Available online at www.tjnsa.com
J. Nonlinear Sci. Appl. 9 (2016), 2376–2388
Research Article

Journal of Nonlinear Science and Applications
Print: ISSN 2008-1898 Online: ISSN 2008-1901

Anti-periodic solutions of Cohen-Grossberg shunting inhibitory cellular neural networks on time scales

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Communicated by Y. J. Cho

Abstract

In this paper, Cohen-Grossberg shunting inhibitory cellular neural networks(CGSICNNs) on time scales are investigated. Some sufficient conditions which ensure the existence and global exponential stability of anti-periodic solutions for a class of CGSICNNs on time scales are established. Numerical simulations are carried out to illustrate the theoretical findings. The results obtained in this paper are of great significance in designs and applications of globally stable anti-periodic Cohen-Grossberg shunting inhibitory cellular neural networks.

Keywords: Cohen-Grossberg shunting inhibitory cellular neural networks, anti-periodic solution, exponential stability, time scales.
2010 MSC: 34C25, 34K13, 34K25.

1. Introduction

It is well known that since the work of Bouzerdout and Pinter [4] in 1993, shunting inhibitory cellular neural networks (CGSICNNs) have been extensively applied in psychophysics, perception, robotics, adaptive pattern recognition, vision and image processing, etc. The applicability and efficiency of such networks hinge upon their dynamics, and therefore the investigation of dynamical behaviors is a preliminary step for any practical design and application of the networks. Recently, considerable effort has been devoted to the study of dynamic behaviors on the existence and stability of the equilibrium point, periodic and almost periodic solutions of SICNNs with time-varying delays and continuously distributed delays (see [5, 14, 19, 27, 35]).

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Received 2015-12-27

We know that the signal transmission process of neural networks can often be described as an anti-periodic solution process. Thus the existence and stability of anti-periodic solutions are an important topic in characterizing the behavior of nonlinear differential equations [7, 8, 10, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34]. Therefore it is worth while to investigate the existence and stability of anti-periodic solutions for BAM neural networks. In this paper, we consider the following Cohen-Grossberg shunting inhibitory cellular neural networks on time scales

$$x_{ij}^{\Delta}(t) = -a_{ij}(x_{ij}(t)) \left[b_{ij}(x_{ij}(t)) + \sum_{C^{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) - L_{ij}(t) \right], \tag{1.1}$$

where $t \in \mathbb{T}$, $\mathbb{T}$ is a periodic time scale, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. C_{ij} denotes the call at the (i, j) position of the lattice, the r-neighborhood $N_r(i, j)$ of C_{ij} is given by $N_r(i, j) = \{C_{ij} : \max\{|k-i|, |l-j|\} \leq r, 1 \leq k \leq m, 1 \leq l \leq n\}$. x_{ij} acts as the activity of the cell $C_{ij}, L_{ij}(t)$ is the external input to $C_{ij}, a_{ij}(x_{ij}(t)) > 0$ and $b_{ij}(x_{ij}(t))$ represent an amplification function at time t and an appropriately behaved function at time t, respectively; $C_{ij}^{kl}(t) \geq 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij}, and the activity function $f(.)$ is a continuous function representing the output or firing rate of the cell C^{kl}, $\tau_{kl}(t) \geq 0$ corresponds to the time delay required in processing and transmitting a signal from the l-th cell to the k-th cell at time t.

The main aim of this article is to establish some sufficient conditions for the existence and exponential stability of anti-periodic solutions of (1.1). Some other models, such as bidirectional associative memory (BAM) networks, cellular neural networks and Hopfield-type neural networks, are special cases of the network model (1.1). To the best of our knowledge, it is the first time to focus on the stability and existence of anti-periodic solutions of (1.1) on time scales.

The remainder of the paper is organized as follows. In Section 2, we introduce some notations and definitions, and state some preliminary results which are needed in later sections. In Section 3, we establish our main results for the existence and exponential stability of anti-periodic solutions of (1.1). In Section 4, we present an example to illustrate the feasibility and effectiveness of our results obtained in previous sections.

2. Preliminaries on time scales

In order to make an easy and convenient reading of this paper, we present some definitions and notations on time scales which can be found in the literatures [1, 2, 3, 6, 9, 11, 12, 13, 20, 28].

Definition 2.1 ([2]). A time scale is an arbitrary nonempty closed subset $\mathbb{T}$ of $\mathbb{R}$, the real numbers. The set $\mathbb{T}$ inherits the standard topology of $\mathbb{R}$.

Definition 2.2 ([2]). The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, the backward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0, \infty)$ are defined, respectively, by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}, \ \rho(t) := \sup\{s \in \mathbb{T} : s < t\}, \ \mu(t) = \sigma(t) - t \text{ for } t \in \mathbb{T}.$$

If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise: left-scattered). If $\mathbb{T}$ has a left-scattered maximum m, then we defined $\mathbb{T}^k$ to be $\mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then we defined $\mathbb{T}_k$ to be $\mathbb{T} \setminus \{m\}$; otherwise, $\mathbb{T}^k = \mathbb{T}$.

Remark 2.3. We denote the $\mathbb{T}$-interval $[a, b]_{\mathbb{T}}$ as $[a, b]_{\mathbb{T}} := \{t \in \mathbb{T} | a \leq t \leq b\}$.

Definition 2.4. A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sides limits exists(finite) at left-dense points in $\mathbb{T}$. The set rd-continuous functions is shown by $C_{rd}^1 = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.5. For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{R}$, we define $f^\Delta(t)$, the delta-derivative of f at t, to be the number(provided it exists) with the property that, given any $\varepsilon > 0$, there is a neighborhood U of t in $\mathbb{T}$ such that

$$|[f(\sigma(t)) - f(s)] - f^\Delta(t)[\sigma(t) - s]| \leq \varepsilon|\sigma(t) - s| \ for\ all\ s \in U.$$

Thus f is said to be delta-differentiable if its delta-derivative exists. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and whose delta-derivative are rd-continuous functions is denoted by $C_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.6. A function $F : \mathbb{T} \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^\Delta(t) = f(t)$, for all $t \in \mathbb{T}$. Then we write $\int_r^s f(t)\Delta t := F(s) - F(r)\ for\ all\ s, t \in \mathbb{T}$.

