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Chaos analysis of the nonlinear duffing oscillators based on the new Adomian polynomials

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Abstract

Numerical recurrence formulae are given to investigate the chaotic motion of the famous Duffing system. The new Adomian polynomial is adopted to treat the cubic nonlinear term. With the numerical simulation of the phase portraits and the Poincare sections, the chaotic behaviors are discussed for varied frequencies, damping coefficients and forces. The results show that the numerical method is reliable to investigate chaotic systems.

Keywords: New Adomian polynomial, duffing systems, chaos.
2010 MSC: 47H10, 54H25.

1. Introduction

The damping oscillator procedure is described by the following famous nonlinear differential equation of second order [10, 11, 14]

$$\frac{d^2x}{dt^2} + \mu\frac{dx}{dt} - x + x^3 = F\cos(wt),$$

subjected to the initial condition $x(t_0) = 0$ and $\dfrac{dx(t_0)}{dt} = 0$. For the solution, the main difficulty is to treat the cubic term x^3. Many methods have been developed to depict chaos and chaos synchronization [8, 12, 17, 18]. The numerical method, such as the predictor–corrector method, the Runge–Kutta method (RKM) are the most often used ones.

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The famous Adomian decomposition method (ADM) was developed by Adomian in the last century [1]. Then, many modified versions have been proposed [1, 2, 9, 13, 15, 16, 19]. Very recently, Duan proposed a new way to calculate Adomian polynomial and greatly improved the efficiencies for solutions [3, 4, 5, 6, 7]. The idea has been extended to solve the initial value problem of the fractional differential equations, two point value problem of the differential ones. The efficiency and the accuracy are compared in [6]. The ADM shows a perspective over the RKM. In view of this point, we adopt the new ADM so that we can better describe the nonlinear dynamics of the oscillator.

In this paper, we use the idea of the ADM to depict the chaotic motion of the Duffing system and also illustrate a general way to solve the initial value problem of differential equations of high order.

2. Basics

According to the theories of the ODEs, for a general differential equations of n-th order

$$\frac{d^n x}{dt^n} + F(t,\ x,\ \frac{dx}{dt},\ \dots,\ \frac{d^{n-1}x}{dt^{n-1}}) = 0,$$

we can have a differential system of first order so that the Duffing system [10] reads

$$\begin{cases} \dfrac{dx}{dt} = y,\ x(t_0) = 0.2, \\ \dfrac{dy}{dt} + \mu y - x + x^3 = F\cos(wt),\ y(t_0) = 0, \end{cases} \tag{2.1}$$

where μ is the damping coefficient, w is the frequency, x is the displacement and y is the velocity.

Much more generally, consider the differential equation of second order

$$\frac{d^2 x}{dt^2} + F(t,\ x,\ \frac{dx}{dt}) = 0,\ x(t_0) = a,\ \frac{dx(t_0)}{dt} = b.$$

We give the algorithm as the following.

Step 1. Assume the transform $\frac{dx}{dt} = y$. One can obtain the following system

$$\begin{cases} \dfrac{dx}{dt} = y, x(t_0) = a, \\ \dfrac{dy}{dt} + F(t,x,y) = 0,\ y(t_0) = b. \end{cases} \tag{2.2}$$

Step 2. Expand $x(t)$ and $y(t)$ in the form of the Taylor series

$$\begin{cases} x = \sum\limits_{i=0}^{\infty} c_{i,1}(t-t_0)^i, \\ y = \sum\limits_{i=0}^{\infty} c_{i,2}(t-t_0)^i. \end{cases}$$

Accordingly, the x_n and the y_n read

$$\begin{cases} x_n = \sum\limits_{i=0}^{n} c_{i,1}(t-t_0)^i, \\ y_n = \sum\limits_{i=0}^{n} c_{i,2}(t-t_0)^i. \end{cases}$$

