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Some random fixed point theorems in generalized convex metric space

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Abstract

In this paper, we consider a new random iteration process to approximate a common random fixed point of a finite family of uniformly quasi-Lipschitzian random mappings in generalized convex metric spaces. Our results presented in this paper extend and improve several recent results.

Keywords: Random iteration process, common random fixed point, uniformly quasi-Lipschitzian random mapping, generalized convex metric spaces.
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1. Introduction

Random fixed point theorems are usually used to obtain the solutions of some nonlinear random systems [2]. Spacek [14] and Hans [7] first discussed some random fixed point theorems in separable metric space. And then, many authors [1, 3, 4, 9, 10, 12, 13] have considered different random iterative algorithms to converge fixed points of contractive type and asymptotically nonexpansive type random mappings in separable normed spaces and Banach spaces. In 1970, the concept of convex metric space was first introduced by Takahashi [15], it pointed out that each linear normed space is a special example of a convex metric space. Recently [5, 6, 8, 17, 18, 19, 20] have used different iteration schemes to obtain fixed points of asymptotically quasi-nonexpansive mappings in convex metric spaces.

In this paper, inspired and motivated by the above facts, we will construct a general random iteration process which converges strongly to a common random fixed point of a finite family of uniformly quasi-Lipschitzian random mappings in generalized convex metric spaces. The results extend and improve the corresponding results in [1, 3, 4, 5, 6, 8, 12, 13, 17, 18, 19, 20].

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2. Preliminaries

Let (Ω, Σ) be a measurable space with Σ being a σ-algebra of subsets of Ω, and let K be a nonempty subset of a metric space (X, d).

Definition 2.1 ([1]).

(i) A mapping $\xi : \Omega \to X$ is measurable if $\xi^{-1}(U) \in \Sigma$ for each open subset U of X;

(ii) The mapping $T : \Omega \times K \to K$ is a random map if and only if for each fixed $x \in K$, the mapping $T(\cdot, x) : \Omega \to K$ is measurable, and it is continuous if for each $\omega \in \Omega$, the mapping $T(\omega, \cdot) : K \to X$ is continuous;

(iii) A measurable mapping $\xi : \Omega \to X$ is a random fixed point of the random map $T : \Omega \times K \to X$ if and only if $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

We denote by $RF(T)$ the set of all random fixed points of a random map T; $T^n(\omega, x)$ the n th iteration $T(\omega, T(\omega, T(\omega, \cdots T(\omega, x) \cdots)))$ of T. And the letter I denotes the random mapping $T : \Omega \times K \to K$ defined by $I(\omega, x) = x$ and $T^0 = I$.

Next, we introduce some random mappings in separable metric spaces.

Definition 2.2. Let K be a nonempty subset of a separable metric space (X, d) and $T : \Omega \times K \to K$ be a random map. The map T is said to be:

(i) an asymptotically nonexpansive random mapping if there exists a sequence of measurable mappings $\{k_n(\omega)\} : \Omega \to [1, \infty)$ with $\lim_{n\to\infty} k_n(\omega) = 0$ such that,

$$d(T^n(\omega, x), T^n(\omega, y)) \leq (1 + k_n(\omega))d(x, y),$$

for each $\omega \in \Omega$ and $x, y \in K$;

(ii) a uniformly L-Lipschitzian random mapping if

$$d(T^n(\omega, x), T^n(\omega, y)) \leq Ld(x, y),$$

for each $x, y \in K$ and L is a positive constant;

(iii) an asymptotically quasi-nonexpansive random mapping if there exists a sequence of measurable mappings $\{k_n(\omega)\} : \Omega \to [1, \infty)$ with $\lim_{n\to\infty} k_n(\omega) = 0$ such that,

$$d(T^n(\omega, \eta(\omega)), \xi(\omega)) \leq (1 + k_n(\omega))d(\eta(\omega), \xi(\omega)),$$

for each $\omega \in \Omega$, where $\xi : \Omega \to K$ is a random fixed point of T and $\eta : \Omega \to K$ is any measurable map;

(iv) a uniformly quasi-Lipschitzian random mapping if

$$d(T^n(\omega, \eta(\omega)), \xi(\omega)) \leq Ld(\eta(\omega), \xi(\omega)),$$

for each $\omega \in \Omega$, where $\xi : \Omega \to K$ is a random fixed point of T , $\eta : \Omega \to K$ is any measurable map and L is a positive constant;

