging to $u_i$ such that $u_{ni} \in S_i(x_n^i)$ $\forall n \in N$. And for all $y_i \in F_i(\tilde{x}^i, u_i)$, by virtue of the continuity and compactness of $F_i$, there exists $y_{ni}$ satisfying $y_{ni} \to y_i$, $y_{ni} \in F_i(x_n^i, u_{ni})$ $\forall n \in N$. Moreover, $y_{ni} - \bar{y}_{ni} + t_n e_i \in Y_i \backslash (-\text{int}C_i)$, we have that $y_i - \bar{y}_i \in Y_i \backslash (-\text{int}C_i)$ by the closedness of $Y_i \setminus (-\text{int}C_i)$, for all $y_i \in F_i(\tilde{x}^i, u_i)$ and all $u_i \in S_i(\tilde{x}^i)$. The uniqueness follows immediately from (2.1). Thus, we complete the proof. □

Now we obtain an characterization for generalized well-posedness for (NGPSP) by using the Kuratowski measure.

**Theorem 2.3.** *If (NGPSP) is well-posed in the generalized sense, then*

$$Q_\epsilon \neq \emptyset, \ \forall \epsilon > 0 \quad \textit{and} \quad \lim_{\epsilon \to 0} \alpha(Q_\epsilon) = 0. \tag{2.4}$$

*Moreover, assume the following conditions hold:*

(i) $F_i : X^i \times X_i \to 2^{Y_i}$ *is continuous with compact values;*

(ii) $S_i : X^i \to 2^{X_i}$ *is closed, subcontinuous and lower semicontinuous on* $X^i$*,*

*then the converse holds.*

*Proof.* Let (NGPSP) be well-posed in the generalized sense, then $Q$ is nonempty and compact. Obviously, $Q_\epsilon \neq \emptyset$ for any $\epsilon > 0$, since $Q \subseteq Q_\epsilon$. If $\{x_n\}_{i\in I}$ is a sequence of Nash equilibria, it is also approximate solution sequence. By virtue of assumption, $\{x_n\}$ contains a subsequence converging to a Nash equilibrium. In the following, we show that $\lim_n \alpha(Q_{\epsilon_n}) = 0$ for every $\epsilon_n$ decreasing to 0. Observe that

$$H(Q_{\epsilon_n}, Q) = \max\{e(Q_{\epsilon_n}, Q), e(Q, Q_{\epsilon_n})\} = e(Q_{\epsilon_n}, Q).$$

Hence,

$$\alpha(Q_{\epsilon_n}) \leq 2H(Q_{\epsilon_n}, Q) + \alpha(Q) = 2e(Q_{\epsilon_n}, Q) = 2 \sup_{x\in Q_{\epsilon_n}} d(x, Q),$$

where $\alpha(Q) = 0$, since $Q$ is compact. To get (2.4), it is sufficient to show that

$$\lim_n \sup_{x\in Q_{\epsilon_n}} d(x, Q) \leq 0.$$

Suppose that $\lim_n \sup_{x\in Q_{\epsilon_n}} d(x, Q) > \gamma > 0$. Then, there exits a positive integer $k$ and $x_n \in Q_{\epsilon_n}$ satisfying $d(x_n, Q) > \gamma$ for arbitrary $n \geq k$. So, the sequence $\{x_n\}$ has no subsequence converging to a point of $Q$, which contradicts the assumption.

Conversely, we assume that (2.4) holds. We follow the proof of Theorem 2.2 that $Q_\epsilon$ is closed. By the Kuratowski theorem, we have

$$H(Q_\epsilon, Q) \to 0 \quad \text{as } \epsilon \to 0, \tag{2.5}$$

where $Q = \bigcap_{\epsilon>0} Q_\epsilon$ is nonempty and compact.

Let $\{x_n\}$ be any approximate solution sequence for (NGPSP). Then there exists $\epsilon_n \geq 0$ with $\epsilon_n \to 0$ such that $\{x_n\} \subseteq Q_{\epsilon_n}$. Due to (2.5), $d(x_n, Q) \to 0$. Then, there exits a sequence $\{p_n\}_{i\in I} \in Q$ such that $d(x_n, p_n) = \| x_n - p_n \| \to 0$. As $Q$ is compact, the sequence $\{p_n\}$ has a subsequence $\{p_{n_k}\}$ converging to $\bar{p} \in Q$. We could easily obtain that $\{x_n\}$ has a subsequence converging to $\bar{p}$. Therefore, (NGPSP) is well-posed in the generalized sense. □

If for each $i \in I$, $F_i$ is single-valued function, denoted by $f_i$, $Y_i = R$ and $S_i(X^i) = X_i$, the Nash equilibrium problem is to find $\bar{x} = (\bar{x})_{i\in I} \in X$, such that for each $i \in I$, $\bar{x}_i \in X_i$ and $f_i(\bar{x}^i, \bar{x}_i) \leq f_i(\bar{x}^i, z_i)$, for all $z_i \in X_i$.