Definition 2.7 ([13])**.** We say that a time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the period of the time scale.

Definition 2.8 ([11])**.** Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period p. We say that the function $f : \mathbb{T} \to \mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega = np$, $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$ and ω is the smallest number such that $f(t + \omega) = f(t)$.

If $\mathbb{T} = \mathbb{R}$, we say that f is periodic with period $\omega > 0$, if ω is the smallest positive number such that $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$.

A function $r : \mathbb{T} \to \mathbb{R}$ is called regressive if $1 + \mu(t)r(t) \neq 0$ for all $t \in \mathbb{T}^k$.

If r is regressive function, then the generalized exponential function e_r is defined by

$$e_r(t, s) = \exp\Big\{\int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau\Big\}, \text{for } s, t \in \mathbb{T},$$

with the cylindrical transformation

$$\xi_h(z) = \begin{cases} \frac{\log(1+hz)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q := p + q + \mu pq, \ominus p := -\frac{p}{1 + \mu p}, p \ominus q := p \oplus (\ominus q).$$

Then the generalized exponential function has the following properties.

Lemma 2.9 ([2])**.** *Assume that $p, q : \mathbb{T} \neq \mathbb{R}$ are two regressive functions, then*

(i) $e_0(t, s) \equiv 1$ *and* $e_p(t, t) \equiv 1$*;*

(ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$*;*

(iii) $e_p(t, s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s, t)$*;*

(iv) $e_p(t, s)e_p(s, r) = e_p(t, r)$*;*

(v) $e_p(t, s)e_q(t, s) = e_{p\oplus q}(t, s)$*;*

(vi) $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t, s)$.

Lemma 2.10 ([2])**.** *Assume that $f, g : \mathbb{T} \neq \mathbb{R}$ are delta differentiable at $t \in \mathbb{T}^k$, then*

(i) $(\nu_1 f + \nu_2 g)^\Delta = \nu_1 f^\Delta + \nu_2 g^\Delta$*, for any constants* ν_1, ν_2.

(ii) $(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t))$*;*

(iii) if $f^{\Delta} \geq 0$, then f is nondecreasing.

Lemma 2.11 ([28])**.** *Assume that $p(t) \geq 0$ for $t \geq s$, then $e_p(t,s) \geq 1$.*

Lemma 2.12 ([28])**.** *Assume that $p \in \mathbb{R}$ is ω-periodic, then $e_p(t+n\omega, s) = e_p(t+\omega, s)^n$, for $n \in N$.*

Definition 2.13 ([6])**.** A function f from $\mathbb{T}$ to $\mathbb{R}$ is positively regressive if $1 + \mu(t)f(t) > 0$ for all $t \in \mathbb{T}$.

Denote R^+ is the set of positively regressive functions from $\mathbb{T}$ to $\mathbb{R}$, and denote $\mathbb{T}^+ = R_+ \cap \mathbb{T}$.

Lemma 2.14 ([6])**.** *Suppose that $p \in R^+$, then*

(i) $e_p(t,s) > 0$, for all $t, s \in \mathbb{T}$;

(ii) if $p(t) \leq q(t)$ for all $t \geq s, t, s \in \mathbb{T}$, then $e_p(t,s) \leq e_q(t,s)$ for all $t \geq s$.

Definition 2.15. Let $x^*(t) = (x_{11}^*(t), x_{11}^*(t), \ldots, x_{1m}^*(t), x_{21}^*(t), x_{22}^*(t), \ldots, x_{mn}^*(t)^T$ be an ω-anti-periodic solution of (1.1) with initial value $\varphi^*(t) = (\varphi_{11}^*(t), \varphi_{11}^*(t), \ldots, \varphi_{1m}^*(t), \varphi_{21}^*(t), \varphi_{22}^*(t), \ldots, \varphi_{mn}^*(t)^T$. If there exists a positive constant λ with $-\lambda \in R^+$ such that for $\delta \in [-\tau, \infty)_{\mathbb{T}}$, there exists $N > 1$, such that the solution $x(t) = (x_{11}(t), x_{11}(t), \ldots, x_{1m}(t), x_{21}(t), x_{22}(t), \ldots, x_{mn}(t)^T$ of (1.1) with initial value $\varphi(t) = (\varphi_{11}^*(t), \varphi_{11}(t), \ldots, \varphi_{1m}(t), \varphi_{21}(t), \varphi_{22}(t), \ldots, \varphi_{mn}(t)^T$ satisfies

$$|x_{ij}(t) - x_{ij}^*(t)| \leq N||\varphi - \varphi^*||_1 e_{-\lambda}(t, \delta), t \in [-\tau, \delta)_{\mathbb{T}}, t \geq \delta,$$

where $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$ and $||\varphi - \varphi^*||_1 = \sup_{-\tau \leq s \leq 0} \max_{1 \leq i \leq m, 1 \leq j \leq n} |\varphi_{ij}(s) - \varphi_{ij}^*(s)|$. Then $x^*(t)$ is said to be exponentially stable.

Definition 2.16 ([13])**.** For each $t \in \mathbb{T}$, let N be a neighborhood of t. Then we define the generalized derivative (or Dini derivative), $D^+u^{\Delta}(t)$, to mean that, given $\varepsilon > 0$, there exists a right neighborhood $N_\varepsilon \subset N$ of t such that

$$\frac{u(\sigma(t)) - u(s)}{u(t,s)} < D^+u^{\Delta}(t) + \varepsilon$$

for $s \in N_s, s > t$, where $\mu(t,s) \equiv \sigma(t) - s$.

