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Fixed point theorems for generalized F-contractions in b-metric-like spaces

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Abstract

In this paper, we introduce some new F-contractions in b-metric-like spaces and investigate some fixed point theorems for such F-contractions. Presented theorems generalize related results in the literature. An example is also given to support our main result.

Keywords: Fixed point, F-contraction, b-metric-like spaces.
2010 MSC: 47H10, 54H25.

1. Introduction and Preliminaries

In 2012, Wardowski [21] introduced the notion of F-contraction and proved a new fixed point theorem about F-contraction. Wardowski defined the F-contraction as follows.

Definition 1.1. Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be an F-contraction if there exists $\tau > 0$ such that for all $x, y \in X$,

$$d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y)), \tag{1.1}$$

where $F : (0, +\infty) \to (-\infty, +\infty)$ is a mapping satisfying the following conditions:

(F1) F is strictly increasing, that is for all $\alpha, \beta \in (0, +\infty)$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$,

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(F2) for any sequence $\{\alpha_n\}$ of positive real numbers, the following holds:

$$\lim_{n\to+\infty} \alpha_n = 0 \text{ if and only if } \lim_{n\to+\infty} F(\alpha_n) = -\infty,$$

(F3) there exists $k \in (0,1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

After that, F-contraction was generalized and many fixed point theorems concerning F-contraction were investigated [3, 5, 6, 11, 16, 19].

On the other hand, the concept of metric spaces has been generalized by many authors, such as partial metric spaces [18], b-metric spaces [12], metric-like spaces [7], partial b-metric spaces [20], quasi-partial metric spaces [15] and b-dislocated metric spaces [13] were introduced and many results in these spaces were obtained [1, 2, 8, 10, 14, 16, 17]. Recently, the notion of b-metric-like spaces were introduced by Alghamdi [4] and some fixed point theorems were studied in such spaces [4, 9].

The aim of this paper is to introduce some new generalized type of F-contractions and prove some fixed point theorems about such new F-contractions in b-metric-like spaces. Our results generalize and improve related results in the literature. An example is presented to support our main result. Throughout this paper, the letters $\mathbb{N}$, $\mathbb{N}^+$, $\mathbb{R}$, $\mathbb{R}_0^+$ and $\mathbb{R}^+$ will denote the set of all nonnegative integer numbers, the set of all positive integer numbers, the set of all real numbers, the set of all nonnegative real numbers and the set of all positive real numbers, respectively.

Let us recall some definitions and facts about partial metric spaces and b-metric-like spaces.

Definition 1.2 ([18])**.** A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}_0^+$ such that for all $x, y, z \in X$:
(P_1) $x = y \Leftrightarrow p(x, y) = p(x, x) = p(y, y)$,
(P_2) $p(x, x) \le p(x, y)$,
(P_3) $p(x, y) = p(y, x)$,
(P_4) $p(x, y) \le p(x, z) + p(z, y) - p(z, z)$.

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X. It is clear that, if $p(x, y) = 0$, then from (P_1) and (P_2), $x = y$. But if $x = y$, $p(x, y)$ may not be 0.

In a partial metric space, the concepts of convergence, completeness and continuity are defined as follows.

Definition 1.3 ([18])**.** Let (X, p) be a partial metric space. Then:

(*i*) A sequence $\{x_n\}$ in a partial metric space (X, p) converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n\to+\infty} p(x, x_n)$.

(*ii*) A sequence $\{x_n\}$ in a partial metric space (X, p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to+\infty} p(x_m, x_n)$.

Definition 1.4 ([4])**.** A b-metric-like on a nonempty set X is a function $\sigma : X \times X \to \mathbb{R}_0^+$ such that for all $x, y, z \in X$ and a constant $s \ge 1$, the following three conditions hold true:

(σ_1) if $\sigma(x, y) = 0$ then $x = y$;

(σ_2) $\sigma(x, y) = \sigma(y, x)$;

(σ_3) $\sigma(x, z) \le s(\sigma(x, y) + \sigma(y, z))$.

The pair (x, σ) is then called a b-metric-like space.

Example 1.5 ([9])**.** Let $X = \{0, 1, 2\}$ and let

$$\sigma(x, y) = \begin{cases} 2, & x = y = 0, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

Then (X, σ) is a b-metric-like space with the constant $s = 2$.

In [4], some concepts in b-metric-like spaces were introduced.

Each b-metric-like σ on X generalizes a topology τ_σ on X whose base is the family of open σ-balls $B_\sigma(x,\varepsilon) = \{y \in X : |\sigma(x,y) - \sigma(x,x)| < \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

A sequence $\{x_n\}$ in the b-metric-like space (X,σ) converges to a point $x \in X$ if and only if $\sigma(x,x) = \lim_{n\to+\infty} \sigma(x,x_n)$.

A sequence $\{x_n\}$ in the b-metric-like space (X,σ) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to+\infty} \sigma(x_m,x_n)$.

A b-metric-like space is called complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_σ to a point $x \in X$ such that $\lim_{n\to+\infty} \sigma(x,x_n) = \sigma(x,x) = \lim_{n,m\to+\infty} \sigma(x_m,x_n)$.

Definition 1.6. Suppose that (X,σ) is a b-metric-like space. A mapping $T : X \to X$ is said to be continuous at $x \in X$, if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $T(B_\sigma(x,\delta)) \subseteq B_\sigma(Tx,\varepsilon))$. We say that T is continuous on X if T is continuous at all $x \in X$.

Remark 1.7 ([9]). Let (X,σ) be a b-metric-like space and let $f : X \to X$ be a continuous mapping. Then

$$\lim_{n\to+\infty} \sigma(x_n,x) = \sigma(x,x) \Rightarrow \lim_{n\to+\infty} \sigma(fx_n,fx) = \sigma(fx,fx).$$

2. Main results

In this section, we will introduce some generalized F-contractions and investigate some fixed point theorems for such generalized F-contractions. We begin with the following definitions.