(v) an semicompact random mapping if for any sequence of measurable mappings $\{\xi_n(\omega)\} : \Omega \to K$, with $\lim_{n\to\infty} d(T(\omega, \xi_n(\omega)), \xi_n(\omega)) = 0$, for each $\omega \in \Omega$, there exists a subsequence $\{\xi_{n_j}\}$ of $\{\xi_n\}$ which converges pointwise to ξ, where $\xi : \Omega \to K$ is a measurable mapping.

Remark 2.3.

(i) If T is an asymptotically nonexpansive random mapping, then T is a uniformly L-Lipschitzian random mapping ($L = \sup_{n\geq 1}\{k_n\}$). And if $RF(T) \neq \emptyset$, then every asymptotically nonexpansive random mapping is an asymptotically quasi-nonexpansive random mapping.

(ii) If T is an asymptotically quasi-nonexpansive random mapping, then T is a uniformly quasi-Lipschitzian random mapping ($L = \sup_{n\geq 1} k_n$). And if $RF(T) \neq \emptyset$, then every uniformly L-Lipschitzian random mapping is a uniformly quasi-Lipschitzian random mapping.

Definition 2.4 ([15])**.** A convex structure in a metric space (X, d) is a mapping $W : X \times X \times [0, 1] \to X$ satisfying, for each $x, y, u \in X$ and each $\lambda \in [0, 1]$,

$$d(u, W(x, y; \lambda)) \leq \lambda d(u, x) + (1 - \lambda)d(u, y).$$

A metric space together with a convex structure is called a convex metric space. A nonempty subset K of X is said to be convex if $W(x, y; \lambda) \in K$ for all $(x, y; \lambda) \in K \times K \times [0, 1]$.

The above definition can be extended as follows:

Definition 2.5 ([17, 19])**.** Let X be a metric space,$I = [0, 1]$,$\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ be real sequence in $[0, 1]$ with $\alpha_n + \beta_n + \gamma_n = 1$. A mapping $W : X^3 \times I^3 \to X$ is said to be a convex structure on X, if it satisfies the following conditions: for any $(x, y, z; \alpha_n, \beta_n, \gamma_n) \in X^3 \times I^3$ and $u \in X$,

$$d(u, W(x, y, z; \alpha_n, \beta_n, \gamma_n)) \leq \alpha_n d(u, x) + \beta_n d(u, y) + \gamma_n d(u, z).$$

A metric space together with a convex structure is called a generalized convex metric space.

A nonempty subset K of X is said to be convex if $W(x, y, z; \alpha_n, \beta_n, \gamma_n) \in K$ for all $(x, y, z; \alpha_n, \beta_n, \gamma_n) \in X^3 \times I^3$. The mapping $W : K^3 \times I^3 \to K$ is said to be a random convex structure if for any measurable mappings $\xi, \eta, \zeta : \Omega \to K$ and each fixed $\alpha, \beta, \gamma \in [0, 1], \alpha+\beta+\gamma = 1$, the mapping $W(\xi(\cdot), \eta(\cdot), \zeta(\cdot); \alpha, \beta, \gamma) : \Omega \to K$ is measurable.

In convex metric spaces, Khan [8] introduced the following iteration process for common fixed points of asymptotically quasi-nonexpansive mappings $\{T_i : i \in J = \{1, 2, \cdots, k\}\}$: any initial point $x_1 \in K$:

$$\begin{cases} x_{n+1} = W(T_k^n y_{(k-1)n}, x_n; \alpha_{kn}), \\ y_{(k-1)n} = W(T_{k-1}^n y_{(k-2)n}, x_n; \alpha_{(k-1)n}), \\ y_{(k-2)n} = W(T_{k-2}^n y_{(k-3)n}, x_n; \alpha_{(k-2)n}), \\ \quad\vdots \\ y_{1n} = W(T_1^n y_{0n}, x_n; \alpha_{1n}), \end{cases} \tag{2.1}$$