Definition 2.17 ([12])**.** For each $t \in \mathbb{T}$, let N be a neighborhood of t. Then, for $V \in C_{rd}[\mathbb{T}, \mathbb{R}^n, \mathbb{R}^+]$, define $D^+V^{\Delta}(t, x(t))$ to mean that, given $\varepsilon > 0$, there exists a right neighborhood $N_\varepsilon \subset N$ of t such that

$$\frac{1}{\mu(t,s)}[V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t,s)f(t, x(t))] < D^+V^{\Delta}(t, x(t)) + \varepsilon$$

for each $s \in N_\varepsilon, s > t$, where $\mu(t,s) \equiv (t)\sigma(t) - s$ and $x(t)$ is any solution of (1.1). If t is right-dense and $V(t, x(t))$ is continuous at t, this reduces

$$D^+V^{\Delta}(t, x(t)) = \frac{V(\sigma(t), x(\sigma(t))) - V(t, x(\sigma(t)))}{\sigma(t) - t}.$$

Lemma 2.18. *Let $\mathbb{T}$ be an ω-periodic time scale. Then $\mu(t)$ is an ω-periodic function.*

Let $x_{ij} \in C(\mathbb{T}, \mathbb{R})$, $x_{ij}(t)$ is said to be ω-anti-periodic, if $x_{ij}(t+\omega) = -x_{ij}(t)$ for all $t \in \mathbb{T}$, $\omega > 0$ is a constant. Denote $\mathbb{R}_+ = [0, 1)$. Throughout this article, we assume that

(H1) $C_{ij}^{kl}, L_{ij} \in C(\mathbb{T}, \mathbb{R}), C_{ij}^{kl}(t+\omega) = C_{ij}^{kl}(t), L_{ij}(t+\omega) = -L_{ij}(t)$ and $\tau_{kl} \in C(\mathbb{T}, \mathbb{R}), \tau_{kl}(t+\omega) = \tau_{kl}(t)$, where $\omega > 0$ is a constant, $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, 1 \leq k \leq m, 1 \leq l \leq n$.

(H2) $a_{ij} \in C(\mathbb{T}, \mathbb{R}_+), a_{ij}(-u) = a_{ij}(u)$, and there exist positive constants $\underline{a}_{ij}$ and $\overline{a}_{ij}$ such that $\underline{a}_{ij} \leq a_{ij}(u) \leq \overline{a}_{ij}$ for all $u \in \mathbb{R}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

(H3) $b_{ij} \in C(\mathbb{T},\mathbb{R}), b_{ij}(-u) = b_{ij}(u)$, and there exist a positive constant $\underline{b}_{ij}$ such that $\underline{b}_{ij}|u| \leq \text{sign}(u)b_{ij}(u)$ for all $u \in \mathbb{R}, i = 1,2,\ldots,m, j = 1,2,\ldots,n$.

(H4) $f \in C(\mathbb{R},\mathbb{R}), f(-u) = -f(u), f(0) = 0$, and there exist positive constants $L > 0$ and M_0 such that $|f(u) - f(v)| \leq L|u - v|, |f(u)| \leq M_0$ for all $u, v \in \mathbb{R}, i = 1,2,\ldots,m, j = 1,2,\ldots,n$.

For convenience, we introduce some notations as follows.

$$\tau = \max_{1\leq k\leq m, 1\leq l\leq n} \max_{t\in[0,\omega]_{\mathbb{T}}} |\tau_{kl}|, \overline{C}_{ij}^{kl} = \max_{t\in[0,\omega]_{\mathbb{T}}} |C_{ij}^{kl}(t)|, \bar{L}_{ij} = \max_{t\in[0,\omega]_{\mathbb{T}}} |L_{ij}(t)|.$$

For $x(t) = (x_{11}(t),\ldots,x_{1m}(t),x_{21}(t),x_{22}(t),\ldots,x_{mn}(t)^T \in C(\mathbb{T},\mathbb{R}^n)$, we define the norm

$$||x|| = \max_{1\leq i\leq m, 1\leq j\leq n} |x_{ij}(t)|.$$

The initial value of (1.1) is as follows

$$x_{ij}(s) = \varphi_{ij}(s), s \in [-\tau, 0]_{\mathbb{T}},$$

where $\varphi_{ij}(s) \in C([-\tau,0]_{\mathbb{T}},\mathbb{R}), i = 1,2,\ldots,m, j = 1,2,\ldots,n$.

3. Main results

In this section, we will state and prove our main results of this paper. In order to obtain our main results, we make the following assumptions.

(H5) There exists a positive constant $\gamma > 0$ such that

$$-\underline{a}_{ij}\underline{b}_{ij}\gamma + \bar{a}_{ij} \sum_{C^{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl} L\gamma^2 + \bar{a}_{ij}\bar{L}_{ij} \leq 0.$$

(H6) There exists a positive constant L_{ij}^a such that

$$|a_{ij}(u) - a_{ij}(v)| \leq L_{ij}^a|u - v|, i = 1,2,\ldots,m, j = 1,2,\ldots,n$$

for all $u, v \in \mathbb{R}$.

(H7) There exists a positive constant r_{ij}^{ab} such that for all $u, v \in \mathbb{R}$

$$|a_{ij}(u)b_{ij}(u) - a_{ij}(v)b_{ij}(v)| \geq r_{ij}^{ab}|u - v|, i = 1,2,\ldots,m, j = 1,2,\ldots,n.$$

(H8) The following condition holds.

$$-r_{ij}^{ab} + L_{ij}^a \sum_{C^{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}\gamma + L_{ij}^a\bar{I}_{ij} + \bar{a}_{ij} \sum_{C^{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl} L < 0,$$

where $i = 1,2,\ldots,m, j = 1,2,\ldots,n$.