Definition 2.1. Let $\mathbb{F}$ be the family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ such that
(F1) F is strictly increasing, that is, for all $\alpha,\beta \in \mathbb{R}^+$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$,
(F2) for any sequence $\{\alpha_n\}$ of positive real numbers, the following holds:

$$\lim_{n\to+\infty} \alpha_n = 0 \text{ if and only if } \lim_{n\to+\infty} F(\alpha_n) = -\infty.$$

Definition 2.2. Let (X,σ) be a b-metric-like space. A self-mapping $T : X \longrightarrow X$ is said to be a generalized F-contraction of type (I) if there exist $\tau > 0$ and $F \in \mathbb{F}$ such that

$$\begin{aligned} &\frac{1}{2s}\sigma(x,Tx) < \sigma(x,y) \Rightarrow \\ &\quad \tau + F(\sigma(Tx,Ty)) \leq \alpha F(\sigma(x,y)) + \beta F(\sigma(x,Tx)) + \gamma F(\sigma(y,Ty)) + tF(\frac{\sigma(x,Ty)}{2s}) \\ &\qquad + hF(\frac{\sigma(y,Tx)}{2s}) \end{aligned} \tag{2.1}$$

for all $x,y \in X$ with $\sigma(Tx,Ty) > 0$, where $\alpha,\beta,\gamma,h,t \in [0,1]$ such that $\alpha+\beta+\gamma+h+t = 1$ and $1-t-\gamma > 0$.

Theorem 2.3. *Let (X,σ) be a complete b-metric-like space and T a generalized F-contraction of type (I). Then, T has a fixed point $v \in X$; that is, $Tv = v$.*

Proof. Let x_0 be an arbitrary point in X. Set $Tx_0 = x_1$ and $Tx_1 = x_2$. Continuing this process, we can construct sequence $\{x_n\}$ in X such that

$$x_{n+1} = Tx_n, \quad n \in \mathbb{N}. \tag{2.2}$$

If there exists $n_0 \in \mathbb{N}$ such that $\sigma(x_{n_0}, x_{n_0+1}) = 0$, then x_{n_0} is the fixed point of T which completes the proof. Consequently, we suppose $\sigma(x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N}$. Hence, we have

$$\frac{1}{2s}\sigma(x_n,Tx_n) < \sigma(x_n,Tx_n) \ \ \forall n \in \mathbb{N}. \tag{2.3}$$

By (2.1), we get

$$\begin{aligned}\tau + F(\sigma(Tx_n, T^2x_n)) \leq & \alpha F(\sigma(x_n, Tx_n)) + \beta F(\sigma(x_n, Tx_n)) + \gamma F(\sigma(Tx_n, T^2x_n)) \\ & + tF(\frac{\sigma(x_n, T^2x_n)}{2s}) + hF(\frac{\sigma(Tx_n, Tx_n)}{2s}) \ \forall n \in \mathbb{N}.\end{aligned} \tag{2.4}$$

Now, we prove the following inequality:

$$\sigma(x_{n+1}, Tx_{n+1}) < \sigma(x_n, Tx_n) \ \forall n \in \mathbb{N}. \tag{2.5}$$

Suppose, on the contrary, that there exists $n_0 \in \mathbb{N}$ such that $\sigma(x_{n_0+1}, Tx_{n_0+1}) \geq \sigma(x_{n_0}, Tx_{n_0})$, due to (2.4), we have

$$\begin{aligned}\tau + F(\sigma(Tx_{n_0}, T^2x_{n_0})) \leq & \alpha F(\sigma(x_{n_0}, Tx_{n_0})) + \beta F(\sigma(x_{n_0}, Tx_{n_0})) + \gamma F(\sigma(Tx_{n_0}, T^2x_{n_0})) \\ & + tF(\frac{\sigma(x_{n_0}, T^2x_{n_0})}{2s}) + hF(\frac{\sigma(Tx_{n_0}, Tx_{n_0})}{2s}) \\ \leq & \alpha F(\sigma(x_{n_0}, Tx_{n_0})) + \beta F(\sigma(x_{n_0}, Tx_{n_0})) + \gamma F(\sigma(Tx_{n_0}, T^2x_{n_0})) \\ & + tF(\frac{s\sigma(x_{n_0}, Tx_{n_0}) + s\sigma(Tx_{n_0}, T^2x_{n_0})}{2s}) + hF(\frac{2s\sigma(Tx_{n_0}, x_{n_0})}{2s}) \\ \leq & \alpha F(\sigma(x_{n_0}, Tx_{n_0})) + \beta F(\sigma(x_{n_0}, Tx_{n_0})) + \gamma F(\sigma(Tx_{n_0}, T^2x_{n_0})) \\ & + tF(\sigma(Tx_{n_0}, T^2x_{n_0})) + hF(\sigma(Tx_{n_0}, x_{n_0})),\end{aligned}$$

which yields

$$\tau + (1 - \gamma - t)F(\sigma(Tx_{n_0}, T^2x_{n_0})) \leq (\alpha + \beta + h)F(\sigma(Tx_{n_0}, x_{n_0})),$$

that is,

$$F(\sigma(Tx_{n_0}, T^2x_{n_0})) \leq F(\sigma(Tx_{n_0}, x_{n_0})) - \frac{\tau}{1 - \gamma - t},$$

which together with (F1) implies $\sigma(Tx_{n_0}, T^2x_{n_0}) < \sigma(Tx_{n_0}, x_{n_0})$, that is, $\sigma(x_{n_0+1}, Tx_{n_0+1}) < \sigma(Tx_{n_0}, x_{n_0})$. It is a contradiction with $\sigma(x_{n_0+1}, Tx_{n_0+1}) \geq \sigma(x_{n_0}, Tx_{n_0})$, so (2.5) holds. Therefore, $\{\sigma(x_n, Tx_n)\}$ is a decreasing sequence of real numbers which is bounded from below. Suppose that there exists $A \geq 0$ such that

$$\lim_{n \to +\infty} \sigma(x_n, Tx_n) = A = inf\{\sigma(x_n, Tx_n) : n \in \mathbb{N}\}. \tag{2.6}$$

Now, we show $A = 0$. Suppose, on the contrary, that $A > 0$. For every $\varepsilon > 0$, there exists $m \in \mathbb{N}$ such that

$$\sigma(x_m, Tx_m) < A + \varepsilon. \tag{2.7}$$

By F(1), we have

$$F(\sigma(x_m, Tx_m)) < F(A + \varepsilon). \tag{2.8}$$

From (2.3), we have

$$\frac{1}{2s}\sigma(x_m, Tx_m) < \sigma(x_m, Tx_m). \tag{2.9}$$

Since T is a generalized F-contraction of type (I), we get

$$\begin{aligned}\tau + F(\sigma(Tx_m, T^2x_m)) \leq & \alpha F(\sigma(x_m, Tx_m)) + \beta F(\sigma(x_m, Tx_m)) + \gamma F(\sigma(Tx_m, T^2x_m)) \\ & + tF(\frac{\sigma(x_m, T^2x_m)}{2s}) + hF(\frac{\sigma(Tx_m, Tx_m)}{2s})\end{aligned}$$

$$\begin{aligned}&\le \alpha F(\sigma(x_m,Tx_m))+\beta F(\sigma(x_m,Tx_m))+\gamma F(\sigma(Tx_m,T^2x_m))\\&\quad+tF(\frac{s\sigma(x_m,Tx_m)+s\sigma(Tx_m,T^2x_m)}{2s})+hF(\frac{2s\sigma(Tx_m,x_m)}{2s})\\&\le \alpha F(\sigma(x_m,Tx_m))+\beta F(\sigma(x_m,Tx_m))+\gamma F(\sigma(Tx_m,T^2x_m))\\&\quad+tF(\sigma(x_m,Tx_m))+hF(\sigma(Tx_m,x_m)),\end{aligned}$$

which implies

$$(1-\gamma)F(\sigma(Tx_m,T^2x_m))\le(\alpha+\beta+t+h)F(\sigma(x_m,Tx_m))-\tau. \tag{2.10}$$