where $y_{0n} = x_n$ and $\{\alpha_{in}\}$ are real sequences in $[0, 1]$. And in generalized convex metric spaces, Wang and Liu [19] considered an Ishikawa type iteration process with errors to approximate the fixed point of two uniformly quasi-Lipschitzian mappings T and S as follows:

$$\begin{cases} x_{n+1} = W(x_n, S^n y_n, u_n; a_n, b_n, c_n), \\ y_n = W(x_n, T^n x_n, v_n; a'_n, b'_n, c'_n), \end{cases} \tag{2.2}$$

where $\{a_n\}, \{b_n\}, \{b_n\}, \{a'_n\}, \{b'_n\}, \{c'_n\}$ are real sequences in $[0, 1]$ with $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$ and $\{u_n\}, \{v_n\}$ are two bounded sequence.

From (2.1) and (2.2), we investigate the following random iteration process in generalized convex metric space.

Definition 2.6. Let $\{T_i : i \in J = \{1, 2, \cdots, k\}\}$ be a finite family of uniformly quasi-Lipschitzian random mappings from $\Omega \times K$ to K , where K is a nonempty closed convex subset of a separable generalized convex metric space (X, d). Let $\xi_1(\omega) : \Omega \to K$ be a measurable map, the sequence $\{\xi_n(\omega)\}$ is defined as follows:

$$\begin{cases} \xi_{n+1}(\omega) = W(\xi_n(\omega), T_k^n(\omega, \eta_{(k-1)n}(\omega)), u_{kn}(\omega); \alpha_{kn}, \beta_{kn}, \gamma_{kn}), \\ \eta_{(k-1)n}(\omega) = W(\xi_n(\omega), T_{k-1}^n(\omega, \eta_{(k-2)n}(\omega)), u_{(k-1)n}(\omega); \alpha_{(k-1)n}, \beta_{(k-1)n}, \gamma_{(k-1)n}), \\ \quad\vdots \\ \eta_{2n}(\omega) = W(\xi_n(\omega), T_2^n(\omega, \eta_{1n}(\omega)), u_{2n}(\omega); \alpha_{2n}, \beta_{2n}, \gamma_{2n}), \\ \eta_{1n}(\omega) = W(\xi_n(\omega), T_1^n(\omega, \eta_{0n}(\omega)), u_{1n}(\omega); \alpha_{1n}, \beta_{1n}, \gamma_{1n}), \end{cases} \tag{2.3}$$

where $\eta_{0n}(\omega) = \xi_n(\omega)$, for any given $i \in J$, $\{\alpha_{in}\}, \{\beta_{in}\}, \{\gamma_{in}\}$ are real sequences in $[0,1]$ with $\alpha_{in}+\beta_{in}+\gamma_{in} = 1$ and $\{u_{in}(\omega)\} : \Omega \to K$ is a sequence of measurable mappings which is bounded in K for all $n \in \mathbb{N}$.

We need the following lemma for proving the main results.

Lemma 2.7 ([16]). *Let X be a separable metric space and Y a metric space. If $f : \Omega \times X \to Y$ is measurable in $\omega \in \Omega$ and continuous in $x \in X$, and if $x : \Omega \to X$ is measurable, then $f(\cdot, x(\cdot)) : \Omega \to Y$ is measurable.*

Lemma 2.8 ([11]). *Let $\{p_n\}, \{q_n\}, \{r_n\}$ be sequences of nonnegative real numbers satisfying the following conditions:*

$$p_{n+1} \leq (1 + q_n)p_n + r_n, \qquad \sum_{n=0}^{\infty} q_n < \infty, \qquad \sum_{n=0}^{\infty} r_n < \infty.$$

We have (i) $\lim_{n\to\infty} p_n$ exists; (ii) if $\liminf_{n\to\infty} p_n = 0$, then $\lim_{n\to\infty} p_n = 0$.

3. Main results

In this section, we give some conditions for the convergence of the random iteration process (2.3) to the common random fixed point of a finite family random uniformly quasi-Lipchitzian random mappings $\{T_i, i \in J\}$. We first prove the following lemma.