Step 3. Following the idea in [6], one can derive a recurrence formula

$$\begin{cases} c_{j+1,1} = \dfrac{1}{j+1} c_{j,2}, c_{0,1} = a, \\ c_{j+1,2} = \dfrac{-1}{j+1} F(t,\ c_{j,1},\ c_{j,2},\ A_j),\ c_{0,2} = b,\ 0 \le j, \end{cases}$$

which gives the solution Eq. (2.2). Here A_j is calculated by Duan [3, 4, 5, 6, 7]

$$A_j = \frac{1}{j}\sum_{i=0}^{j-1}(i+1)c_{i+1,1}\frac{dA_{j-1-i}}{dc_{0,1}}.$$

Step 4. Assume $x_n = \varphi_1(t, t_0, c_{0,1}, c_{0,2})$ and $y_n = \varphi_2(t, t_0, c_{0,1}, c_{0,2})$ which are the n-th order approximation. For $t = t_0$, we can have $x_n(t_0) = c_{0,1},\ y_n(t_0) = c_{0,2}$. Once more, we choose t_1 near t_0 and $x_n(t_1),\ y_n(t_1)$ as the initial values. As a result, we can successively obtain all the numerical solutions $x_n(t_i)$ and $y_n(t_i)$. With the given length size and the node number to h and N, $t_n = t_0 + nh$, one can obtain the values

$$x_n^* = \varphi_1(t_n, t_{n-1}, x_{n-1}^*), y_n^* = \varphi_1(t_n, t_{n-1}, y_{n-1}^*),$$

where the initial iteration reads

$$x_0^* = x(t_0),\ y_0^* = y(t_0).$$

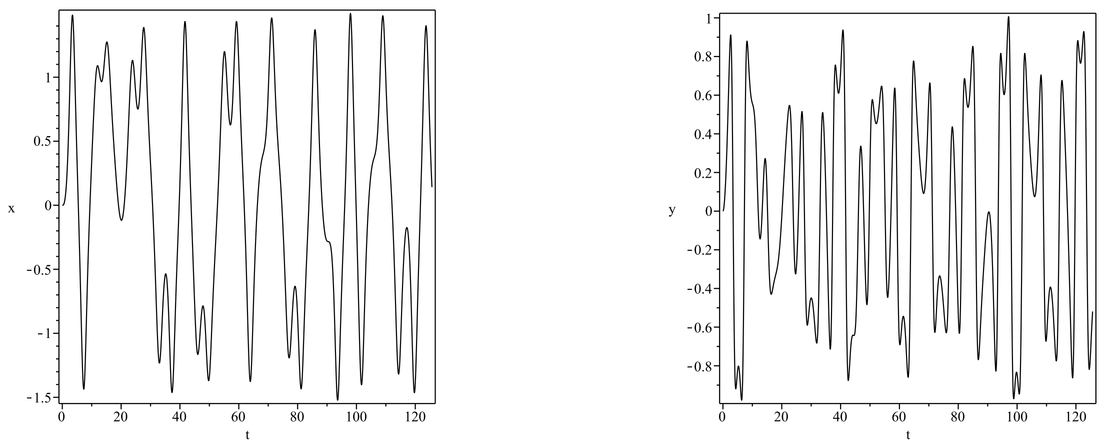
3. Chaos Analysis

For Eq. (2.1), we can give the following numerical recurrence formula as

$$\begin{cases} c_{j+1,1} = \dfrac{1}{j+1}c_{j,2},\ c_{0,1} = 0.2, \\ c_{j+1,2} = \dfrac{-1}{j+1}(\mu c_{j,2} - c_{j,1} + A_j - f_j),\ c_{0,2} = 0,\ 0 \le j, \end{cases}$$

where f_i is the coefficient and $F\cos(wt) = \sum\limits_{j=0}^{n} f_j(t-t_0)^j$.

Now we can discuss the chaotic motion with respect to t in Fig. 1a and Fig. 1b. Let $j = 10$, $n = 2000$, $h = \dfrac{2\pi}{100w}$. The Poincare section and the phase trajectory are plotted in Fig. 2a and Fig. 2b from which one can determine the system's chaotic state. Due to the sensitivity of the chaos, we can see minor changes in the $x(t_0)$ and the $y(t_0)$ lead to great variations in the motion.