Taking $\alpha+\beta+\gamma+h+t=1$ into account, we get, by (2.10),

$$F(\sigma(Tx_m,T^2x_m))\le F(\sigma(x_m,Tx_m))-\frac{\tau}{1-\gamma}. \tag{2.11}$$

Since $\frac{1}{2s}\sigma(Tx_m,T^2x_m)<\sigma(Tx_m,T^2x_m)$, from (2.1) we have

$$\begin{aligned}\tau+F(\sigma(T^2x_m,T^3x_m))&\le \alpha F(\sigma(Tx_m,T^2x_m))+\beta F(\sigma(Tx_m,T^2x_m))+\gamma F(\sigma(T^2x_m,T^3x_m))\\&\quad+tF(\frac{\sigma(Tx_m,T^3x_m)}{2s})+hF(\frac{\sigma(T^2x_m,T^2x_m)}{2s})\\&\le \alpha F(\sigma(Tx_m,T^2x_m))+\beta F(\sigma(Tx_m,T^2x_m))+\gamma F(\sigma(T^2x_m,T^3x_m))\\&\quad+tF(\frac{s\sigma(Tx_m,T^2x_m)+s\sigma(T^2x_m,T^3x_m)}{2s})+hF(\frac{2s\sigma(T^2x_m,Tx_m)}{2s})\\&\le \alpha F(\sigma(Tx_m,T^2x_m))+\beta F(\sigma(Tx_m,T^2x_m))+\gamma F(\sigma(T^2x_m,T^3x_m))\\&\quad+tF(\sigma(Tx_m,T^2x_m))+hF(\sigma(T^2x_m,Tx_m)).\end{aligned}$$

This yields

$$F(\sigma(T^2x_m,T^3x_m))\le F(\sigma(Tx_m,T^2x_m))-\frac{\tau}{1-\gamma}.$$

Continuing the above process and taking (2.8) into account, we have

$$\begin{aligned}F(\sigma(T^nx_m,T^{n+1}x_m))&\le F(\sigma(T^{n-1}x_m,T^nx_m))-\frac{\tau}{1-\gamma}\\&\le F(\sigma(T^{n-2}x_m,T^{n-1}x_m))-\frac{2\tau}{1-\gamma}\\&\ \vdots\\&\le \sigma(x_m,Tx_m))-\frac{n\tau}{1-\gamma}\\&<F(A+\varepsilon)-\frac{n\tau}{1-\gamma}.\end{aligned} \tag{2.12}$$

Letting $n\to+\infty$ in (2.12), we get $\lim_{n\to+\infty}F(\sigma(T^nx_m,T^{n+1}x_m))=-\infty$ which together with F(2) implies $\lim_{n\to+\infty}\sigma(T^nx_m,T^{n+1}x_m)=0$. So, there exists $N_1\in\mathbb{N}$ such that $\sigma(T^nx_m,T^{n+1}x_m)<A$ for all $n>N_1$, that is, $\sigma(x_{m+n},Tx_{m+n})<A$ for all $n>N_1$, which is a contradiction with the definition of A, therefore,

$$\lim_{n\to+\infty}\sigma(x_n,Tx_n)=0. \tag{2.13}$$

Now, we prove

$$\lim_{n,m\to+\infty}\sigma(x_n,x_m)=0. \tag{2.14}$$

Suppose, on the contrary, that there exists $\varepsilon > 0$ and sequences $\{p(n)\}$ and $\{q(n)\}$ of natural numbers such that

$$p(n) > q(n) > n, \quad \sigma(x_{p(n)}, x_{q(n)}) \geq \varepsilon \text{ and } \sigma(x_{p(n)-1}, x_{q(n)}) < \varepsilon \ \forall n \in \mathbb{N}. \tag{2.15}$$

Applying the triangle inequality, we get

$$\begin{aligned} \sigma(x_{p(n)}, x_{q(n)}) &\leq s\sigma(x_{p(n)}, x_{p(n)-1}) + s\sigma(x_{p(n)-1}, x_{q(n)}) \\ &< s\sigma(x_{p(n)}, x_{p(n)-1}) + s\varepsilon \\ &= s\sigma(Tx_{p(n)-1}, x_{p(n)-1}) + s\varepsilon \quad \forall n \in \mathbb{N}. \end{aligned} \tag{2.16}$$

Owing to (2.13), there exists $N_2 \in \mathbb{N}$ such that

$$\sigma(x_{p(n)-1}, Tx_{p(n)-1}) < \varepsilon, \quad \sigma(x_{p(n)}, Tx_{p(n)}) < \varepsilon, \quad \sigma(x_{q(n)}, Tx_{q(n)}) < \varepsilon \ \forall n > N_2, \tag{2.17}$$

which together with (2.16) shows

$$\sigma(x_{p(n)}, x_{q(n)}) < 2s\varepsilon \ \forall n > N_2, \tag{2.18}$$

hence

$$F(\sigma(x_{p(n)}, x_{q(n)})) < F(2s\varepsilon) \ \forall n > N_2. \tag{2.19}$$

From (2.15) and (2.17), we get

$$\frac{1}{2s}\sigma(x_{p(n)}, Tx_{p(n)}) < \frac{\varepsilon}{2s} < \sigma(x_{p(n)}, x_{q(n)}) \ \forall n > N_2. \tag{2.20}$$

Using the triangle inequality, we have

$$\varepsilon \leq \sigma(x_{p(n)}, x_{p(n)}) \leq s\sigma(x_{p(n)}, x_{p(n)+1}) + s^2\sigma(x_{p(n)+1}, x_{q(n)+1}) + s^2\sigma(x_{q(n)+1}, x_{q(n)}). \tag{2.21}$$