Lemma 3.1. *Let K be a nonempty closed convex subset of a separable generalized convex metric space (X, d). Let $\{T_i : i \in J = \{1, 2, \cdots, k\}\} : \Omega \times K \to K$ be a finite family of uniformly quasi-Lipchitzian random mappings with $L_i > 0$. Suppose that the sequence $\{\xi_n(\omega)\}$ is as in (2.3) and $\sum_{n=0}^{\infty}(\beta_{kn} + \gamma_{kn}) < \infty$. If $F = \bigcap_{i=1}^{k} RF(T_i) \neq \emptyset$. Then*

(i) *there exist two positive constants M_0, M_1, such that*

$$d(\xi_{n+1}(\omega), \xi(\omega)) \leq (1 + \theta_n M_0)d(\xi_n(\omega), \xi(\omega)) + \theta_n M_1, \tag{3.1}$$

where $\theta_n = \beta_{kn} + \gamma_{kn}$, for all $\xi(\omega) \in F$ and $n \in \mathbb{N}$;

(ii) *there exists a positive constant M_2, such that*

$$d(\xi_{n+m}(\omega), \xi(\omega)) \leq M_2 d(\xi_n(\omega), \xi(\omega)) + M_1 M_2 \sum_{j=n}^{n+m-1} \theta_j, \tag{3.2}$$

for all $\xi(\omega) \in F$ and $n, m \in \mathbb{N}$.

Proof. (i) Let $\xi(\omega) \in F$. Since $\{u_{in}(\omega)\}$ are bounded sequences in K for all $i \in J$, there exists $M > 0$ such that

$$M = \max_{1\leq i\leq k} d(u_{in}(\omega), \xi(\omega)).$$

Let $L = \max_{1\leq i\leq k}\{L_i\} > 0$. By (2.3), we have

$$\begin{aligned} d(\eta_{1n}(\omega), \xi(\omega)) =& d(\xi_n(\omega), W(T_1^n(\omega, \eta_{0n}(\omega)), u_{1n}(\omega); \alpha_{1n}, \beta_{1n}, \gamma_{1n}), \xi(\omega)) \\ \leq& \alpha_{1n}d(\xi_n(\omega), \xi(\omega)) + \alpha_{1n}d(T_1^n(\omega, \eta_{0n}(\omega)), \xi(\omega)) + \gamma_{1n}d(u_{1n}(\omega), \xi(\omega)) \\ \leq& \alpha_{1n}d(\xi_n(\omega), \xi(\omega)) + \beta_{1n}Ld(\xi_n(\omega), \xi(\omega)) + \gamma_{1n}M \\ \leq& (1 + L)d(\xi_n(\omega), \xi(\omega)) + M. \end{aligned}$$

Assume that

$$d(\eta_{in}(\omega), \xi(\omega)) \leq (1 + L)^i d(\xi_n(\omega), \xi(\omega)) + \sum_{j=0}^{i-1} L^j M$$

holds for some $1 \le i \le k-1$. Then

$$\begin{aligned}
d(\eta_{(i+1)n}(\omega),\xi(\omega)) =& d(\xi_n(\omega), W(T^n_{i+1}(\omega,\eta_{in}(\omega)), u_{(i+1)n}(\omega);\alpha_{(i+1)n},\beta_{(i+1)n},\gamma_{(i+1)n}),\xi(\omega))\\
\le& \alpha_{(i+1)n} d(\xi_n(\omega),\xi(\omega)) + \beta_{(i+1)n} d(T^n_{i+1}(\omega,\eta_{in}(\omega)),\xi(\omega))\\
&+\gamma_{(i+1)n} d(u_{(i+1)n}(\omega),\xi(\omega))\\
\le& \alpha_{(i+1)n} d(\xi_n(\omega),\xi(\omega)) + \beta_{(i+1)n} L d(\eta_{in}(\omega),\xi(\omega)) + \gamma_{(i+1)n} d(u_{(i+1)n}(\omega),\xi(\omega))\\
\le& \alpha_{(i+1)n} d(\xi_n(\omega),\xi(\omega)) + \beta_{(i+1)n} L[(1+L)^i d(\xi_n(\omega),\xi(\omega)) + \sum_{j=0}^{i-1} L^j M]\\
&+\gamma_{(i+1)n} d(u_{(i+1)n},\xi(\omega))\\
\le& [\alpha_{(i+1)n} + \beta_{(i+1)n} L(1+L)^i] d(\xi_n(\omega),\xi(\omega)) + \beta_{(i+1)n} L \sum_{j=0}^{i-1} L^j M]\\
&+\gamma_{(i+1)n} M\\
\le& (1+L(1+L)^i) d(\xi_n(\omega),\xi(\omega)) + \sum_{j=1}^{i} L^j M + M\\
\le& (1+L)^{i+1} d(\xi_n(\omega),\xi(\omega)) + \sum_{j=0}^{i} L^j M.
\end{aligned}$$