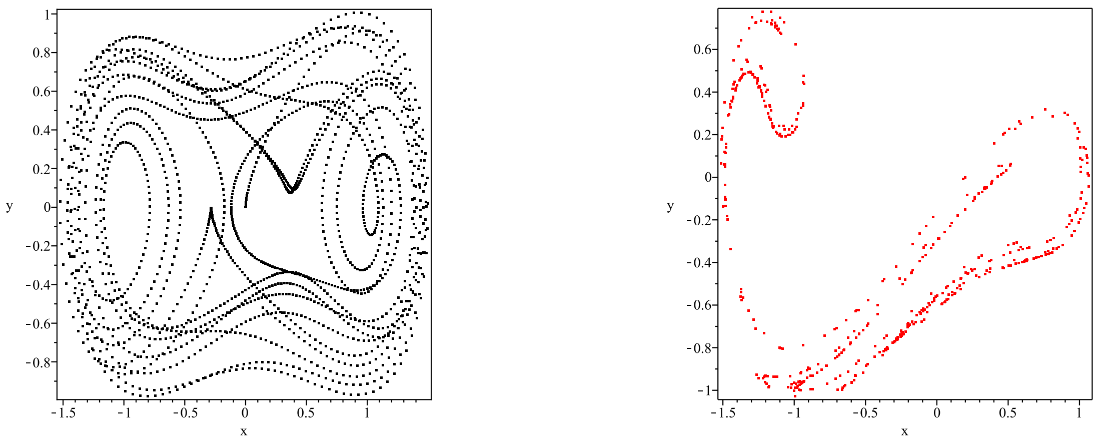


(a) Chaotic motion versus time t

(b) Chaotic velocity versus time t

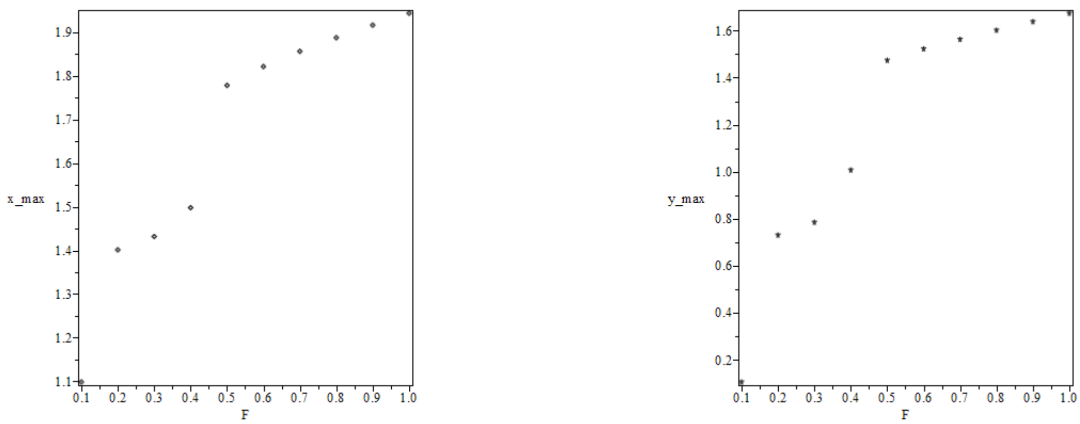
Figure 1

Based on the numerical formulae, we also can consider the maximal x and y versus the variations of the exciting force after $t = 50$ seconds. We can observe that in Figs. 3a and 3b, for F between 0.4 and 0.5, the maximal x and y vary sharply.



(a) Poincare section (b) Phase trajectory

Figure 2



(a) Maximal displacement versus force (b) Maximal velocity versus force

Figure 3

Here we only discuss the chaos behaviors of the Duffing system. For the error analysis, readers are referred to [6].

4. Concluding remarks

This paper applies the new Adomian polynomial for the famous Duffing equation. The second order equation is equivalently given as differential equations. Numerical recurrence formulae are given. Furthermore, the chaos including the phase trajectory and the Poincare sections are shown. The relationship between the maximal displacement, the maximal velocity and variations of the force are discussed. We can conclude that the new Adomian polynomial can be more easily used in the chaotic system and depict the nonlinear dynamics more accurately than the classical one [1].

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