Lemma 3.1. *Let the conditions* (H1)-(H5) *hold. Suppose that* $\tilde{x}(t) = (\tilde{x}_{11}(t),\tilde{x}_{11}(t),\ldots,\tilde{x}_{1m}(t),\tilde{x}_{21}(t), \tilde{x}_{22}(t),\ldots,\tilde{x}_{mn}(t)^T$ *of* (1.1) *with initial value* $\tilde{\varphi}(t) = (\tilde{\varphi}_{11}(t),\tilde{\varphi}_{11}(t),\ldots,\tilde{\varphi}_{1m}(t),\tilde{\varphi}_{21}(t),\tilde{\varphi}_{22}(t),\ldots,\tilde{\varphi}_{mn}(t)^T$ *satisfies*

$$\tilde{x}_{ij}(s) = \tilde{\varphi}_{ij}(s), |\tilde{\varphi}_{ij}(s)| < \gamma, s \in [-\tau,0]_{\mathbb{T}}, i = 1,2,\ldots,m, j = 1,2,\ldots,n. \tag{3.1}$$

Then

$$|\tilde{x}_{ij}(t)| < \gamma, t \in \mathbb{T}^+, i = 1,2,\ldots,m, j = 1,2,\ldots,n. \tag{3.2}$$

Proof. By way of contradiction, assume that (3.2) does not hold. Then, there exist

$$ij \in \{11, 12, 1m, 21, 22, \dots, mn\}$$

and the first time $t_0 \in \mathbb{T}^+$ such that

$$|\tilde{x}_{ij}(t_0)| = \gamma, |\tilde{x}_{ij}(t)| < \gamma, t \in [-\tau, t_0)_{\mathbb{T}},$$

$$|\tilde{x}_{hl}(t)| < \gamma, t \in [-\tau, t_0)_{\mathbb{T}}, \text{for } hl \neq ij,$$

where $h = 1, 2, \dots, m, l = 1, 2, \dots, n$. Let

$$W_{ij}(t) = |x_{ij}(t)|, i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

It follows from (H1)-(H5) that

$$\begin{aligned}
0 &\leq D^+ W_{ij}^{\Delta}(t_0) \\
&\leq -\underline{a}_{ij}\underline{b}_{ij}|\tilde{x}_{ij}(t_0)| + \bar{a}_{ij} \sum_{C^{kl} \in N_r(i,j)} \overline{C}_{ij}^{kl} L |x_{kl}(t_0 - \tau_{kl}(t_0))||\tilde{x}_{ij}(t_0)| + \bar{a}_{ij}\bar{L}_{ij} \\
&\leq -\underline{a}_{ij}\underline{b}_{ij}\gamma + \bar{a}_{ij} \sum_{C^{kl} \in N_r(i,j)} \overline{C}_{ij}^{kl} L\gamma^2 + \bar{a}_{ij}\bar{L}_{ij} \leq 0,
\end{aligned}$$

which is a contradiction and hence (3.2) holds. The proof of Lemma 3.1 is complete. □

Remark 3.2. In view of the boundedness of this solution, it follows that $\tilde{x}_{ij}(t)$ can be defined on $[-\tau, \infty)_{\mathbb{T}}$ provided that the initial value is bounded.

Lemma 3.3. *Suppose that* (H1)–(H5) *hold. Let* $x^*(t) = (x_{11}^*(t), x_{12}^*(t), \dots, x_{1m}^*(t), x_{21}^*(t), x_{22}^*(t), \dots, x_{mn}^*(t))^T$ *be the solution of* (1.1) *with initial value* $\varphi^* = (\varphi_{11}^*(t), \varphi_{12}^*(t), \dots, \varphi_{1m}^*(t), \varphi_{21}^*(t), \varphi_{22}^*(t), \dots, \varphi_{mn}^*(t))^T$, *and* $x(t) = (x_{11}(t), x_{12}(t), \dots, x_{1m}(t), x_{21}(t), x_{22}(t), \dots, x_{mn}(t))^T$ *be the solution of* (1.1) *with initial value* $\varphi = (\varphi_{11}(t), \varphi_{12}(t), \dots, \varphi_{1m}(t), \varphi_{21}(t), \varphi_{22}(t), \dots, \varphi_{mn}(t))^T$. *Then there exists constant* $\lambda > 1$ *such that for every* $\delta \in [-\tau, 0]_{\mathbb{T}}$, *there exists* $N = N(\delta) \geq 1$ *such that* $x(t)$ *satisfies*

$$|x_{ij}(t) - x_{ij}^*(t)| \leq N\|\varphi - \varphi^*\|_1 e_{\ominus\lambda}(t, \delta), t \in [0, \infty)_{\mathbb{T}}, i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

Proof. Let $z(t) = x(t) - x^*(t)$. Then

$$\begin{aligned}
z_{ij}^{\Delta}(t) &= -a_{ij}(x_{ij}(t)) \left[b_{ij}(x_{ij}(t)) + \sum_{C^{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) - L_{ij}(t) \right] \\
&\quad + a_{ij}(x_{ij}^*(t)) \left[b_{ij}(x_{ij}^*(t)) + \sum_{C^{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}^*(t - \tau_{kl}(t))) x_{ij}^*(t) - L_{ij}(t) \right] \\
&= -\left[a_{ij}(x_{ij}(t)) b_{ij}(x_{ij}(t)) - a_{ij}(x_{ij}^*(t)) b_{ij}(x_{ij}^*(t)) \right] \\
&\quad - \left[a_{ij}(x_{ij}(t)) \sum_{C^{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t) \right. \\
&\quad \left. - a_{ij}(x_{ij}^*(t)) \sum_{C^{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}^*(t - \tau_{kl}(t))) x_{ij}^*(t) \right] \\
&\quad + [a_{ij}(x_{ij}(t)) - a_{ij}(x_{ij}^*(t))] L_{ij}(t),
\end{aligned} \tag{3.3}$$

where $i = 1, 2, \dots, n$. Next, define a Lyapunov functional as

$$V_{ij}(t) = e_\lambda(t,\delta)|z_{ij}(t)|, \delta \in [-\tau, 0]_{\mathbb{T}}, t \in \mathbb{T}, i = 1, 2, \dots, m, j = 1, 2, \dots, n. \tag{3.4}$$