*Remark* 2.4. In [13], the following Nash games problems with real-valued payoff is studied. A point $\bar{x} = (\bar{x}_1, \bar{x}_2) \in X_1 \times X_2$ is a Nash equilibrium if: $f_1(\bar{x}_1, \bar{x}_2) \leq f_1(z_1, \bar{x}_2), \forall z_1 \in X_1$; and $f_2(\bar{x}_1, \bar{x}_2) \leq f_1(\bar{x}_1, z_2), \forall z_2 \in X_2$, where $f_i : X_1 \times X_2 \to R \cup \{+\infty\}$.

For the above Nash-type games problems with real-valued payoff, we introduce the following similar definitions of approximate solutions sequence and approximate solution set. Let $\hat{Q}$ denotes the solution set of the above problem.

**Definition 2.5.** A sequence $\{x_n\}_{i\in I} \subseteq X$ is called an approximate solution sequence for (NGPSP) if there exists $\{\epsilon_n\} \subseteq R^1_+$ with $\epsilon_n \to 0$ such that

$$d(x_{ni}, S_i(x^i_n)) \leq \epsilon_n, \ \forall n \in N, \ i \in I,$$

and

$$f_i(x_n^i, x_{ni}) \leq f_i(x_n^i, z_i) + \epsilon_n \quad \text{for all } z_i \in X_i,\ \forall n \in N,\ i \in I,$$

the approximate solution set

$$\hat{Q}_\epsilon = \{\bar{x} = (\bar{x}_i, \bar{x}^i) : d(\bar{x}_i, S_i(\bar{x}^i)) \leq \epsilon,\ f_i(\bar{x}^i, \bar{x}_i) \leq f_i(\bar{x}^i, z_i) + \epsilon,\ \text{for all } z_i \in X_i, \forall i \in I\}.$$

By using the above definitions, we obtain the following well-posed results for Nash-type games problems with real-valued payoff.

**Theorem 2.6.** *If $X_i (i \in I)$ is compact and the following conditions are satisfied:*

(i) *$f_i : X^i \times X_i \to R$ is lower semicontinuous on $X$;*

(ii) *For every $z_i \in X_i$, the function $f_i(., z_i)$ is upper semicontinuous on $X_i$ $(i \in I)$;*

(iii) *$S_i : X^i \to 2^{X_i}$ is $(\tau, \sigma)$-closed and $(\tau, \sigma)$-subcontinuous,*

*then (NGPSP) is well-posed in the generalized sense.*

*Proof.* Let $\{x_n = (x_{n1}, x_{n2})\} \subseteq X$ be an approximate solution sequence for (NGPSP). Then, there exists $\{\epsilon_n\} \subseteq R_+^1$ with $\epsilon_n \to 0$ such that

$$d(x_{ni}, S_i(x_n^i)) \leq \epsilon_n, \qquad \forall n \in N, \quad i \in I,$$

and

$$f_i(x_n^i, x_{ni}) \leq f_i(x_n^i, z_i) + \epsilon_n \quad \text{for all } \ z_i \in X_i, \qquad \forall n \in N, \quad i \in I. \tag{2.6}$$

By virtue of the compactness of $X_i$, $\{x_n\}$ has a subsequence converging to $\check{x} = (\check{x}_1, \check{x}_2)$. We only need to prove that $\check{x} \in \hat{Q}$. Using the proof similar to that of Theorem 2.2 , we get that $\check{x}_i \in S_i(\check{x}^i)$. By the assumption (i), (ii) and (2.6), we get that

$$\begin{aligned} f_1(\check{x}_2, \check{x}_1) &\leq \liminf_n f_1(x_{n2}, x_{n1}) \leq \liminf_n (f_1(x_{n2}, z_1) + \epsilon_n) \\ &\leq \limsup_n (f_1(x_{n2}, z_1) + \epsilon_n) \leq f_1(\check{x}_2, z_1), \forall z_1 \in X_1. \end{aligned}$$

By the same way, $f_2(\check{x}_1, \check{x}_2) \leq f_2(\check{x}_1, z_2), \forall z_1 \in X_2$. So, we complete the proof. □

*Remark* 2.7. In [13], under some closedness, convexity and lower semicontinuous of real-valued payoff functions assumptions, some well-posed results with parameter $\alpha$ are also obtained for the above Nash-type games problems with real-valued payoff. However, the above theorem doesn't need the convexity assumptions of real-valued payoff functions. Hence, our results are different from ones in the literatures.

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