Therefore, by induction, for all $1 \le i \le k$, we have

$$d(\eta_{in}(\omega),\xi(\omega)) \le (1+L)^i d(\xi_n(\omega),\xi(\omega)) + \sum_{j=0}^{i-1} L^j M.$$

Then, it follows from (2.3) that

$$\begin{aligned}
d(\xi_{n+1}(\omega),\xi(\omega)) =& d(\xi_n(\omega), W(T^n_k(\omega,\eta_{kn}(\omega)), u_{kn}(\omega);\alpha_{kn},\beta_{kn},\gamma_{kn}),\xi(\omega))\\
\le& \alpha_{kn} d(\xi_n(\omega),\xi(\omega)) + \beta_{kn} L d(\eta_{(k-1)n}(\omega),\xi(\omega)) + \gamma_{kn} d(u_{kn}(\omega),\xi(\omega))\\
\le& \alpha_{kn} d(\xi_n(\omega),\xi(\omega)) + \beta_{kn} L[(1+L)^{k-1} d(\xi_n(\omega),\xi(\omega)) + \sum_{j=0}^{k-2} L^j M] + \gamma_{kn} M\\
\le& [\alpha_{kn} + \beta_{kn} L(1+L)^{k-1}] d(\xi_n(\omega),\xi(\omega)) + \beta_{kn} L \sum_{j=0}^{k-2} L^j M + \gamma_{kn} M\\
\le& [1+\theta_n(1+L)^k] d(\xi_n(\omega),\xi(\omega)) + \theta_n(\sum_{j=0}^{k-2} L^j M + M)\\
\le& (1+\theta_n M_0) d(\xi_n(\omega),\xi(\omega)) + \theta_n M_1,
\end{aligned}$$

where $\theta_n = \beta_{kn} + \gamma_{kn}$, $M_0 = (1+L)^k$ and $M_1 = \sum_{j=0}^{k-1} L^j M$.

(ii)Notice that $1 + x \le e^x$ for all $x \ge 0$. By using this we have:

$$\begin{aligned}
d(\xi_{n+m}(\omega),\xi(\omega)) \le& (1+\theta_{n+m-1} M_0) d(\xi_{n+m-1}(\omega),\xi(\omega)) + \theta_{n+m-1} M_1\\
\le& e^{\theta_{n+m-1} M_0}[(1+\theta_{n+m-2}) M_0 d(\xi_{n+m-2}(\omega),\xi(\omega)) + \theta_{n+m-2} M_1] + \theta_{n+m-1} M_1\\
\le& e^{(\theta_{n+m-1}+\theta_{n+m-2}) M_0} d(\xi_{n+m-2}(\omega),\xi(\omega)) + e^{\theta_{n+m-1} M_0} M_1 (\theta_{n+m-1} + \theta_{n+m-2})\\
&\qquad\cdots\cdots
\end{aligned}$$

$$\leq e^{M_0 \sum_{j=1}^{\infty} \theta_j} d(\xi_n(\omega), \xi(\omega)) + e^{M_0 \sum_{j=1}^{\infty} \theta_j} M_1 \sum_{j=n}^{n+m-1} \theta_j$$
$$\leq M_2 d(\xi_n(\omega), \xi(\omega)) + M_1 M_2 \sum_{j=n}^{n+m-1} \theta_j,$$

where $M_2 = e^{M_0 \sum_{j=1}^{\infty} \theta_j}$. □

Theorem 3.2. *Let K be a nonempty closed convex subset of a separable complete generalized convex metric space (X, d) with a random convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of continuous uniformly quasi-Lipchitzian random mappings with $L_i > 0$. Suppose that the sequence $\{\xi_n(\omega)\}$ is as in (2.3) and $\sum_{n=1}^{\infty}(\beta_{kn} + \gamma_{kn}) < \infty$. If $F = \bigcap_{i=1}^{k} RF(T_i) \neq \emptyset$, then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$ if and only if $\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0$, where $d(x, F) = \inf\{d(x, y) : \forall y \in F\}$.*