It follows from (3.3) and (3.4) that

$$\begin{aligned}
D^+(V_{ij}^\Delta(t)) \leq & \lambda e_\lambda(t,\delta)|z_{ij}(t) + e_\lambda(\sigma(t),\delta)\mathrm{sign}(z_{ij}) \\
& \times \Bigg\{ - \big[a_{ij}(x_{ij}(t))b_{ij}(x_{ij}(t)) - a_{ij}(x^*_{ij}(t))b_{ij}(x^*_{ij}(t))\big] \\
& - \Bigg[a_{ij}(x_{ij}(t)) \sum_{C^{kl}\in N_r(i,j)} C^{kl}_{ij}(t) f(x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) \\
& -a_{ij}(x^*_{ij}(t)) \sum_{C^{kl}\in N_r(i,j)} C^{kl}_{ij}(t) f(x^*_{kl}(t-\tau_{kl}(t)))x^*_{ij}(t) \Bigg] \\
& + [a_{ij}(x_{ij}(t)) - a_{ij}(x^*_{ij}(t))]L_{ij}(t) \Bigg\} \\
\leq & e_\lambda(\sigma(t),\delta) \Bigg\{ - (r^{ab}_{ij} - \lambda)|z_{ij}| + L^a_{ij}|z_{ij}| + L^a_{ij}M_0\gamma|z_{ij}| \\
& + \bar{a}_{ij} \Bigg(\sum_{C^{kl}\in N_r(i,j)} \bar{C}^{kl}_{ij} M_0|z_{ij}| + \sum_{C^{kl}\in N_r(i,j)} \bar{C}^{kl}_{ij} M_0\gamma|z_{kl}(t-\tau_{kl}(t))| \Bigg) \Bigg\} \\
\leq & [1 + (\mu(t)\lambda] \Bigg\{ - (r^{ab}_{ij} - \lambda)V_{ij}(t) + L^a_{ij}V_{ij}(t) \\
& + L^a_{ij}M_0\gamma V_{ij}(t) + \bar{a}_{ij} \Bigg(\sum_{C^{kl}\in N_r(i,j)} \bar{C}^{kl}_{ij} M_0 V_{ij}(t) \\
& + \sum_{C^{kl}\in N_r(i,j)} \bar{C}^{kl}_{ij} M_0\gamma e_\lambda(t, t-\tau_{kl}(t)) V_{kl}(t-\tau_{kl}(t))| \Bigg) \Bigg\},
\end{aligned} \tag{3.5}$$

where $i = 1, 2, \dots, m, j = 1, 2, \dots, n$. Set

$$\|\varphi - \varphi^*\|_1 = \max_{1\leq i\leq m, 1\leq j\leq n} \max_{s\in[-\tau,0]_{\mathbb{T}}} |\varphi_{ij}(s) - \varphi^*_{ij}(s)| > 0, i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

From (3.4), for every $\delta \in [-\tau, 0]_{\mathbb{T}}$, we can choose a constant $N = N(\delta) \geq 1$ such that

$$V_{ij}(t) = e_\lambda(t,\delta)|z_{ij}| < N||\varphi - \varphi^*||_1, t \in [-\tau, 0]_{\mathbb{T}}, i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

We claim that for every $\delta \in [-\tau, 0]_{\mathbb{T}}$,

$$V_{ij}(t) = e_\lambda(t,\delta)|z_{ij}| < N||\varphi - \varphi^*||_1, t \in [0, +\infty]_{\mathbb{T}}, i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

Otherwise, there must exist some $ij \in \{11, 12, \dots, 1m, 21, 22, \dots, mn\}$ and the first time $t_1 > 0$ such that

$$V_{ij}(t_1) = N\|\varphi - \varphi^*\|_1, V_{ij}(t) < N\|\varphi - \varphi^*\|_1, t \in [-\tau, t_1), i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

$$V_{hl}(t) < N\|\varphi - \varphi^*\|_1, hl \neq ij, t \in [-\tau, t_1), h = 1, 2, \dots, m, l = 1, 2, \dots, n.$$

By Lemma 2.18, we have $e_\lambda(t_1, t_1 - k_0\omega) = e_\lambda(0, -k_0\omega)$. It follows from (3.5) that

$$\begin{aligned}
D^+(V_{ij}^{\Delta}(t_1)) \le [1 + (\mu(t_1)\lambda] \Bigg\{ &- (r_{ij}^{ab} - \lambda)V_{ij}(t_1) + L_{ij}^a V_{ij}(t_1) \\
&+ L_{ij}^a M_0 \gamma V_{ij}(t_1) + \bar{a}_{ij} \Bigg(\sum_{C^{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} M_0 V_{ij}(t_1) \\
&+ \sum_{C^{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} M_0 \gamma e_\lambda(t_1, t_1 - \tau_{kl}(t_1)) V_{kl}(t_1 - \tau_{kl}(t_1))| \Bigg) \Bigg\} \\
\le [1 + (\mu(t_1)\lambda] N ||\varphi - \varphi^*||_1 \Bigg\{ &- (r_{ij}^{ab} - \lambda) + L_{ij}^a + L_{ij}^a M_0 \gamma \\
&+ \bar{a}_{ij} \Bigg(\sum_{C^{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} M_0 + \sum_{C^{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} M_0 \gamma e_\lambda(t_1, t_1 - k_0\omega) \Bigg) \Bigg\} \\
\le [1 + (\mu(t_1)\lambda] N ||\varphi - \varphi^*||_1 \Bigg\{ &- (r_{ij}^{ab} - \lambda) + L_{ij}^a + L_{ij}^a M_0 \gamma \\
&+ \bar{a}_{ij} \Bigg(\sum_{C^{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} M_0 + \sum_{C^{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} M_0 \gamma e_\lambda(0, -k_0\omega) \Bigg) \Bigg\} \\
\le [1 + (\mu(t_1)\lambda] \beta_{ij}(\lambda) N ||\varphi - \varphi^*||_1 & \\
\le 0, i = 1, 2, \dots, m, j = 1, 2, \dots, n. &
\end{aligned} \tag{3.6}$$