Letting $n \to +\infty$ in (2.21), by (2.13), we obtain $\frac{\varepsilon}{s^2} \leq \liminf_{n\to+\infty} \sigma(x_{p(n)+1}, x_{q(n)+1})$, hence, there exists $N_3 \in \mathbb{N}$, such that $\sigma(x_{p(n)+1}, x_{q(n)+1}) > 0$ for $n > N_3$, that is $\sigma(Tx_{p(n)}, Tx_{q(n)}) > 0$ for $n > N_3$. By (2.1) and (2.19), we have

$$\begin{aligned} \tau + F(\sigma(Tx_{p(n)}, Tx_{q(n)})) &\leq \alpha F(\sigma(x_{p(n)}, x_{q(n)})) + \beta F(\sigma(x_{p(n)}, Tx_{p(n)})) + \gamma F(\sigma(x_{q(n)}, Tx_{q(n)})) \\ &\quad + tF(\frac{\sigma(x_{p(n)}, Tx_{q(n)})}{2s}) + hF(\frac{\sigma(x_{q(n)}, Tx_{p(n)})}{2s}) \\ &\leq \alpha F(\sigma(x_{p(n)}, x_{q(n)})) + \beta F(\sigma(x_{p(n)}, Tx_{p(n)})) + \gamma F(\sigma(x_{q(n)}, Tx_{q(n)})) \\ &\quad + tF(\frac{\sigma(x_{p(n)}, x_{q(n)}) + \sigma(x_{q(n)}, Tx_{q(n)})}{2}) \\ &\quad + hF(\frac{\sigma(x_{q(n)}, x_{p(n)}) + \sigma(x_{p(n)}, Tx_{p(n)})}{2}) \end{aligned} \tag{2.22}$$

for $n > max\{N_2, N_3\}$.

Taking (2.17), (2.18) and (2.19) into account, (2.22) yields

$$\begin{aligned} \tau + F(\sigma(Tx_{p(n)}, Tx_{q(n)}) &< \alpha F(2s\varepsilon) + \beta F(\sigma(x_{p(n)}, Tx_{p(n)})) + \gamma F(\sigma(x_{q(n)}, Tx_{q(n)})) \\ &\quad + tF(\frac{2s\varepsilon + \varepsilon}{2}) + hF(\frac{2s\varepsilon + \varepsilon}{2}) \end{aligned} \tag{2.23}$$

for $n > max\{N_2, N_3\}$.

Letting $n \to +\infty$ in (2.23), we obtain

$$\lim_{n\to+\infty} F(\sigma(Tx_{p(n)}, Tx_{q(n)})) = -\infty,$$

which yields $\lim_{n\to+\infty} \sigma(Tx_{p(n)}, Tx_{q(n)}) = 0$, which together with

$$\sigma(x_{p(n)}, x_{q(n)}) \leq s\sigma(x_{p(n)}, x_{p(n)+1}) + s^2\sigma(x_{p(n)+1}, x_{q(n)+1}) + s^2\sigma(x_{q(n)+1}, x_{q(n)})$$

shows $\lim_{n\to+\infty} \sigma(x_{p(n)}, x_{q(n)}) = 0$, this is a contradiction with (2.15), so (2.14) holds, therefore, $\{x_n\}$ is a Cauchy sequence in X. Since (X, σ) is complete, there exists $v \in X$ such that

$$\sigma(v, v) = \lim_{n\to+\infty} \sigma(x_n, v) = \lim_{n,m\to+\infty} \sigma(x_n, x_m) = 0. \tag{2.24}$$

It is easy to prove that the following fact holds,

$$\frac{\sigma(x_n, Tx_n)}{2s} < \sigma(x_n, v) \text{ or } \frac{\sigma(Tx_n, T^2x_n)}{2s} < \sigma(Tx_n, v). \tag{2.25}$$

Suppose, on the contrary, that there exists $m_0 \in \mathbb{N}$ such that

$$\frac{\sigma(x_{m_0}, Tx_{m_0})}{2s} \geq \sigma(x_{m_0}, v) \text{ and } \frac{\sigma(Tx_{m_0}, T^2x_{m_0})}{2s} \geq \sigma(Tx_{m_0}, v). \tag{2.26}$$

By (2.5) and (2.26), we get

$$\begin{aligned}\sigma(x_{m_0}, Tx_{m_0}) &\leq s\sigma(x_{m_0}, v) + s\sigma(v, Tx_{m_0})\\ &\leq \frac{\sigma(x_{m_0}, Tx_{m_0})}{2} + \frac{\sigma(Tx_{m_0}, T^2x_{m_0})}{2}\\ &< \frac{\sigma(x_{m_0}, Tx_{m_0})}{2} + \frac{\sigma(x_{m_0}, Tx_{m_0})}{2}\\ &= \sigma(x_{m_0}, Tx_{m_0}).\end{aligned}$$

This is a contradiction. Hence (2.25) holds and it yields

$$\begin{aligned}\tau + F(\sigma(Tx_n, Tv)) \leq& \alpha F(\sigma(x_n, v)) + \beta F(\sigma(x_n, Tx_n)) + \gamma F(\sigma(v, Tv)) + tF(\frac{\sigma(x_n, Tv)}{2s})\\ &+ hF(\frac{\sigma(v, Tx_n)}{2s}),\end{aligned} \tag{2.27}$$

or

$$\begin{aligned}\tau + F(\sigma(T^2x_n, Tv)) \leq& \alpha F(\sigma(Tx_n, v)) + \beta F(\sigma(Tx_n, T^2x_n)) + \gamma F(\sigma(v, Tv))\\ &+ tF(\frac{\sigma(Tx_n, Tv)}{2s}) + hF(\frac{\sigma(v, T^2x_n)}{2s}).\end{aligned} \tag{2.28}$$

Next, we discuss the following cases.
Case 1: Suppose that (2.27) holds. From (2.27), we have

$$\begin{aligned}\tau + F(\sigma(Tx_n, Tv)) \leq& \alpha F(\sigma(x_n, v)) + \beta F(\sigma(x_n, Tx_n)) + \gamma F(\sigma(v, Tv))\\ &+ tF(\frac{\sigma(x_n, v) + \sigma(v, Tv)}{2}) + hF(\frac{\sigma(v, x_n) + \sigma(x_n, Tx_n)}{2}).\end{aligned} \tag{2.29}$$

Owing to (2.13) and (2.24), for some $\varepsilon_0 > 0$, there exists $N_4 \in \mathbb{N}$ such that

$$\sigma(v, x_n) < \varepsilon_0 \text{ and } \sigma(x_n, Tx_n) < \varepsilon_0, \tag{2.30}$$

for $N > N_4$.