Proof. The necessity is obvious. Thus, we only need prove the sufficiency. From Lemma 3.1, we know that

$$d(\xi_{n+1}(\omega), F) \leq (1 + \theta_n M_0) d(\xi_n(\omega), F) + \theta_n M_1. \tag{3.3}$$

Since $\sum_{n=1}^{\infty}(\beta_{kn} + \gamma_{kn}) < \infty$, from Lemma 2.8, we know $\lim_{n\to\infty} d(\xi_n(\omega), F)$ exists. By hypothesis,

$$\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0,$$

we have

$$\lim_{n\to\infty} d(\xi_n(\omega), F) = 0.$$

Next, We show that $\{\xi_n\}$ is a Cauchy sequence. Indeed, for any $\varepsilon > 0$, there exists a constant N_0 such that for all $n \geq N_0$, we have

$$d(\xi_n, F) \leq \frac{\varepsilon}{4M_1} \quad \text{and} \quad \sum_{n=N_0}^{\infty} \theta_n \leq \frac{\varepsilon}{4M_0 M_1}.$$

In particular, there exists a $p_1(\omega) \in F$ and a constant $N_1 > N_0$, such that

$$d(\xi_{N_1}(\omega), p_1(\omega)) \leq \frac{\varepsilon}{4M_1}.$$

It follows from Lemma 3.1 that for $n > N_1$ we have

$$\begin{aligned} d(\xi_{n+m}(\omega), \xi_n(\omega)) \leq & d(\xi_{n+m}(\omega), p_1(\omega)) + d(p_1(\omega), \xi_n(\omega)) \\ \leq & 2M_1 d(\xi_{N_1}(\omega), p_1(\omega)) + M_0 M_1 (\sum_{j=N_1}^{n+m-1} \theta_j + \sum_{j=N_1}^{n-1} \theta_j) \\ \leq & 2M_1 \frac{\varepsilon}{4M_1} + 2M_0 M_1 \frac{\varepsilon}{4M_0 M_1} = \varepsilon. \end{aligned}$$

This implies that $\{\xi_n\}$ is a Cauchy sequence in closed convex subset of a complete generalized convex metric spaces. Therefore $\{\xi_n(\omega)\}$ converges to a point of K. Suppose $\lim_{n\to\infty} \xi_n(\omega) = p(\omega)$, for each $\omega \in \Omega$. Since T_i are continuous, by Lemma 2.7, we know that for any measurable mapping $f : \Omega \to K$, $T_i^n(\omega, f(\omega)) : \Omega \to K$ are measurable mappings. Thus, $\{\xi_n(\omega)\}$ is a sequence of measurable mappings. Hence, $p(\omega) : \Omega \to K$ is also measurable. Next, we will prove that $p(\omega) \in F$. Notice that

$$d(p(\omega), F) \leq d(\xi_n(\omega), p(\omega)) + d(\xi_n(\omega), F).$$

Since $\lim_{n\to\infty} d(\xi_n(\omega), F) = 0$ and $\lim_{n\to\infty} d(\xi_n(\omega), p(\omega)) = 0$, we conclude that $d(p(\omega), F) = 0$. Therefore, $p(\omega) \in F$.

□

Remark 3.3.

(i) Theorem 3.2 extends the corresponding results in [1, 3, 4, 6, 9, 10, 12, 13, 15] to the generalized convex metric space, which is more general space;

(ii) Theorem 3.2 extends the corresponding results in [8, 9, 10, 11, 16, 17, 18, 19, 20] to the finite family of uniformly quasi-Lipschitzian random mappings,which are stochastic generalization of uniformly quasi-Lipschitzian mappings.