From the above inequality, we get $0 \le D^+(V_{ij}^{\Delta}(t_1)) < 0$, which is a contradiction. Then

$$V_{ij}(t) \le N||\varphi - \varphi^*||_1, t \in [0, +\infty]_{\mathbb{T}}, \delta \in (-\tau, 0]_{\mathbb{T}}, i = 1, 2, \dots, m, j = 1, 2, \dots, n,$$

which implies

$$|z_{ij}(t)| \le N||\varphi - \varphi^*||_1 e_{\ominus\lambda}(t, \delta).$$

Thus we have

$$|x_{ij}(t) - x_{ij}^*(t)| \le N\|\varphi - \varphi^*\|_1 e_{\ominus\lambda}(t, \delta), t \in [0, \infty)_{\mathbb{T}}, \delta \in (-\tau, 0]_{\mathbb{T}}, i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

The proof of Lemma 3.3 is complete. □

Theorem 3.4. *Assume that* (H1)–(H8) *are satisfied. Then* (1.1) *has exactly one* T*-anti-periodic solution* $x^*(t)$*. Moreover, this solution is globally exponentially stable.*

Proof. Let $v(t) = (v_{11}(t), v_{12}(t), \dots,\ v_{1m}(t), v_{21}(t), v_{22}(t), \dots,\ v_{mn}(t),)^T$ is a solution of (1.1) with initial conditions

$$v_{ij}(s) = \varphi_{ij}^v(s), |\varphi_{ij}^v(s)| < \gamma, s \in (-\tau, 0]_{\mathbb{T}},\ i = 1, 2, \dots, m, j = 1, 2, \dots, n. \tag{3.7}$$

Thus according to Lemma 3.1, the solution $v(t)$ is bounded and

$$|v_{ij}(t)| < \gamma, \text{for all } \ t \in (-\tau, \infty]_{\mathbb{T}}, i = 1, 2, \dots, m, j = 1, 2, \dots, n. \tag{3.8}$$

From (1.1), we obtain

$$\begin{aligned}
&[(-1)^{k+1}v_{ij}(t+(k+1)T)]^{\Delta} \\
&\quad = (-1)^{k+1}\bigg\{ -a_{ij}(x_{ij}(t+(k+1)T))\Big[b_{ij}(x_{ij}(t+(k+1)T)) \\
&\qquad + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f(x_{kl}(t+(k+1)T-\tau_{kl}(t))) \\
&\qquad \times x_{ij}(t+(k+1)T)-L_{ij}(t)\Big]\bigg\} \\
&\quad = (-1)^{k+1}\bigg\{ -a_{ij}(x_{ij}(t+(k+1)T))\Big[b_{ij}(x_{ij}(t+(k+1)T)) \\
&\qquad + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t+(k+1)T)f(x_{kl}(t+(k+1)T-\tau_{kl}(t+(k+1)T))) \\
&\qquad \times x_{ij}(t+(k+1)T)-(-1)^{k+1}L_{ij}(t)\Big]\bigg\} \\
&\quad = -a_{ij}((-1)^{k+1}x_{ij}(t+(k+1)T))\Big[b_{ij}((-1)^{k+1}x_{ij}(t+(k+1)T)) \\
&\qquad + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t+(k+1)T)f((-1)^{k+1}x_{kl}(t+(k+1)T-\tau_{kl}(t+(k+1)T))) \\
&\qquad \times(-1)^{k+1}x_{ij}(t+(k+1)T)-L_{ij}(t)\Big],
\end{aligned} \tag{3.9}$$

where $i=1,2,\ldots,m, j=1,2,\ldots,n$. Thus $(-1)^{k+1}v(t+(k+1)T)$ are the solutions of (1.1) on $\mathbb{R}$ for any natural number k. Then, from Lemma 3.3, there exists a constant $N>0$ such that

$$\begin{aligned}
&|(-1)^{k+1}v_{ij}(t+(k+1)T)-(-1)^{k}v_{ij}(t+kT)| \\
&\quad \le Ne_{-\lambda}(t+kT,\delta)\sup_{s\in[-\tau,0]_{\mathbb{T}}}\max_{1\le i\le m,1\le j\le n}|v_{ij}(s+T)+v_{ij}(s)| \\
&\quad \le 2Ne_{-\lambda}(t+kT,\delta)\gamma, \text{for all } t+kT\in\mathbb{T}.
\end{aligned} \tag{3.10}$$

Thus, for any natural number m, we have

$$(-1)^{m+1}v_{ij}(t+(m+1)T) = v_{ij}(t)+\sum_{k=0}^{m}[(-1)^{k+1}v_{ij}(t+(k+1)T)-(-1)^{k}v_{ij}(t+kT)]. \tag{3.11}$$

Hence,

$$|(-1)^{m+1}v_{ij}(t+(m+1)T)| \le |v_{ij}(t)|+\sum_{k=0}^{m}|(-1)^{k+1}v_{ij}(t+(k+1)T)-(-1)^{k}v_{ij}(t+kT)|, \tag{3.12}$$

where $i=1,2,\ldots,n$. In view of (3.10), we can choose a sufficiently large constant $M>0$ and a positive constant θ such that

$$|(-1)^{k+1}v_{ij}(t+(k+1)T)-(-1)^{k}v_{ij}(t+kT)| \le \theta(e_{-\lambda}(t+T,\delta))^{k}, \tag{3.13}$$

for all $k>M$, $i=1,2,\ldots,n$, on any compact set of $\mathbb{R}$. Obviously, together with (3.12) and (3.13), $\{(-1)^{m}v(t+mT)\}$ uniformly converges to a continuous function

$$x^*(t)=(x_{11}^*(t),x_{12}^*(t),\ldots,x_{1n}^*(t),x_{21}^*(t),x_{22}^*(t),\ldots,x_{mn}^*(t))^T$$

on any compact set of $\mathbb{R}$.