With the help of (2.29) and (2.30), we get

$$\tau + F(\sigma(Tx_n, Tv)) \leq \alpha F(\sigma(x_n, v)) + \beta F(\sigma(x_n, Tx_n)) + \gamma F(\sigma(v, Tv)) + tF(\frac{\varepsilon_0 + \sigma(v, Tv)}{2}) + hF(\varepsilon_0)$$

for $N > N_4$.

Taking $n \to +\infty$ in the above inequality, we have $\lim_{n\to+\infty} F(\sigma(Tx_n, Tv)) = -\infty$ which yields

$$\lim_{n\to+\infty} \sigma(Tx_n, Tv) = 0. \tag{2.31}$$

On the other hand, we have $\sigma(v, Tv) \leq s\sigma(v, Tx_n) + s\sigma(Tx_n, Tv) = s\sigma(v, x_{n+1}) + s\sigma(Tx_n, Tv)$. By letting $n \to +\infty$ in the above inequality, by (2.24) and (2.31), we get $\sigma(v, Tv) = 0$, it means $v = Tv$. Thus v is the fixed point of T.

Case 2: Let (2.28) hold. From (2.28), we have

$$\begin{aligned} F(\sigma(T^2x_n, Tv)) <& \tau + F(\sigma(T^2x_n, Tv)) \\ \leq& \alpha F(\sigma(Tx_n, v)) + \beta F(\sigma(Tx_n, T^2x_n)) + \gamma F(\sigma(v, Tv)) + tF(\frac{\sigma(Tx_n, Tv)}{2s}) \\ &+ hF(\frac{\sigma(v, T^2x_n)}{2s}) \\ \leq& \alpha F(\sigma(Tx_n, v)) + \beta F(\sigma(Tx_n, T^2x_n)) + \gamma F(\sigma(v, Tv)) + tF(\frac{\sigma(Tx_n, v) + \sigma(v, Tv)}{2}) \\ &+ hF(\frac{\sigma(v, Tx_n) + \sigma(Tx_n, T^2x_n)}{2}) \\ =& \alpha F(\sigma(x_{n+1}, v)) + \beta F(\sigma(x_{n+1}, Tx_{n+1})) + \gamma F(\sigma(v, Tv)) \\ &+ tF(\frac{\sigma(x_{n+1}, v) + \sigma(v, Tv)}{2}) + hF(\frac{\sigma(v, x_{n+1}) + \sigma(x_{n+1}, Tx_{n+1})}{2}). \end{aligned} \tag{2.32}$$

(2.30) and (2.32) yield

$$\begin{aligned} F(\sigma(T^2x_n, Tv)) <& \alpha F(\sigma(x_{n+1}, v)) + \beta F(\sigma(x_{n+1}, Tx_{n+1})) + \gamma F(\sigma(v, Tv)) \\ &+ tF(\frac{\varepsilon_0 + \sigma(v, Tv)}{2}) + hF(\varepsilon_0) \end{aligned} \tag{2.33}$$

for $N > N_4$.

Taking $n \to +\infty$ in (2.33), we have $\lim_{n\to+\infty} F(\sigma(T^2x_n, Tv)) = -\infty$ which yields

$$\lim_{n\to+\infty} \sigma(T^2x_n, Tv) = 0. \tag{2.34}$$

On the other hand, we have $\sigma(v, Tv) \leq s\sigma(v, T^2x_n) + s\sigma(T^2x_n, Tv) = s\sigma(v, x_{n+2}) + s\sigma(T^2x_n, Tv)$. By letting $n \to +\infty$ in the above inequality, from (2.24) and (2.34), we get $\sigma(v, Tv) = 0$, it means $v = Tv$. Thus v is the fixed point of T and this completes the proof. □

Definition 2.4. Let (X, σ) be a b-metric-like space. A self-mapping $T : X \longrightarrow X$ is said to be a generalized F-contraction of type (II) if there exist $\tau > 0$ and $F \in \mathbb{F}$ such that

$$\frac{1}{2s}\sigma(x, Tx) < \sigma(x, y) \Rightarrow \tau + F(\sigma(Tx, Ty)) \leq \alpha F(\sigma(x, y)) + \beta F(\sigma(x, Tx)) + \gamma F(\sigma(y, Ty))$$

for all $x, y \in X$ with $\sigma(Tx, Ty) > 0$, where $\gamma \in [0, 1)$ and $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta + \gamma = 1$.

By taking $t = h = 0$ in Theorem 2.3, we can get the following corollary.

Corollary 2.5. *Let (X, σ) be a complete b-metric-like space and T a generalized F-contraction of type (II). Then, T has a fixed point $v \in X$; that is, $Tv = v$.*

Remark 2.6. Replacing b-metric-like space by b-metric space in Corollary 2.5, we can get Theorem 9 in [5].

Corollary 2.7. *Let (X, σ) be a complete b-metric-like space and T a self-mapping on X. Assume that there exist $\tau > 0$ and $F \in \mathbb{F}$ such that, for all $x, y \in X$ with $\sigma(Tx, Ty) > 0$,*

$$\frac{1}{2s}\sigma(x, Tx) < \sigma(x, y) \Rightarrow \tau + F(\sigma(Tx, Ty)) \leq F(\sigma(x, y)).$$

Then, T has a fixed point $v \in X$; that is, $Tv = v$.

Proof. The proof is easy by taking $\alpha = 1, \beta = \gamma = 0$ in Corollary 2.5. □

Corollary 2.8. *Let (X, σ) be a complete partial space and T a generalized F-contraction of type (I). Then, T has a unique fixed point.*