By Remark 3.3, we get the following result:

Corollary 3.4. *Let K be a nonempty closed convex subset of a separable complete generalized convex metric space (X, d) with a random convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of continuous asymptotically quasi-nonexpansive random mappings with $k_{in}(\omega) : \Omega \to [0, \infty)$ for each $\omega \in \Omega$. Suppose that the sequence $\{\xi_n(\omega)\}$ is as in (2.3) and $\sum_{n=1}^{\infty}(\beta_{kn} + \gamma_{kn}) < \infty$. If $F = \bigcap_{i=1}^{k} RF(T_i) \neq \emptyset$, then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$ if and only if $\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0$, where $d(x, F) = \inf\{d(x, y) : \forall y \in F\}$.*

Remark 3.5. In Corollary 3.4, we remove the condition: " $\sum_{n=1}^{\infty} k_{in} < \infty, i \in J$", which is required in many other paper (see, e.g., [1, 12, 15, 16, 17, 18, 19, 20]. And the condition "$\sum_{n=1}^{\infty}(\beta_{kn} + \gamma_{kn}) < \infty$" is replaced with "$\sum_{n=1}^{\infty} \beta_{in} < \infty, \sum_{n=1}^{\infty} \gamma_{in} < \infty, i \in J$".

Theorem 3.6. *Let K be a nonempty closed convex subset of a separable complete generalized convex metric space (X, d) with a random convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of continuous uniformly quasi-Lipchitzian random mappings with $L_i > 0$. Suppose that the sequence $\{\xi_n(\omega)\}$ is as in (2.3) , $\sum_{n=1}^{\infty}(\beta_{kn} + \gamma_{kn}) < \infty$ and $F = \bigcap_{i=1}^{k} RF(T_i) \neq \emptyset$. If for some given $1 \leq l \leq k$,*

(i) $\lim_{n\to\infty} d(T_l(\omega, \xi_n(\omega)), \xi_n(\omega)) = 0$,

(ii) *there exists a positive constant M_3 such that*

$$d(T_l(\omega, \xi_n(\omega)), \xi_n(\omega)) \geq M_3 d(\xi_n(\omega), F),$$

then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$.

Proof. From the conditions (i) and (ii), it implies that

$$\lim_{n\to\infty} d(\xi_n(\omega), F) = 0.$$

Thus, from the proof of Theorem 3.2 , we know that $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$ □

Theorem 3.7. *Let K be a nonempty closed convex subset of a separable complete generalized convex metric space (X, d) with a random convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of continuous uniformly quasi-Lipchitzian random mappings with $L_i > 0$. Suppose that the sequence $\{\xi_n(\omega)\}$ is as in (2.3) , $\sum_{n=1}^{\infty}(\beta_{kn} + \gamma_{kn}) < \infty$ and $F = \bigcap_{i=1}^{k} RF(T_i) \neq \emptyset$. If*

(i) *for all $1 \leq i \leq k$, $\lim_{n\to\infty} d(T_i(\omega, \xi_n(\omega)), \xi_n(\omega)) = 0$;*

(ii) *for some $1 \leq l \leq k$, T_l is semicompact,*

then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$.

Proof. Since T_l is semicompact and $\lim_{n\to\infty} d(T_l(\omega,\xi_n(\omega)),\xi_n(\omega)) = 0$, there exist a subsequence $\{\xi_{n_j}(\omega)\} \subset \{\xi_n(\omega)\}$ such that $\lim_{j\to\infty} \xi_{n_j}(\omega) = \xi^*(\omega) \in K$. Since T_i are continuous, Thus, $\{\xi_n\}$ is a sequence of measurable mappings. Hence, $\xi^*(\omega) : \Omega \to K$ is also measurable.

By $d(T_i(\omega,\xi^*(\omega)),\xi^*(\omega)) = \lim_{n\to\infty} d(T_i(\omega,\xi_{n_j}(\omega)),\xi_{n_j}(\omega)) = 0$, we know that $\xi^*(\omega) \in F$. From Lemma 3.1, we have

$$d(\xi_{n+1}(\omega),\xi^*(\omega)) \le (1+\theta_n M_0)d(\xi_n(\omega),\xi^*(\omega)) + \theta_n M_1.$$

Since $\sum_{n=1}^{\infty} \theta_n < \infty$, from Lemma 2.8, there exists a constant $\alpha \ge 0$ such that

$$\lim_{n\to\infty} d(\xi_n(\omega),\xi^*(\omega)) = \alpha.$$

From $\lim_{j\to\infty} \xi_{n_j}(\omega) = \xi^*(\omega)$, we know that

$$\alpha = 0.$$

Thus, $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$. □

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