Now we show that $x^*(t)$ is T-anti-periodic solution of (1.1). Firstly, $x^*(t)$ is T-anti-periodic, since

$$\begin{aligned} x^*(t+T) &= \lim_{m\to\infty}(-1)^m v(t+T+mT) \\ &= -\lim_{(m+1)\to\infty}(-1)^{m+1}v(t+(m+1)T) \\ &= -x^*(t). \end{aligned} \tag{3.14}$$

In the sequel, we prove that $x^*(t)$ is a solution of (1.1). Because of the continuity of the right-hand side of (1.1), Eq. (3.9) implies that $\{((-1)^{m+1}v(t+(m+1)T))'\}$ uniformly converges to a continuous function on any compact subset of R. Thus, letting $m\to\infty$, we can easily obtain

$$(x_{ij}^*)^{\Delta}(t) = -a_{ij}(x_{ij}^*(t))\left[b_{ij}(x_{ij}^*(t)) + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f(x_{kl}^*(t-\tau_{kl}(t)))x_{ij}^*(t) - L_{ij}(t)\right], \tag{3.15}$$

where $i=1,2,\dots,m, j=1,2,\dots,n$. Therefore, $x^*(t)$ is a solution of (1.1). Finally, by applying Lemma 3.3, it is easy to check that $x^*(t)$ is globally exponentially stable. This completes the proof. □

4. An example

In this section, we will give an example to illustrate the feasibility and effectiveness of our main results obtained in Section 3. Let $\mathbb{T}=\mathbb{Z}, i,j=2$. Considering the following system

$$\begin{cases} x_{11}^{\Delta}(k) = -a_{11}(x_{11}(k))\left[b_{11}(x_{11}(k)) + \sum\limits_{C^{kl}\in N_r(1,1)} C_{11}^{kl}(k)f(x_{kl}(k-\tau_{kl}(k)))x_{11}(k) - L_{11}(k)\right], \\ x_{12}^{\Delta}(k) = -a_{12}(x_{12}(k))\left[b_{12}(x_{12}(k)) + \sum\limits_{C^{kl}\in N_r(1,2)} C_{12}^{kl}(k)f(x_{kl}(k-\tau_{kl}(k)))x_{12}(k) - L_{12}(k)\right], \\ x_{21}^{\Delta}(k) = -a_{21}(x_{21}(k))\left[b_{21}(x_{21}(k)) + \sum\limits_{C^{kl}\in N_r(2,1)} C_{21}^{kl}(k)f(x_{kl}(k-\tau_{kl}(k)))x_{21}(k) - L_{21}(k)\right], \\ x_{22}^{\Delta}(k) = -a_{22}(x_{22}(k))\left[b_{22}(x_{22}(k)) + \sum\limits_{C^{kl}\in N_r(2,2)} C_{22}^{kl}(k)f(x_{kl}(k-\tau_{kl}(k)))x_{22}(k) - L_{22}(k)\right], \end{cases} \tag{4.1}$$

where $\Delta x_{ij}(k) = x_{ij}(k+1) - x_{ij}(k), i,j=1,2, k\in\mathbb{Z}, f(u)=\sin u, \tau_{kl}(k) = 0.4\sin^2(\pi k), r=1$ and

$$\begin{bmatrix} a_{11}(u) & a_{12}(u) \\ a_{21}(u) & a_{22}(u) \end{bmatrix} = \begin{bmatrix} 5-\cos u & 6-\cos u \\ 7+\cos u & 8+\cos u \end{bmatrix},$$

$$\begin{bmatrix} b_{11}(u) & b_{12}(u) \\ b_{21}(u) & b_{22}(u) \end{bmatrix} = \begin{bmatrix} u & u \\ u & u \end{bmatrix},$$

$$\begin{bmatrix} C_{11}^{kl}(u) & C_{12}^{kl}(u) \\ C_{21}^{kl}(u) & C_{11}^{kl}(u) \end{bmatrix} = \begin{bmatrix} 0.05-0.05\cos u & 0.05-0.05\cos u \\ 0.05-0.05\cos u & 0.05-0.05\cos u \end{bmatrix},$$

$$\begin{bmatrix} L_{11}(k) & L_{12}(k) \\ L_{21}(k) & L_{22}(k) \end{bmatrix} = \begin{bmatrix} \sin\pi k & \sin\pi k \\ \sin\pi k & \sin\pi k \end{bmatrix}.$$

It is easy to show that $r_{11}^{ab}=4, r_{12}^{ab}=5, r_{21}^{ab}=6, r_{22}^{ab}=7, L_{ij}^{a}=1, L=1,$

$$\begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}, \begin{bmatrix} \underline{b}_{11} & \underline{b}_{12} \\ \underline{b}_{21} & \underline{b}_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}, \begin{bmatrix} \bar{L}_{11} & \bar{L}_{12} \\ \bar{L}_{21} & \bar{L}_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

$$\begin{bmatrix} \sum\limits_{C^{kl}\in N_1(1,1)} \overline{C}_{11}^{kl} & \sum\limits_{C^{kl}\in N_1(1,2)} \overline{C}_{12}^{kl} \\ \sum\limits_{C^{kl}\in N_1(2,1)} \overline{C}_{21}^{kl} & \sum\limits_{C^{kl}\in N_1(2,2)} \overline{C}_{22}^{kl} \end{bmatrix} = \begin{bmatrix} 0.04 & 0.04 \\ 0.04 & 0.04 \end{bmatrix}.$$