Proof. Since every partial metric space is a b-metric-like space [4], the existence of fixed point of the mapping T is guaranteed by Theorem 2.3. Now, we prove the uniqueness of the fixed point of mapping T. Suppose u, v are fixed point of T such that $u \neq v$, then we get $p(u, v) > 0$. If $p(v, v) = 0$, then $0 = \frac{p(v,v)}{2s} = \frac{p(v,Tv)}{2s} < p(v, u)$. If $p(v, v) > 0$, then $0 = \frac{p(v,Tv)}{2s} = \frac{p(v,v)}{2s} < p(v, v) \leq p(v, u)$, hence we have

$$\begin{aligned}\tau + F(p(v, u)) =& \tau + F(p(Tv, Tu))\\ \leq& \alpha F(p(v, u)) + \beta F(p(v, Tv)) + \gamma F(p(u, Tu)) + tF(\frac{p(v, Tu)}{2s}) + hF(\frac{p(u, Tv)}{2s})\\ <& \alpha F(p(v, u)) + \beta F(p(v, v)) + \gamma F(p(u, u)) + tF(p(v, u)) + hF(p(u, v)),\end{aligned}$$

taking $P(2)$ in to account, the above inequalities yields

$$\begin{aligned}F(p(v, u)) < \tau + F(p(v, u)) <& \alpha F(p(v, u)) + \beta F(p(v, u)) + \gamma F(p(v, u)) + tF(p(v, u)) + hF(p(u, v))\\ =& (\alpha + \beta + \gamma + t + h)F(p(u, v))\\ =& F(p(u, v)),\end{aligned}$$

which is a contradiction. Thus, T has a unique fixed point. □

Definition 2.9. Let (X, σ) be a b-metric-like space. A self-mapping $T : X \longrightarrow X$ is said to be a generalized F-contraction of type (III) if there exist $\tau > 0$ and $F \in \mathbb{F}$ such that

$$\begin{aligned}&\sigma(Tx, Ty) > 0 \Rightarrow\\ \tau + F(\sigma(Tx, Ty)) \leq& \alpha F(\sigma(x, y)) + \beta F(\sigma(x, Tx)) + \gamma F(\sigma(y, Ty))\\ &+ tF(\frac{\sigma(x, Ty)}{2s}) + hF(\frac{\sigma(y, Tx)}{2s})\end{aligned} \tag{2.35}$$

for all $x, y \in X$, where $\gamma \in [0, 1)$, $\alpha, \beta, t, h \in [0, 1]$ such that $\alpha + \beta + \gamma + t + h = 1, 1 - \gamma - t > 0$.

Theorem 2.10. *Let (X, σ) be a complete b-metric-like space and T a continuous generalized F-contraction of type (III). If $\sigma(Tx, Tx) \leq \sigma(x, x)$, then, T has a fixed point $v \in X$; that is, $Tv = v$.*

Proof. As in the proof of Theorem 2.3, choosing $x_0 \in X$, we construct sequence $\{x_n\}$ by $x_n = Tx_n = T^n x_0$ and we can suppose

$$0 < \sigma(x_n, Tx_n) = \sigma(Tx_{n-1}, Tx_n) \ \forall n \in \mathbb{N}. \tag{2.36}$$

From (2.35) and (2.36), we have

$$\begin{aligned}\tau + F(\sigma(Tx_{n-1}, Tx_n)) \leq & \alpha F(\sigma(x_{n-1}, x_n)) + \beta F(\sigma(x_{n-1}, Tx_{n-1})) + \gamma F(\sigma(x_n, Tx_n)) \\ & + tF(\frac{\sigma(x_{n-1}, Tx_n)}{2s}) + hF(\frac{\sigma(x_n, Tx_{n-1})}{2s}).\end{aligned} \tag{2.37}$$

We claim

$$\sigma(x_n, Tx_n) < \sigma(x_{n-1}, Tx_{n-1}) \ \forall n \in \mathbb{N}^+. \tag{2.38}$$

Suppose, on the contrary, that there exists $n_0 \in \mathbb{N}$ such that $\sigma(x_{n_0}, Tx_{n_0})) \geq \sigma(x_{n_0-1}, Tx_{n_0-1}))$, which together with (2.37) yields

$$\begin{aligned}\tau + F(\sigma(x_{n_0}, Tx_{n_0})) = & \tau + F(\sigma(Tx_{n_0-1}, Tx_{n_0})) \\ \leq & \alpha F(\sigma(x_{n_0-1}, x_{n_0})) + \beta F(\sigma(x_{n_0-1}, Tx_{n_0-1})) + \gamma F(\sigma(x_{n_0}, Tx_{n_0})) \\ & + tF(\frac{\sigma(x_{n_0-1}, Tx_{n_0})}{2s}) + hF(\frac{\sigma(x_{n_0}, Tx_{n_0-1})}{2s}) \\ \leq & \alpha F(\sigma(x_{n_0-1}, x_{n_0})) + \beta F(\sigma(x_{n_0-1}, Tx_{n_0-1})) + \gamma F(\sigma(x_{n_0}, Tx_{n_0})) \\ & + tF(\frac{s\sigma(x_{n_0-1}, x_{n_0}) + s\sigma(x_{n_0}, Tx_{n_0})}{2s}) \\ & + hF(\frac{s\sigma(x_{n_0}, x_{n_0-1}) + s\sigma(x_{n_0-1}, Tx_{n_0-1})}{2s}) \\ = & \alpha F(\sigma(x_{n_0-1}, Tx_{n_0-1})) + \beta F(\sigma(x_{n_0-1}, Tx_{n_0-1})) + \gamma F(\sigma(x_{n_0}, Tx_{n_0})) \\ & + tF(\frac{s\sigma(x_{n_0-1}, Tx_{n_0-1}) + s\sigma(x_{n_0}, Tx_{n_0})}{2s}) \\ & + hF(\frac{s\sigma(Tx_{n_0-1}, x_{n_0-1}) + s\sigma(x_{n_0-1}, Tx_{n_0-1})}{2s}) \\ \leq & \alpha F(\sigma(x_{n_0-1}, Tx_{n_0-1})) + \beta F(\sigma(x_{n_0-1}, Tx_{n_0-1})) + \gamma F(\sigma(x_{n_0}, Tx_{n_0})) \\ & + tF(\sigma(x_{n_0}, Tx_{n_0})) + hF(\sigma(x_{n_0-1}, Tx_{n_0-1})).\end{aligned} \tag{2.39}$$

By (2.39), we get

$$\tau + (1 - \gamma - t)F(\sigma(x_{n_0}, Tx_{n_0})) \leq (\alpha + \beta + h)F(\sigma(x_{n_0-1}, Tx_{n_0-1})),$$

which shows

$$F(\sigma(x_{n_0}, Tx_{n_0})) \leq F(\sigma(x_{n_0-1}, Tx_{n_0-1})) - \frac{\tau}{1 - \gamma - t}. \tag{2.40}$$

Applying (2.40) and F(1), we have $\sigma(x_{n_0}, Tx_{n_0}) < \sigma(x_{n_0-1}, Tx_{n_0-1})$, this is a contradiction. Hence, (2.38) holds.