Let $\gamma=2$. Then we have

$$-\underline{a}_{11}\underline{b}_{11}\gamma + \bar{a}_{11}\sum_{C^{kl}\in N_1(1,1)} \overline{C}_{11}^{kl} L\gamma^2 + \bar{a}_{11}\bar{L}_{11} = -4\times 1\times 2 + 6\times 0.04\times 1\times 2^2 + 6\times 1 = -1.04 < 0,$$

$$-\underline{a}_{12}\underline{b}_{12}\gamma + \bar{a}_{12}\sum_{C^{kl}\in N_1(1,2)} \overline{C}_{12}^{kl} L\gamma^2 + \bar{a}_{12}\bar{L}_{12} = -5\times 1\times 2 + 5\times 0.04\times 1\times 2^2 + 5\times 1 = -4.2 < 0,$$

$$-\underline{a}_{21}\underline{b}_{21}\gamma + \bar{a}_{21}\sum_{C^{kl}\in N_1(2,1)} \overline{C}_{21}^{kl} L\gamma^2 + \bar{a}_{21}\bar{L}_{21} = -6\times 1\times 2 + 6\times 0.04\times 1\times 2^2 + 6\times 1 = -5.04 < 0,$$

$$-\underline{a}_{22}\underline{b}_{22}\gamma + \bar{a}_{22}\sum_{C^{kl}\in N_1(2,2)} \overline{C}_{ij}^{kl} L\gamma^2 + \bar{a}_{22}\bar{L}_{22} = -7\times 1\times 2 + 7\times 0.04\times 1\times 2^2 + 7\times 1 = -5.88 < 0.$$

and

$$\begin{aligned} &-r_{11}^{ab} + L_{11}^{a}\sum_{C^{kl}\in N_1(1,1)} \overline{C}_{11}^{kl}\gamma + L_{11}^{a}\bar{I}_{11} + \bar{a}_{11}\sum_{C^{kl}\in N_1(1,1)} \overline{C}_{11}^{kl} L \\ &\quad = -4 + 1\times 0.04\times 2 + 1\times 1 + 6\times 0.04\times 1 = -2.68 < 0, \end{aligned}$$

$$\begin{aligned} &-r_{12}^{ab} + L_{12}^{a}\sum_{C^{kl}\in N_1(1,2)} \overline{C}_{12}^{kl}\gamma + L_{12}^{a}\bar{I}_{12} + \bar{a}_{12}\sum_{C^{kl}\in N_1(1,2)} \overline{C}_{12}^{kl} L \\ &\quad = -5 + 1\times 0.04\times 2 + 1\times 1 + 5\times 0.04\times 1 = -3.72 < 0, \end{aligned}$$

$$\begin{aligned} &-r_{21}^{ab} + L_{21}^{a}\sum_{C^{kl}\in N_1(2,1)} \overline{C}_{21}^{kl}\gamma + L_{21}^{a}\bar{I}_{21} + \bar{a}_{21}\sum_{C^{kl}\in N_1(2,1)} \overline{C}_{21}^{kl} L \\ &\quad = -6 + 1\times 0.04\times 2 + 1\times 1 + 6\times 0.04\times 1 = -4.68 < 0, \end{aligned}$$

$$\begin{aligned} &-r_{22}^{ab} + L_{22}^{a}\sum_{C^{kl}\in N_1(2,2)} \overline{C}_{22}^{kl}\gamma + L_{22}^{a}\bar{I}_{22} + \bar{a}_{22}\sum_{C^{kl}\in N_1(2,2)} \overline{C}_{22}^{kl} L \\ &\quad = -7 + 1\times 0.04\times 2 + 1\times 1 + 7\times 0.04\times 1 = -5.68 < 0. \end{aligned}$$

Then all the conditions in Theorem 3.4 are satisfied, then system (4.1) has a at least one 2-periodic solution, which is exponentially stable. The results are verified by the numerical simulations in Figure 1 and Figure 2.

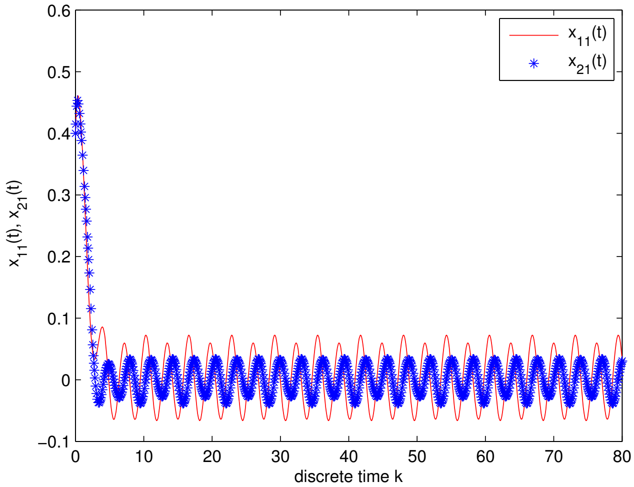

Figure 1: Time response of state variables $x_{11}(t)$ and $x_{21}(t)$.

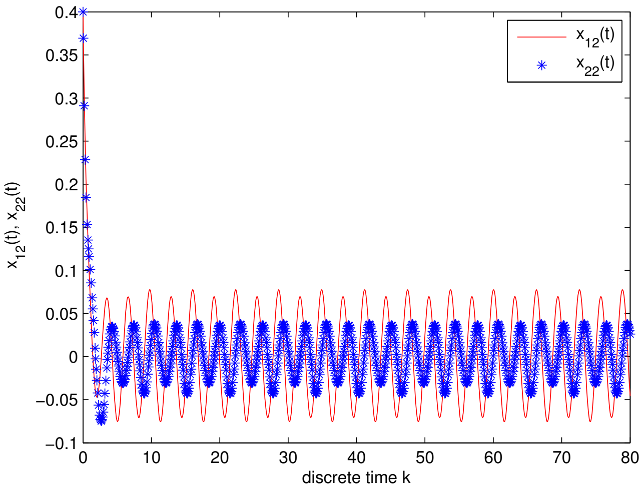

Figure 2: Time response of state variables $x_{12}(t)$ and $x_{22}(t)$.

Acknowledgments

The authors are grateful to supporting by National Natural Science Foundation of China (No.11261010), Natural Science and Technology Foundation of Guizhou Province(J[2015]2025 and J[2015]2026), 125 Special Major Science and Technology of Department of Education of Guizhou Province ([2012]011) and Natural Science Foundation of the Education Department of Guizhou Province(KY[2015]482).

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