Applying (2.35) and (2.38), we obtain

$$\begin{aligned}\tau + F(\sigma(x_n, Tx_n)) = & \tau + F(\sigma(Tx_{n-1}, Tx_n)) \\ \leq & \alpha F(\sigma(x_{n-1}, x_n)) + \beta F(\sigma(x_{n-1}, Tx_{n-1})) + \gamma F(\sigma(x_n, Tx_n)) \\ & + tF(\frac{\sigma(x_{n-1}, Tx_n)}{2s}) + hF(\frac{\sigma(x_n, Tx_{n-1})}{2s}) \\ \leq & \alpha F(\sigma(x_{n-1}, x_n)) + \beta F(\sigma(x_{n-1}, Tx_{n-1})) + \gamma F(\sigma(x_n, Tx_n)) \\ & + tF(\sigma(x_{n-1}, x_n)) + hF(\sigma(x_n, x_{n-1})) \\ = & \alpha F(\sigma(x_{n-1}, Tx_{n-1})) + \beta F(\sigma(x_{n-1}, Tx_{n-1})) + \gamma F(\sigma(x_n, Tx_n)) \\ & + tF(\sigma(x_{n-1}, Tx_{n-1})) + hF(\sigma(Tx_{n-1}, x_{n-1})),\end{aligned}$$

which yields

$$F(\sigma(x_n, Tx_n)) \leq F(\sigma(x_{n-1}, Tx_{n-1})) - \frac{\tau}{1-\gamma}.$$

Continuing this process, we get

$$F(\sigma(x_n, Tx_n)) \leq F(\sigma(x_0, Tx_0)) - \frac{n\tau}{1-\gamma}. \tag{2.41}$$

Letting $n \to +\infty$, (2.41) shows $\lim\limits_{n\to+\infty} F(\sigma(x_n, Tx_n)) = -\infty$, hence

$$\lim_{n\to+\infty} \sigma(x_n, Tx_n) = 0. \tag{2.42}$$

Now, we prove

$$\lim_{n,m\to+\infty} \sigma(x_n, x_m) = 0. \tag{2.43}$$

Suppose, on the contrary, that there exists $\varepsilon > 0$ and sequences $\{p(n)\}$ and $\{q(n)\}$ of natural numbers such that

$$p(n) > q(n) > n,\ \sigma(x_{p(n)}, x_{q(n)}) \geq \varepsilon \text{ and } \sigma(x_{p(n)-1}, x_{q(n)}) < \varepsilon\ \forall n \in \mathbb{N}. \tag{2.44}$$

Applying the triangle inequality, we get

$$\begin{aligned} \sigma(x_{p(n)-1}, x_{q(n)-1}) \leq & s\sigma(x_{p(n)-1}, x_{q(n)}) + s\sigma(x_{q(n)}, x_{q(n)-1}) \\ < & s\sigma(x_{q(n)}, x_{q(n)-1}) + s\varepsilon \\ = & s\sigma(Tx_{q(n)-1}, x_{q(n)-1}) + s\varepsilon\ \forall n \in \mathbb{N}. \end{aligned} \tag{2.45}$$

Owing to (2.42), there exists $N_1 \in \mathbb{N}$ such that

$$\sigma(x_{p(n)-1}, Tx_{p(n)-1}) < \varepsilon, \quad \sigma(x_{q(n)-1}, Tx_{q(n)-1}) < \varepsilon\ \forall n > N_1, \tag{2.46}$$

which together with (2.45) shows

$$\sigma(x_{p(n)-1}, x_{q(n)-1}) < 2s\varepsilon\ \forall n > N_1, \tag{2.47}$$

hence

$$F(\sigma(x_{p(n)-1}, x_{q(n)-1})) < F(2s\varepsilon)\ \forall n > N_1. \tag{2.48}$$

From (2.44), we get $\varepsilon \leq \sigma(x_{p(n)}, x_{q(n)}) = \sigma(Tx_{p(n)_1}, Tx_{q(n)-1})\ \forall n > N_1$, which together with (2.35) yields

$$\begin{aligned} \tau + F(\sigma(Tx_{p(n)-1}, Tx_{q(n)-1}) \leq & \alpha F(\sigma(x_{p(n)-1}, x_{q(n)-1})) + \beta F(\sigma(x_{p(n)-1}, Tx_{p(n)-1})) \\ & + \gamma F(\sigma(x_{q(n)-1}, Tx_{q(n)-1})) + tF(\frac{\sigma(x_{p(n)-1}, Tx_{q(n)-1})}{2s}) \\ & + hF(\frac{\sigma(x_{q(n)-1}, Tx_{p(n)-1})}{2s}) \\ \leq & \alpha F(\sigma(x_{p(n)-1}, x_{q(n)-1})) + \beta F(\sigma(x_{p(n)-1}, Tx_{p(n)-1})) \\ & + \gamma F(\sigma(x_{q(n)-1}, Tx_{q(n)-1})) \\ & + tF(\frac{\sigma(x_{p(n)-1}, x_{q(n)-1}) + \sigma(x_{q(n)-1}, Tx_{q(n)-1})}{2}) \\ & + hF(\frac{\sigma(x_{q(n)-1}, x_{p(n)-1}) + \sigma(x_{p(n)-1}, Tx_{p(n)-1})}{2}) \end{aligned} \tag{2.49}$$

for all $n > N_1$.

Taking (2.46), (2.47) and (2.48) into account, (2.49) yields

$$\begin{aligned} \tau + F(\sigma(Tx_{p(n)-1}, Tx_{q(n)-1}) <& \alpha F(2s\varepsilon) + \beta F(\sigma(x_{p(n)-1}, Tx_{p(n)-1})) + \gamma F(\sigma(x_{q(n)-1}, Tx_{q(n)-1})) \\ &+ tF(\frac{2s\varepsilon+\varepsilon}{2}) + hF(\frac{2s\varepsilon+\varepsilon}{2}). \end{aligned} \tag{2.50}$$

Letting $n \to +\infty$ in (2.50), we obtain $\lim\limits_{n\to+\infty} F(\sigma(Tx_{p(n)-1}, Tx_{q(n)-1})) = -\infty$, which yields $\lim\limits_{n\to+\infty} \sigma(Tx_{p(n)-1}, Tx_{q(n)-1}) = 0$ by F(2), that is, $\lim\limits_{n\to+\infty} \sigma(x_{p(n)}, x_{q(n)}) = 0$ which is a contradiction with (2.44), so (2.43) holds, therefore, $\{x_n\}$ is a Cauchy sequence in X. Since (X, σ) is complete, there exists $v \in X$ such that

$$\sigma(v, v) = \lim_{n\to+\infty} \sigma(x_n, v) = \lim_{n,m\to+\infty} \sigma(x_n, x_m) = 0. \tag{2.51}$$

Since T is continuous, we have

$$\sigma(Tv, Tv) = \lim_{n\to+\infty} \sigma(Tx_n, Tv) = \lim_{n\to+\infty} \sigma(x_{n+1}, Tv). \tag{2.52}$$

Due to $\sigma(Tv, Tv) \le \sigma(v, v)$, from (2.51) and (2.52), we have

$$\lim_{n\to+\infty} \sigma(x_n, Tv) = 0. \tag{2.53}$$

Since $\sigma(v, Tv) \le \sigma(v, x_n) + \sigma(x_n, Tv)$, by (2.53), we get $\sigma(v, Tv) = 0$, which gives $v = Tv$, therefore, T has a fixed point, this completes the proof. □

Definition 2.11. Let (X, σ) be a b-metric-like space. A self-mapping $T : X \longrightarrow X$ is said to be generalized F-contraction of type (IV) if there exists $\tau > 0$ and $F \in \mathbb{F}$ such that

$$\sigma(Tx, Ty) > 0 \Rightarrow \tau + F(\sigma(Tx, Ty)) \le \alpha F(\sigma(x, y)) + \beta F(\sigma(x, Tx)) + \gamma F(\sigma(y, Ty))$$

for all $x, y \in X$, where $\gamma \in [0, 1)$ and $\alpha, \beta \in [0, 1]$.

By taking $t = h = 0$ in Theorem 2.10, we can get the following corollary.

Corollary 2.12. *Let (X, σ) be a complete b-metric-like space and T a continuous generalized F-contraction of type (IV). If $\sigma(Tx, Tx) \le \sigma(x, x)$, then, T has a fixed point $v \in X$; that is, $Tv = v$.*

Remark 2.13. Replacing b-metric-like space by b-metric space and metric-like space in Corollary 2.12, respectively, we can get Theorem 14 in [5].

Corollary 2.14. *Let (X, σ) be a complete b-metric-like space and T a continuous self-mapping on X. If there exists $\tau > 0$ and $F \in \mathbb{F}$ such that for all $x, y \in X$,*

$$\sigma(Tx, Ty) > 0 \Rightarrow \tau + F(\sigma(Tx, Ty)) \le F(\sigma(x, y)).$$

Then, T has a fixed point $v \in X$; that is, $Tv = v$.

Proof. The proof can be finished by taking $\alpha = 1, \beta = \gamma = 0$. □

Corollary 2.15. *Let (X, σ) be a complete partial space and T a continuous generalized F-contraction of type (III). If $\sigma(Tx, Tx) \le \sigma(x, x)$, then, T has a unique fixed point.*

Proof. The proof is similar to the proof Corollary 2.8. □

Now, we introduce an example to illustrate the validity of our main result.

Example 2.16. Let $X = [0,1]$ and let $\sigma : X \times X \to R_0^+$ be defined by $\sigma(x,y) = (max\{x,y\})^2$. Define a mapping $T : X \to X$ as follows:

$$Tx = \begin{cases} \frac{x}{4}, & x \in [0,1), \\ \frac{1}{8}, & x = 1. \end{cases}$$

It is easy to prove that (X,σ) is a complete b-metric-like space with constant $s = 2$. Define the function $F(\alpha) = ln\alpha$ for $\alpha \in \mathbb{R}^+$, then we get

$$\tau + F(\sigma(Tx,Ty)) \leq F(\sigma(x,y)) \Leftrightarrow ln\frac{\sigma(x,y)}{\sigma(Tx,Ty)} \geq \tau.$$

First, we can observe that

$$\frac{\sigma(x,Tx)}{4} < \sigma(x,y) \Leftrightarrow \{(x = 1 \wedge y \in [0,1]) \vee (x < 1 \wedge y = 1) \vee (x < y < 1) \vee (x \leq y < 1)\}.$$

For $x = 1 \wedge y \in [0,1])$, we have $\sigma(x,y) = \sigma(1,y) = 1$ and

$$\sigma(Tx,Ty) = \sigma(\frac{1}{8},Ty) = \begin{cases} \frac{1}{64}, & y \in [0,\frac{1}{2}], \\ \frac{y^2}{16}, & y \in (\frac{1}{2},1), \\ \frac{1}{64}, & y = 1. \end{cases}$$

Hence, we get

$$\frac{\sigma(x,y)}{\sigma(Tx,Ty)} = \begin{cases} 64, & y \in [0,\frac{1}{2}], \\ \frac{16}{y^2}, & y \in (\frac{1}{2},1), \\ 64, & y = 1. \end{cases} \tag{2.54}$$

For $x < 1 \wedge y = 1$, we have $\sigma(x,y) = \sigma(x,1) = 1$ and

$$\sigma(Tx,Ty) = \sigma(Tx,\frac{1}{8}) = \begin{cases} \frac{1}{64}, & x \in [0,\frac{1}{2}], \\ \frac{x^2}{16}, & x \in (\frac{1}{2},1). \end{cases}$$

Hence, we get

$$\frac{\sigma(x,y)}{\sigma(Tx,Ty)} = \begin{cases} 64, & x \in [0,\frac{1}{2}], \\ \frac{16}{x^2}, & x \in (\frac{1}{2},1). \end{cases} \tag{2.55}$$

For $x < y < 1$ we have $\sigma(x,y) = y^2$ and $\sigma(Tx,Ty) = \sigma(\frac{x}{4},\frac{y}{4}) = \frac{y^2}{16}$. Hence, we get

$$\frac{\sigma(x,y)}{\sigma(Tx,Ty)} = 16. \tag{2.56}$$

For $y \leq x < 1$ we have $\sigma(x,y) = x^2$ and $\sigma(Tx,Ty) = \sigma(\frac{x}{4},\frac{y}{4}) = \frac{x^2}{16}$. Hence, we get

$$\frac{\sigma(x,y)}{\sigma(Tx,Ty)} = 16. \tag{2.57}$$

From (2.54) $-$ (2.57), we can obtain that if $0 < \tau \leq ln16$ then $ln\frac{\sigma(x,y)}{\sigma(Tx,Ty)} \geq \tau$. Thus, $\tau + F(\sigma(Tx,Ty)) \leq F(\sigma(x,y))$. Therefore T satisfies the conditions of Corollary 2.7 with $0 < \tau \leq ln16$. Hence, all the required hypotheses of Corollary 2.7 are satisfied. Thus, T has a fixed point.

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